Discussion on a general formula of sediment diffusion coefficient and sediment fluctuating intensities in the sediment-laden flow

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Received 21 August 2014; revised 11 November 2016

According to the research achievement of velocity fluctuating intensities, the formula of sediment fluctuating intensities is raised analogically and is verified with the experimental data. The results show that the formula structure could preferably conform to the distribution tendency. A general expression about SDC (sediment diffusion coefficient) is deduced to describe the three types of SDC. Meanwhile, a simple-structure SSC (suspended sediment concentration) profile formula is obtained for depicting the situation largest SSC absent from the bottom. The classic field data as well as experimental results of three types SDC and the measured data in the hyper concentration flow are applied to calibrate the SDC profile formula, and the data of flume experiments and pipeline tests are utilized to validate the SSC profile formula. With a discussion of the value range of relevant parameters of the SDC and SSC equations in different water-sediment environments, the conclusion is gained that the parameters in higher concentration flow are relatively steady while the mutative amplitude of the ones in lower concentration is comparative larger.

[Keywords: general formula of sediment diffusion coefficient, sand fluctuating intensities, sediment concentration profile, S-type profile]

Introduction

The expression of SDC (sediment diffusion coefficient) is one of the most essential but still unsolved problems in sediment research, which significantly affects the SSC (suspended sediment concentration) distributions, sand transport rate, and so on. The definition of SDC originated from the replacement of the fluctuation of water-sediment flux according to the analogical relationship between the turbulent momentum fluctuating and the MTC (momentum transfer coefficient). Plenty of research on SDC and SSC profiles has been conducted and some of the achievements utilized in the practical hydraulic and coastal engineering make sense. On the basis of performances gained by sediment research predecessors, the SDC profiles could be divided into three types: the quasi-parabolic type, the parabolic-constant type and the increasing type towards water surface. The representative studies of each type were collected and illustrated below.

A. Quasi-parabolic type. Cellino and Graf studied SDC experiments used the spherical-sand particles of two different diameter in a 16.8m length, 0.60m width, 0.12m depth flume, and the results indicated that the SDC profile of finer sand increased from the bottom and got maximum at middle depth, then deceased to zero.
at water surface, but the SDC profile of coarser sand would reduce to a constant at water surface. Namely, influenced by suspended sediment concentration and middle-diameter of sand particles, the SDC might obey an asymmetric quasi-parabolic distribution rather than the symmetric distribution of Rouse equation. Jobson\textsuperscript{3} confirmed this conclusion generally, putting forward that the expression of SDC should be piecewise functions with the separation place where relative depth was equal to 0.1. B. Parabolic-constant type. The experimental results of Coleman\textsuperscript{4} and field data of Enoree River organized by Anderson\textsuperscript{5} probably proved to be the most classic materials for the SDC parabolic-constant type distribution. However, the width to depth ratio of Coleman\textsuperscript{4} experimental flume was less than three, and the experimental results of such narrow-deep open-channel flow are only limited to some special flow regimes. By analyzing the field data of Enoree River, Van Rijn\textsuperscript{6} raised the segmentation function to express the SDC profile and applied the Prandtl hypothesis to modified SSC profile formula. The SDC of hyper-concentration flows such as Yellow River, Wuding River and Luo River were elucidated and compared by Liu\textsuperscript{7}, which also revealed the parabolic-constant distribution of SDC with separation relative depth equaling to 0.2. Green\textsuperscript{8} came forward piecewise functions consisted of linear function and constants by means of researching measured data of suspended sediment concentration in a deep channel of a large estuary in New Zealand, including both silt and sand. C. Increasing type towards water surface. Based on observed data of tidal-effected channels and estuaries, Nikora and Goring\textsuperscript{9} presented a conclusion that MTC profiles appeared to be parabolic distribution and the profile of SDC manifested a monotonically increasing form profile at higher SSC and parabolic form at lower SSC. Each category of SDC profile above corresponded to a specific expression lack of uniform one, leading to variously complicated profile of SSC. The general expression of SDC had not been raised for all field data. It is worth noting that the position of vertical largest SSC emerged free from the water bottom in unidirectional flows in numerous flume experiments and field observations. Ni\textsuperscript{10}, Wang\textsuperscript{11} adopted the two-phase flow method to ascertain the expression which is too complicated to use in the realistic engineering calculations. The overwhelming majority of research approach belongs to the direct modification of SDC or SSC profile formulas or the formation methods based on the field data for lack of general applicability. The object of study in this paper would present the general formula of SDC to describe these three types and deduce a simple-structure universal SSC profile equation to express vertical distributions of SSC including the situation that the largest concentration of sediment is absent from the bottom

Materials and Methods

Theoretical derivation

Owing to the fundamental concept of SDC that indicates the substituting formulation of the product of sand kinematic fluctuating velocity and suspended sediment fluctuating concentration, the most essential research should be stared from the fluctuation of related physical quantities. With the development of experimental measurements, the situation mentioned by Graf\textsuperscript{12} that experimental studies were mainly based on measurements of the mean concentration of suspended sediments has long gone. Many important measures such as concentration fluctuations or turbulent sediment fluxes have been omitted for short of appropriate instruments\textsuperscript{13}. Nakagawa and Nezu\textsuperscript{14} conducted the turbulent velocity measurement tests and reached a semi-empirical exponential formula to express the fluctuating intensities of velocity in the clear water, which could be written as

\[
\frac{\sqrt{\nu^*}}{u_*} = 2.30 \exp\left(-\frac{y}{h}\right)
\]

\[
\frac{\sqrt{\nu^*}}{u_*} = 1.27 \exp\left(-\frac{y}{h}\right)
\]
Where \( u' \) and \( v' \) are longitudinal and vertical turbulence intensity in clear water respectively, \( u_* \) is friction velocity, \( y \) is vertical coordinate, \( h \) is the depth of water. For uniform open-channel flow, Nezu and Nakagawa\textsuperscript{15} suggested that the turbulence intensities distributed according to an exponential law. Kironoto and Graf\textsuperscript{16} implemented the similar tests on a rough bed and obtained the equation of the fluctuating intensities of velocity with different parameters, which could be shown as

\[
\frac{\sqrt{u'^2}}{u_*} = 2.04 \exp(-0.97 \frac{y}{\delta})
\]

\[
\frac{\sqrt{v'^2}}{u_*} = 1.14 \exp(-0.76 \frac{y}{\delta})
\]

Where \( \delta \) is the distance between the gravel bed and the point where velocity reached maximum value. For the water-sand environment with low sediment concentration, the sediment-laden flows are still belonged to Newton fluid with the viscosity changing slightly. The sizes of eddy in sediment-laden flows are similar to the ones of clear water, meanwhile the process of sediment diffusion could be deemed as the water turbulence transfer. In terms of the field MTC and SDC data of tests by Cellino and Graf\textsuperscript{2}, Graf and Cellino\textsuperscript{17}, Nikora and Goring\textsuperscript{9}, etc, the fluctuating velocity intensities structures of sand particles are able to be assumed approximate as the water turbulence. Nonetheless, the fluctuating characteristics of sediment particles movement and the parameters variations, representing the diffusion extent of the sediment group, are distinguished from the water flow, so as the fluctuating tendency near the bottom. In accordance with the experimental results by Li\textsuperscript{18}, the horizontal and vertical sand particle fluctuating intensities could be expressed respectively as

\[
\frac{\sqrt{u'^2}}{u_*'} = D_1 \eta^{\beta} \exp(-\lambda_{uu} \eta^{\beta})
\]

\[
\frac{\sqrt{v'^2}}{u_*'} = D_2 \eta^{\beta} \exp(-\lambda_{vv} \eta^{\beta})
\]

Where, \( q_1 \) and \( q_2 \) are sediment diffusion decay coefficients in the horizontal and vertical directions respectively, which would be impacted by suspended sediment concentrations and sediment grain sizes, \( u' \) and \( v' \) are longitudinal and vertical sand particle velocity fluctuating components in sediment-laden flow respectively, \( p_1, p_2 \) are the vertical correction factors, \( D_1, D_2, \lambda_{uu}, \lambda_{vv}, \eta \) are empirical constants, \( \eta \) is the relative depth. Supposing that the longitudinal diffusion strength of sediment plays much less significant role than the vertical diffusion in two-dimension vertical sediment transport, the governmental equation could be written as

\[
\frac{\partial S}{\partial t} = -\left( \frac{\partial v_* S}{\partial y} \right) + \omega \frac{\partial S}{\partial y}
\]

Where \( \omega \) is the sediment settling velocity in the sediment-laden flow. The suspended sediment concentration and the vertical component of the sediment kinematic velocity would be written as summation of temporal average section and fluctuation section. Namely, \( S = \bar{S} + S' \), \( v_* = \bar{v}_* + v'_* \). Substituting these equations into the sediment diffusion equation (2), with the extended period temporal average, equation (2) can be shown as

\[
\frac{\partial \bar{S}}{\partial t} = -\left( \frac{\partial \bar{v}'_* S'}{\partial y} \right) + \omega \frac{\partial \bar{S}}{\partial y}
\]
Assumed that the sediment-laden flow was steady, and the fluctuation constitution of SSC could be explained as the instantaneous variation of some sand particles. In the physical significance, the SSC fluctuation belongs to the microscopic phenomenon and the sand particle velocity fluctuation belongs to the microcosmic process. The change of unit volume of sediment concentration would be replaced by the sediment particles fluctuation. Based on the formula structure of $v'_sv'_s$, the mathematical expression of $v'_sS'$ could be written as

$$v'_sS' = u'_s(D'_s)^2 \eta^3 \eta \exp(-2\lambda_{ps} \eta_p)$$  \hspace{1cm} (4a)$$

$$v'_sS' = N \rho_s u_s \eta \eta_p \exp(-\lambda_p \eta_p) = G(\eta)$$  \hspace{1cm} (4b)$$

Where $N$ is the number of sand particles joining the effective fluctuation in unit volume sediment-laden flow, $\rho_s$ is the density of sand particles, $p_s, q_s$ and $\lambda_p$ are empirical constants.

Equation (3) could be rewritten with equation (4b) as

$$-\frac{\partial G(\eta)}{\partial \eta} + \omega_s \frac{\partial \bar{S}}{\partial \eta} = 0$$  \hspace{1cm} (5)$$

Or

$$\frac{\partial \bar{S}}{\partial \eta} = \frac{1}{\omega_s} \frac{\partial G(\eta)}{\partial \eta}$$  \hspace{1cm} (6)$$

By the analogy with the definition of MTC, the concept of vertical SDC would be brought in and denoted as

$$v'_sS' = -\epsilon_s \frac{\partial \bar{S}}{h \partial \eta}$$  \hspace{1cm} (7)$$

Where, $\epsilon_s$ is vertical SDC, the magnitude of which might be similar to that of MTC. Substituting equation (7) into equation (3) and still following the steady assumption, the sediment diffusion governing equation could be shown as

$$h \omega_s \frac{\partial \bar{S}}{\partial \eta} = -\frac{\partial \epsilon_s}{\partial \eta} \frac{\partial \bar{S}}{\partial \eta} - \epsilon_s \frac{\partial^2 \bar{S}}{\partial \eta^2}$$  \hspace{1cm} (8)$$

By the simultaneous equation (5) and (8), a non-homogeneous ordinary differential equation would be gained as

$$\frac{\partial \epsilon_s}{\partial \eta} + \frac{1}{h \omega_s} \frac{\partial G(\eta)}{\partial \eta} = -h \omega_s$$  \hspace{1cm} (9)$$

Adopting the method of variation of constant, the general solution of this ordinary differential equation (9) could be written as

$$\epsilon_s = \frac{-h \omega_s G(\eta) + C}{\frac{\partial G(\eta)}{\partial \eta}}$$  \hspace{1cm} (10)$$

Where $C_1$ is the integral constant. In consideration of a common situation that the suspended sediment concentration near the bottom is higher, regarded as the source-land of turbulence diffusion, the SDC at the bottom would be assumed to equal to zero, and the limit format of SDC would be indicated as

$$\lim_{\eta \rightarrow 0} \epsilon_s = \lim_{\eta \rightarrow 0} \frac{-h \omega_s G(\eta) + C}{\frac{\partial G(\eta)}{\partial \eta}} = 0$$  \hspace{1cm} (11)$$

In order to keep the validity of this limitation, the parameter $C_1$ should be equal to zero. Substituting formula of $G(\eta)$ into equation (11), the general form of SDC would be expressed as

$$\epsilon_s = \frac{-h \omega_s G(\eta)}{\lambda_p q_s \eta_p^q_s - p_s}$$  \hspace{1cm} (12)$$

SDC would obey different distribution-types with each kind of sediment-laden flows. In equation (12), the parameters $p_s$ and $q_s$ demonstrate great importance that make the SDC profile as diverse types, and the typical results are shown in Fig. 1.
If transforming the formula of SDC, equation (12) could be extended with the infinite series law, the expression could be indicated as

$$
\varepsilon_s = -\frac{h_o}{P_s} \sum_{n=0}^{\infty} \left( \eta \left( \frac{\lambda_s q \eta^{q_s}}{P_s} \right)^n \right)
$$

Assuring the second order accuracy of SDC, the first two terms of equation (13) would be retained, and the usual quasi-parabolic formula would be gained as

$$
\varepsilon_s = -\frac{h_o}{P_s} \left( \eta + \frac{\lambda_s q \eta^{q_s+1}}{P_s} + \cdots \right)
$$

Assuring the second order accuracy of SDC, the first two terms of equation (13) would be retained, and the usual quasi-parabolic formula would be gained as

$$
\varepsilon_s = -\frac{h_o}{P_s} \left( \eta + \frac{\lambda_s q \eta^{q_s+1}}{P_s} \right)
$$

The corresponding SSC profile could be solved and written as

$$
\overline{s} = \frac{N \rho u \eta^p}{\omega_s} \exp(-\lambda_s \eta^{q_s})
$$

Equation (15) is the general expression for the situations that sand particles fluctuating velocity intensities satisfy with equation (1). If importing the reference concentration $\overline{s}_a$ and reference depth $\eta_a$, equation (15) could be rewritten as

$$
\frac{\overline{s}}{\overline{s}_a} = \left( \frac{\eta}{\eta_a} \right)^{\nu_s} \exp\left[ -\lambda_s (\eta^{q_s} - \eta_a^{q_s}) \right]
$$

The famous Rouse equation could be regarded as one special case of equation (16). When $q_s$ equals to one, meanwhile $-p_s$ and $\lambda_s$ equal to the Rouse number, equation (16) could be transformed to the Rouse equation after the Taylor expansion of the exponential function for just first two terms.

$$
\frac{\overline{s}}{\overline{s}_a} = \left( \frac{\eta}{\eta_a} \right)^{\nu_s} \left( 1 - \eta_a^{q_s} \right)^{\nu_s} = \left( \frac{\eta}{\eta_a} \right)^{\nu_s} \left( 1 - \eta \right)^{\nu_s}
$$

Then the parameter $p_s$ is the opposite number of Rouse number. From the whole derivation process of SSC profile formula, the SSC profile is the macro responses of microcosmic sand particle movement, and is related to the sediment fluctuation directly. Because of the interaction effect among sand particles in the sand group and the quasi-fluid approximation, the congruent relationship would generate the change in a certain degree.

**Results**

**Verification and Comparison**

A series of experimental results conducted in sediment-laden flow and field data of actual rivers would be adopted to verify the equations deduced above. In order to validate the precision and rationality of equation (1) about fluctuating intensities of sand particles, the experiments conducted by Li18 would be available for verifying the turbulent features of sand particles.
or sand groups. The experimental results proceeded by Graf and Cellino\textsuperscript{17}, Nikora and Goring\textsuperscript{9} and field data in Yellow River and Enoree River, which may be the representative data of these three kinds of SDC, would be utilized to test the universality of equation (1), (12) and (16). Cellino and Graf\textsuperscript{2} applied the experiments to study the sediment-laden flow in open-channels under non-capacity and capacity conditions. For the sake of detecting these equations in different SSC conditions, the test method of Cellino and Graf\textsuperscript{2} would be applied and the variation analysis would be appreciable. The data that the largest SSC were free from the bottom would be collected to verify equation (16) and to illustrate the general applicability.

**Test one**

Li\textsuperscript{18} wielded the particle tracking velocimetry system (short for PTV) to measure the fluctuating characteristics and SSC profiles of sand particles with different median diameters under the unidirectional uniform flow conditions. With the purpose of avoiding secondary flow in the experiments, the width-depth ratio was staying around four basically. Li\textsuperscript{18} measured the fluctuating intensities of the six different median diameter sands including 0.1mm, 0.3mm, 0.5mm, 0.7mm, 1.0mm and 1.5mm, and adopted the empirical distribution formula of clear water turbulences presented by Nezu\textsuperscript{15} to fit the fluctuation of sediment. But the calculated results would be distorted near the bottom due to the monotonic feature of the distribution formulas. Nevertheless the vertical correction factors would modify the tendency of calculations for fluctuating intensities towards the bottom in equation (1) and avoid the inconsistent situation. The comparisons of the horizontal and the vertical fluctuating intensities between experimental results and computed values of equation (1) are shown in Fig.3 and Fig.4 respectively. The ordinate origins in figures (Fig. 3(a), 3(b), Fig. 4(a), 4(b)) are shifted 0.5 toward the horizontal coordinate to demonstrate the accuracy and clarify the comparison. From the figures about the distributions of sand particle fluctuating intensities, it is obvious that the horizontal fluctuations might be more complex than the vertical ones\textsuperscript{19}, and the coarser sand particle would obey the empirical relationship of turbulences in clear water. Based on these verifications, equation (1) is able to express the situation that the largest fluctuation was absent from the bottom. Fig. 5(a) and 5(b) exhibit the comparisons between field data and calculated values of equation (16) in vertical distribution of SSC bring out well agreement. With the increasing sizes of median, would be more and more evident, prominence of the asymmetry of SSC profile increases with the sizes of median diameter, while the finer sand particles would keep the suspended states in the flow more easily.

![Fig. 3(a)](image1) — Compared field data with calculated values in horizontal fluctuating intensities when \(d_{50}=0.1\)mm, 0.3mm, 0.5mm

![Fig. 3(b)](image2) — Compared field data with calculated values in horizontal fluctuating intensities when \(d_{50}=0.7\)mm, 1.0mm, 1.5mm

![Fig. 4(a)](image3) — Compared field data with calculated values in vertical fluctuating intensities when \(d_{50}=0.1\)mm, 0.3mm, 0.5mm
Test two

Graf and Cellino\textsuperscript{17} conducted the measurements in a re-circulating tilting channel, and all measurements were performed at the centerline of the cross section. Two kinds of sand particles, of which the median diameter were, the sand particles with the median diameter 0.135mm and 0.230mm were adopted in these experiments. Choosing the three groups of sand I (0.135mm) and the two groups of sand II (0.230mm) to test equation (4b) with the least square method of calibrating the relevant parameters, the comparison between measured data and computed values would be exhibited in Fig. 6(a) for sand I and Fig. 6(b) for sand II. equation (4b) could be capable of expressing the various tendencies of fluctuation features in the light of the contrast of these five experimental groups and the largest fluctuating sediment flux of the coarser sand particles may be absent from the bottom being different from the finer sands. The experimental results about SDC profiles by Graf and Cellino\textsuperscript{17} belonged to the quasi-parabolic type and these observed data have been adopted to verify the SDC profile expression by some scholars. In terms of the relevant parameters in the fluctuating sediment flux profiles calculations, the comparisons between measured data and computed value of equation (12) about the SDC profiles in sand I (0.135mm) and sand II (0.230mm) test groups are displayed in Fig. 7(a) and Fig. 7(b) respectively. The test groups in Fig. 6(a) would also be shifted towards the horizontal coordinate in sequence. The computed results of equation (12) could conform to the measured data in the both sand test groups. Though the SDC profiles in the experiments of Graf and Cellino\textsuperscript{17} mostly obeyed the quasi-parabolic type, the coarser sands perform a great degree of the vertical asymmetry than the finer sands.
Test three

Nikora and Goring\(^9\) conducted the field experiments in February 1997 in a straight reach of the Balmoral Irrigation Canal (North Canterbury, New Zealand). The coarse bed materials on the banks were changed to rounded greywacke gravel in the central flat part of the channel. Due to the concentration in the Hurunui River (which fed the Balmoral Canal), two distinct sets of measurement for SSC reflecting higher concentration or lower concentration water-sediment environments were available. The particle size distributions for suspended sediment varied insignificantly with the depth, covering approximately 0.03 to 0.10 mm with mean size 0.06-0.07 mm and 0.05-0.06 mm for two sets respectively. The comparisons for lower and higher concentration SSC profiles between field data, selecting four experimental groups, and computed results by equation (16) are revealed in Fig. 8(a) and 8(b), and the calculations could coincide to the measured data preferably at the both distinct sets. The SSC profiles pertain to the macroscopic performance of sediment movement, while the sensitivity about the change of relevant parameters would be weaker than the microscopic phenomenon as the sediment fluctuation. In the sake of verifying the accuracy of sediment fluctuating intensity computed by equation (4b), the Fig. 9(a) and 9(b) exhibit the comparisons between field data and calculated values, while the non-dimensional sediment fluctuations in lower and higher concentration could be described with equation (4b) more objectively.

The experimental results about SDC profiles of Nikora and Goring\(^9\) may be the most commonly cited to express the evidences about the increasing type of the SDC distribution. For the purpose of better contrasting the precision of equation (12), the calculations of Rouse equation are brought in the comparisons. For lower concentration test groups, Fig. 10(a) and 10(b) explain that the SDC profiles basically obey the quasi-parabolic type, and the advantages of equation (12) were only embodied in the asymmetric depiction. For higher concentration test groups, the SDC profiles obey the increasing type, while the calculations of Rouse equation failed to apply in these situations, and the tendency of SDC profiles could be expressed by equation (12). The comparisons between field data and calculated values by Rouse equation and equation (12) would be emerged in Fig. 11(a) and 11(b), illustrating that the SDC profiles would happen to disperse in a certain degree near the water surface.
Test four

The SDC profiles data of the Enoree River and the experimental results conducted by Coleman were considered as the classic data of parabolic-constant type of SDC profiles declaring the viewpoint that the SDC near the water surface should be stabilized to a constant. Van Rijn once modified the SDC formula structure of Rouse equation and put forward the piecewise function for satisfying the field data of the Enoree River. The comparisons between measured SDC in Enoree River and calculated results of equation (12) were presented in Fig. 12 so as to prove the universality of the formula structure of equation (12). The comparisons between measured SDC of experiments conducted by Coleman and calculated results of equation (12) were exhibited in Fig. 13. The contrasts manifested that equation (12) is capable of owning the ability of expressing the parabolic-constant type SDC. At the same time, the SDC observed data in Yellow River (which belongs to high concentration water-sediment dynamics) collected by Liu are compared with the computed results of equation (12) for eight kinds of concentrations in Fig. 14(a) and 14(b), and the calculations could coincide with the field data superiorly for each test case which confirmed the availability of equation (12) in the hyper concentration flows.
Some predecessors discovered the phenomenon that the vertical largest SSC was free from the bottom reaching out of the description of Rouse equation, Van Rijn formula and some empirical formulas in terms of the decreasing tendency towards the bed of SSC near the bottom. For these quasi-S type SSC profiles, Wang\(^{11}\) has carried out some research with the two-phase flow method, and gained abundant achievements. But the formula deduced by Wang\(^{11}\) was too complicated for practical engineering to apply. Equation (16) possesses preferably available for this kind of SSC vertical
distributions. Adopting the experimental data processed by Wang, the measured data are compared with calculations of equation (16) about suspended sediment concentration in Fig. 15. In addition, the field data collected by Dai are compared with calculations of equation (16) about suspended sediment concentration in Fig. 16. The results showed that calculations of equation (16) could be accorded with the measured data, while these comparisons could be testified that equation (16) is able to express the S-type SSC profiles with simpler formula structure.

**Discussion**

Multi-groups of experimental and field data are adopted to verify formulas of sand particle fluctuating intensities, sediment diffusion coefficients profiles and suspended sediment concentration profiles, drawing a conclusion that the rational-structure formulas deduced above could reflect the distributions and variation tendencies of every physical quantity more objectively, and confirm the general applicability of the structure of these formulas. The SDC profile formula could express the common three types of SDC ultimately. The vertical distribution formula of SSC which is able to describe the situations the largest SSC absent from or at the bottom are appropriate for the hyper concentration flow. In practical engineering calculations, relevant parameters are key factors for calculations and calibrations determining the types of results.

The values of relevant parameters in the test one have listed in Tab.1, and this test is mainly used for calibrating about sand particle fluctuating intensities. In spite of keeping the dynamic conditions stable for each test, the parameters in the calibrations about sand particle fluctuating intensities are fairly sensitive for the reason that even small adjustment would generate larger discrepancy for SDC and the trend discipline of each parameter variation may be too week to conduct inductions. Especially, different sizes of sand particles and SSC would result in different flow resistance and turbulent structures bringing in obvious influences on SDC and might be hard to limit the value range of parameters. In the test one, parameter $p_1$ would be positive for finer sand particles test groups, and negative for coarser sand particles. This transformation is corresponding to the variation of sand particle fluctuating intensities profiles near the bottom. Namely, sand particle fluctuating intensities profiles appear the situation that largest fluctuating intensities absent from the bed when $p_1$ is positive value, and obey the monotone decreasing function when $p_1$ is negative value.

According to the parameter calibrations, the relevant parameters among horizontal fluctuating intensities, vertical fluctuating intensities and sediment diffusion coefficients are considerably different. When the largest value of fluctuating intensities are absent from the bottom, the corresponding SSC profile might obey the distribution with the largest SSC at the bottom or not.

<table>
<thead>
<tr>
<th>$d_{50}$(mm)</th>
<th>$p_1$</th>
<th>$\lambda_{su}$</th>
<th>$q_1$</th>
<th>$p_2$</th>
<th>$\lambda_{sv}$</th>
<th>$q_2$</th>
<th>$p_s$</th>
<th>$\lambda_s$</th>
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<td>0.1</td>
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<td>-0.468</td>
<td>4.790</td>
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</table>
The value ranges of relevant parameters in the test two, three and four, as a result of verifying the general availability of SDC profile formula deduced for the three types, may concentrate into a relatively small region because of applying the single size or little different sizes sand particles for various dynamic environments in experiments or field measurements. Based on the two kinds of sand particles of test two, different median diameters of sand particles test groups generates obviously different parameters in the verifications, which are listed in Tab.2, and the maximum change range of these parameters is $p_s$ due to the opposite fluctuation tendency of the two kinds sand particles near the bottom. Simultaneously, these parameters centralize in a constant nearby in the test groups with the same size sand particles. In the test three, each parameter essentially preserves the same order of magnitude in the lower concentration and higher concentration test cases, especially $p_s$ around -0.01. The absolute values of calibrated parameters in higher concentration test groups are larger than the ones in the lower concentration, which may be the distinctions between quasi-parabolic type and increasing type in the SDC profile calculations. The value range differences of relevant parameters between test two and test three may be a kind of reflection for experimental results and natural field measurements. The field data coming from the Yellow River in the test four belong to the hyper concentration flow and the values of parameters listed in Tab.3 that have little to do with the increasing SSC are properly steady. When SSC is higher than 300 kg/m$^3$, parameters all converge on a small range around a constant, with $p_s$ around -3.6, $\lambda_s$ around 14.3 and $q_s$ around 1.4. In the test five, the parameters verified with these field data characterized by the maximum absence from the bottom in the SSC profiles conform to the law in other tests that $p_s$ is all equal to positive values. Generally speaking, the differences among parameters calibrated in these tests are rather larger while would be smaller when the similar dynamic conditions and the same size sand particles. The amplitudes of variation about these parameters in the hyper concentration flow are smaller than other lower concentration flows.

<table>
<thead>
<tr>
<th>Case</th>
<th>Test two</th>
<th>Test three</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q50S01</td>
<td>Q70S025</td>
</tr>
<tr>
<td>$p_s$</td>
<td>-0.205</td>
<td>-0.122</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>3.454</td>
<td>2.537</td>
</tr>
<tr>
<td>$q_s$</td>
<td>2.360</td>
<td>1.590</td>
</tr>
</tbody>
</table>
Conclusion

Based on the research achievements of velocity fluctuating intensities, the formula of sediment fluctuating intensities is raised analogically which is different from the velocity fluctuating intensities for opposite tendency near the bottom. The experimental results conducted by Li (1999) are utilized for verifying the rationality of the formula structure. The results show that the formula structure could preferably conform to the distribution tendency.

Compared the velocity flux with sediment flux, a general SDC profile expression is deduced from the sediment diffusion equation, for the purpose of describing the common three types of SDC such as the quasi-parabolic type, parabolic-constant type and increasing towards the water surface type. Further, given that the largest SSC appears absent from the bottom of profiles in certain conditions, a simple-structure SSC profile formula is obtained covering the general distributions of SSC.

The classic field data as well as experimental results of three types SDC and the measured data in the hyper concentration flow are applied to calibrate the general availability of the SDC profile formula. The conclusion indicates that this expression could reflect the trend of every type objectively. The data of flume experiments and pipeline tests with the special distribution features are utilized to validate the SSC profile formula.

The value range of relevant verified parameters of the SDC and SSC equations in different water-sediment environments are listed and discussed. Through comparisons and analysis, the inference is gained that the parameters in higher concentration flow are relatively steady while the mutative amplitude of the ones in lower concentration is comparative larger. Sand particle horizontal or vertical fluctuating intensities profiles or SDC profiles present the situation that maximum value absent from the bed when $p_1$, $p_2$ or $p_3$ are positive values, and obey the monotone decreasing function when $p_1$, $p_2$ or $p_3$ are negative values.

Acknowledgements

This work was supported by the National science and technology support program of China under Grant No. 2016YFC0402403, and the Foundation of special project of scientific research and technology development fund of the Yellow River Institute of Hydraulic Research in China (Grant No. HKY-JBYW-2016-24 and No. HKF201702)

Reference

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