Prediction of bed ripple geometry under controlled wave conditions: wave-flume experiments and MIKE21 numerical simulations

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Laboratory experiments using a wave flume were designed to examine the characteristics of ripple formed on a sandy bed. Ripples were formed on the sediment surface under a variety of surface wave conditions for a period of 33min. Profile measurements were taken all over the flume for each wave conditions. Examples of data collected by the sand ripple profiler after these experiments are presented, showing the sand ripple profiles under variety of wave conditions. In order to evaluate the experimental data, MIKE 21/3 Coupled Model FM numerical model was run with the same initial and boundary conditions.

[Keywords: Bed form, Experimental model, MIKE21, Ripple, Sand beach, Waves]

Introduction
Ripples are the most common bed forms and are the focus of this research. Ripples form in many different environments and have a variety of characteristics. The bed form type depends on the strength and nature of the flow. A steady current, tidal current, waves, or a combination of all three will influence the size, shape, and orientation of the bed forms. The nonlinear complexities of the flow present challenges in predicting ripples, and much research has been done examining bed forms under different flow regimes (e.g., Bagnolds,1946 3; Sleath,1984 4; Wiberg and Harris,1994 5; Nielsen,1992 6).

Materials and Methods
Some important ripple morphology parameters include the mobility number, $\psi$, wave orbital excursion, $A$, and the period parameter, $\chi$. The mobility number (Equation 1) is a ratio of the disturbing forces to the stabilizing forces on a sediment particle under waves action. It is a measure of a sediment particle’s tendency to move due to wave action.

$$\psi = \frac{(A\omega)^2}{(s - 1)gd}$$ (1)

Where $A$ is the wave orbital excursion (Equation 2), $\omega$ is the radial frequency (Equation 3), $s$ is the specific gravity of the sediment (for quartz sand $s = 2.65$), and $d$ is the median grain size diameter.

$$A = \frac{U_0T}{2\pi}$$ (2)

$$\omega = \frac{2\pi}{T}$$ (3)

In which $T$ is the wave period and $U_0$ is the maximum free-stream velocity of the flow oscillation.

Mogridge and Kamphuis (1972) 8 claim that ripple geometry depends on a dimensionless parameter derived from the mobility number and the wave orbital excursion length, called the period parameter (Equation 4).

$$\chi = \frac{d}{(s - 1)gT^2}$$ (4)

Published research on ripples dates back as far as 1882, when Hunt (1882)9 described his observations of the ripple-mark in sand. Following soon after, Candolle (1883)10 stated that ripples form when two liquids of different viscosities come in contact with each other in an oscillatory manner; and Forel (1883)11 observed...
that initial ripple wavelengths formed on a flat bed that are about half as long as the equilibrium wavelengths. The first published ripple experiments were performed by Darwin (1883)\textsuperscript{12}. He rotated a circular tub filled with sand and water in an oscillating motion and discovered that ripples formed radially in the sand. The work of Bagnold (1946) was the next major contribution to the field of ripple dynamics. He defined bed form as “vortex” ripples after observing the separation of flow at the ripple crest and the formation of a vortex in the lee of the ripple. When the flow reverses, the vortex is ejected upwards, causing the sediment to become suspended. He also presented the hypothesis that the ripple length is proportional to the wave orbital excursion length. Other significant investigations on the occurrence, formation, and development of ripples include Costello and Southard (1981)\textsuperscript{13}, Sleath (1976)\textsuperscript{14}, and Sleath (1984).

Field observations are crucial for the characterization of morphologic phenomena (Blondeaux, 2001)\textsuperscript{15}. Some of the first significant data sets of ripple observations include Inman (1957)\textsuperscript{16}, Dingler (1974)\textsuperscript{17}, and Miller and Komar (1980a)\textsuperscript{18}. These data confirm the hypothesis that the ripple wavelength is proportional to the wave orbital excursion length. Other early laboratory experiments have also contributed to the understanding of ripple dynamics (e.g., Carstens and Neilson, 1967\textsuperscript{19}; Mogridge and Kamphuis, 1972; Lofquist, 1978\textsuperscript{20}; Miller and Komar, 1980b\textsuperscript{21}). Empirical expressions to predict ripple height, wavelength, and steepness under different flow conditions have been formulated from laboratory and field measurements. The ripple predictor of Nielsen (1981) is one of the most well-known and verified. He developed formulas for ripple height, wavelength, and steepness under different flow conditions. Grant and Madsen (1982)\textsuperscript{22} used flume data of ripple spacing and height to develop general expressions for ripple height and steepness. A ripple predictor presented by Vongvisessomjai (1984)\textsuperscript{23} determines the geometry based on the grain size diameter and the period parameter, $\chi$ (Equation 1-4). Then, Modridge et al. (1994) and Wiberg and Harris (1994) each presented a ripple model. Much research has been done to examine and expand the validity of these ripple predictor methods. Li and Amos (1998)\textsuperscript{24} compared the methods of Grant and Madsen (1982) and Nielsen (1981) and proposed a modified expression that incorporates the enhanced shear velocity at the ripple crest. Other modifications to the Nielsen (1981)

\begin{equation}
\text{Experimental Model}
\end{equation}

A series of experiments observing the interaction between surface waves and rippled bed morphology was undertaken in an experimental wave flume of the Soil Conservation and Watershed Management Research Institute, Iran. Experiments were designed to investigate equilibrium rippled bed under a variety of surface wave conditions. Figure 1 shows the X-Y positional table of the bed profiler used to measure the sediment-water interface.

![Figure 1. The X-Y positioned table of the ripple measurement system designed and developed by this study.](image)

In this section we will look at the three main components of the equipment used: the wave generation and measurement systems, the sand bed parameters and the ripple measurement system.

![Figure 2. Plan (bottom) and elevation (top) views of the original wave flume setup](image)
The wave flume has a total length of 20m and a width of 5.5m with a depth of 2m (refer to figure 2). Waves are generated via a hydraulic piston-type wave paddle with a rectangular cross section. Irregular waves were generated using a specialized wave synthesizer program known as the SW Wave Maker Wave generation control program. The wave generation software controls the movement of the wave paddle. Three surface wave frequency conditions were used in this study having peak frequencies of: 0.45, 0.55 and 0.83 Hz. The repeating period for each of these runs were 33 minutes.

A 7m long and 1m width sand bed with slope of 0.03 was installed in the flume, starting at a distance of 2m from the wave paddle. Experiments were run with a type of sand with a percentile grain size diameter ($D_{50}$) of 0.2 mm. The flume experiments were established with the whole of the 7m sand bed composed of this sediment type. A sieve analysis was undertaken for sediment (refer to figure 3).

To measure the rippled surface under water, a specialized bed profiling system was designed and developed. The axis along the flume in the direction of wave travel was defined as the primary axis while the secondary axis was defined as the axis across the flume normal to the direction of wave travel.

Bed profiler system consists of a vertical position controlled device driven by a small electric motor with a mechanical transmission actuating a rod. At the lower end of the rod there is a probe which is the bottom sensor, used for the tracking the rod point under water closely to the sediment bed. The vertical displacement of the moving elements out of water is converted into an electric signal representing depth. Bed profiler system has 0.1mm accuracy.

The Numerical Model

Mike 21/3 Coupled Model FM is a dynamic modeling system developed by DHI Water & Environment. It is based on a flexible mesh approach and has been developed for applications within oceanographic, coastal and estuarine environments. It can be applied for investigating the morphological advancement of near beach bathymetry due to the variable hydrodynamic conditions. This model provides an approach to analyze the mutual interaction between waves and currents through dynamic coupling between the Hydrodynamic Module and the Spectral Wave Module. It can also model the process of full feedback among bed level changes caused by wave circulation through dynamic coupling between the Hydrodynamic Module, the Spectral Wave Module and the Sand Transport Module.

Hydrodynamic Module (HD). In this research, a two-dimensional model in HD Module was used to investigate the wave-induced currents. The model is based on equations of shallow water in which the depth-averaged velocities of Navier-Stokes equations are integrated in an incompressible fluid.

The continuity equation and integration of horizontal momentum equations over depth $h = \eta + d$ can be obtained as:

$$\frac{\partial h}{\partial t} + \frac{\partial h\bar{u}}{\partial x} + \frac{\partial h\bar{v}}{\partial y} = hS \quad (5)$$

$$\frac{\partial h\bar{u}}{\partial t} + \frac{\partial h\bar{u}^2}{\partial x} + \frac{\partial h\bar{v}\bar{u}}{\partial y}$$

$$= f\bar{u}h - gh \frac{\partial \eta}{\partial x} - h \frac{\partial p_a}{\partial x} -$$

$$\frac{gh^2}{2\rho_0} \frac{\partial \rho}{\partial x} + \frac{\rho_0}{\rho_0} - \frac{\partial}{\partial y} \left( \frac{\partial s_{xx}}{\partial x} + \frac{\partial s_{xy}}{\partial y} \right) +$$

$$\frac{\partial}{\partial x} \left( hT_{xx} \right) + \frac{\partial}{\partial y} \left( hT_{xy} \right) + hu_e S \quad (6)$$

$$\frac{\partial h\bar{v}}{\partial t} + \frac{\partial h\bar{u}\bar{v}}{\partial x} + \frac{\partial h\bar{v}^2}{\partial y}$$

$$= -f\bar{u}h - gh \frac{\partial \eta}{\partial y} - h \frac{\partial p_a}{\partial y} -$$

$$\frac{gh^2}{2\rho_0} \frac{\partial \rho}{\partial y} + \frac{\rho_0}{\rho_0} - \frac{\partial}{\partial x} \left( \frac{\partial s_{yx}}{\partial x} + \frac{\partial s_{yy}}{\partial y} \right) +$$

$$\frac{\partial}{\partial x} \left( hT_{xy} \right) + \frac{\partial}{\partial y} \left( hT_{yy} \right) + hv_e S \quad (7)$$

$u$ and $v$ are the depth-averaged velocities defined by:
\[ h\hat{u} = \int_{-d}^{\eta} udz, \quad h\hat{v} = \int_{-d}^{\eta} vdz \quad (8) \]

The lateral stresses \( T_{ij} \) consists of viscous friction, turbulent friction, and differential advection which is estimated using an eddy viscosity formulation, based on depth averaged velocity gradients expressed by the following equation:

\[ T_{xx} = 2A \frac{\partial \hat{u}}{\partial x}, \quad T_{xy} = A \left( \frac{\partial \hat{u}}{\partial y} + \frac{\partial \hat{v}}{\partial x} \right), \]
\[ T_{yy} = 2A \frac{\partial \hat{v}}{\partial y} \quad (9) \]

Where \( h(x,y,t) \) is the depth of water; \( \eta(x,y,t) \) is sea surface elevation; \( g \) is gravity acceleration; \( p_a(x,y,t) \) is atmosphere pressure; \( s_{xx}, s_{xy}, s_{yy} \) and \( s_{yy} \) are components of the radiation stress tensor.

**Sediment Transport Module (ST).** In this research, the method which is used in ST Module for calculation of sediment transport is based on the model of combined wave and current, and for this purpose the sediment tables for interpolation were used.

**Spectral Wave Module (SW).** Spectral wave formulation was used in this module, which is based on conservation equation of wave, defined by Komen et al. (1994)\(^29\) and Young (1999)\(^30\) as:

\[ \frac{\partial N}{\partial t} + \nabla \cdot (\hat{\nu}N) = \frac{S}{\sigma} \quad (10) \]

Where \( N(x,y,\sigma,\theta,t) \) is the spectra density, \( \hat{\nu} = (c_x, c_y, c_\sigma, c_\theta) \) is the propagation velocity of a wave group and \( S \) is the source term for the energy balance equation. The propagation velocities of a wave group have been achieved from below relations:

\[ (c_x, c_y) = \frac{d\tilde{x}}{dt} = c_g + \tilde{U} \quad (11) \]
\[ c_\sigma = \frac{d\sigma}{dt} = \frac{d\sigma}{ds} \left[ \frac{d\tilde{U}}{ds} + \tilde{U}, \nabla_s d \right] - c_g \frac{k}{k} \frac{d\tilde{U}}{d\sigma} \quad (12) \]
\[ c_\theta = \frac{d\theta}{dt} = -\frac{1}{k} \left[ \frac{d\sigma}{ds} \frac{d\tilde{U}}{d\sigma} + k \frac{d\tilde{U}}{dm} \right] \quad (13) \]

Where \( s \) is the space co-ordinate in wave direction \( \theta \), and \( m \) is a co-ordinate perpendicular to \( s \).

**Experimental and Numerical Methods and Tests**

The tests were started with the sediment bed in a no-ripple condition by raking the bed to remove any ripples present from previous experiments. Ripples were formed on the sediment surface by running regular waves confirming to a standard JONSWAP spectrum along the flume for a period of 30min. After this time the waves and rippled bed were considered to be at equilibrium. A number of three-dimensional grids were profiled at the test areas that each grid has 2cm horizontal and 2.5cm vertical spaces. This provided 13650 three-dimensional data points taken from the rippled surface.

Figure 4 present initial bathymetry of flume with longitudinal profile used to determine ripple height and distance, and figure 5 shows an example of the data obtained from the equilibrium profiles using the ripple measurement system. The rippled bed presented in figure 5 was formed under regular waves with \( T=2.2s; H_s=11cm \).
To evaluate the effects of wave height on sediment transport in both physical and numerical models, effective parameters are taken as radial wave height parameter ($H_0$) and wave period (T). Table 1 shows wave programs for both experimental and numerical tests. This table lists all the tests undertaken in this research.

Results and Discussion

Experimental and numerical results

Table 2 presents the dimensions of the ripples formed in experimental and numerical models under different wave conditions. In this table $\eta$ is ripple height and $\lambda$ is ripple length. Presented values were calculated from averaged values of longitudinal profile of figure 4.

Results Comparison

Here we compare the empirical ripple prediction model of Nielsen [1981] to previous field measurements [Li and Amos, 1999; Hanes et al., 2001], and to our experimental and numerical data sets. The Nielsen [1981] model for irregular waves is based on field wave-ripple data of Inman [1957] and Dingler [1974] and predicts the ripple height $\eta$ and length $\lambda$ of ripples using the near-bed semi excursion $A$ and the mobility number $\psi$. Nondimensional ripple height is expressed as

$$\frac{\eta}{\lambda} = 21\psi^{-1.85} \quad \psi > 10$$

$$\frac{\eta}{\lambda} = 0.275 - 0.022\psi^{0.5} \quad \psi < 10$$

And nondimensional ripple length is expressed as

$$\frac{\lambda}{\eta} = \exp \left( \frac{693 - 0.37H\psi}{1000 + 0.75H^2\psi} \right)$$

Nielsen [1981] independently fit curves for ripple steepness, giving

$$\frac{\eta}{\lambda} = 0.342 - 0.34\lambda^{0.5}$$
Table 1. Wave and bed features used in both experimental and numerical model

<table>
<thead>
<tr>
<th>Test no.</th>
<th>$d_{50}$ (mm)</th>
<th>$H_0$ (m)</th>
<th>$T$ (s)</th>
<th>Duration (min)</th>
<th>Water Level (m)</th>
<th>Bed slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.075</td>
<td>1.8</td>
<td>30</td>
<td>0.2</td>
<td>0.03</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.11</td>
<td>1.8</td>
<td>30</td>
<td>0.2</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>0.05</td>
<td>1.8</td>
<td>30</td>
<td>0.2</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>0.11</td>
<td>1.2</td>
<td>30</td>
<td>0.2</td>
<td>0.03</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>0.11</td>
<td>2.2</td>
<td>30</td>
<td>0.2</td>
<td>0.03</td>
</tr>
</tbody>
</table>

The Shields parameter $\theta_{2.5}$ is defined by

$$\theta_{2.5} = \frac{1}{2} f_{2.5} \psi,$$

where $f_{2.5}$ is the Swart [1974] friction factor with a roughness of $2.5d_{50}$,

$$f_{2.5} = \exp\left[5.213(2.5d_{50}/A)^{0.194} - 5.977\right]$$  \hspace{1cm} (18)

Based on the laboratory data of Carstens et al. [1969], Grant and Madsen [1982] derived expressions for wave-dominant ripples. Li and Amos [1999] proposed some modification to the Grant and Madsen’s [1982] method:

For $\theta_{cr} < \theta_{ws} < \theta_{bf}$,

$$\eta = 0.101A_b \left(\frac{\theta_{ws}}{\theta_{cr}}\right)^{-0.16}$$

$$\lambda = 3.60\eta \left(\frac{\theta_{ws}}{\theta_{cr}}\right)^{0.04}$$  \hspace{1cm} (19)

And for $\theta_{ws} > \theta_{bf}$,

$$\eta = 0.356A_b S_*^{-0.8} \left(\frac{\theta_{ws}}{\theta_{cr}}\right)^{-1.5}$$

$$\lambda = 3.03\eta S_*^{-0.6} \left(\frac{\theta_{ws}}{\theta_{cr}}\right)$$  \hspace{1cm} (20)

Where $A_b$ is the near bed wave orbital amplitude, $\theta_{ws}$ is the skin-friction wave Shields parameter, $\theta_{cr}$ is the critical Shields parameter for bed load transport, $\theta_{bf}$ is the critical Shields parameter for ripple break off, $S_*$ is a dimensionless sediment parameter.

Some of the measured ripple heights and ripple wavelengths with their wave and current input data of the Li and Amos [1999] were chosen to compare with our data.

The Nielsen [1981] model curves for ripple height, length, and steepness are shown in figure 8 along with our measured and modeled ripple dimensions. Dimensionless wave ripple (a) height and (b) length versus mobility number and (c) ripple steepness versus Shields parameter of our experimental and numerical model, Nielsen [1981] curve, Hanes et al. [2001], Li and Amos [1999] are compared in this figure.

Figure 8. Dimensionless wave ripple (a) height and (b) length versus mobility number and (c) ripple steepness versus Shields parameter. Blue curves are Nielsen (1981) curves.
For better comparison of physical model results with empirical relationships, ripple height changes chart and also ripple distances changes chart are plotted in profile of figure 9 and 10.

Figure 9 shows ripple height changes and figure 10 shows ripple distances changes in this profile after wave with 11cm height and 2.2s period.

Plotted charts show good agreement between results of laboratory and results of prior empirical relationships. According to the plotted charts, good agreements between result in some parts of the plot shows that empirical relationships work better in these sections. It means that in the start of flume where it is deeper, results of empirical relationships are more close to experimental model but it deviates at intermediate depths, also where we become far from flume start, where depth decreases, these results again become close to each other.

Table 2. The dimensions of the ripples formed in experimental and numerical models

<table>
<thead>
<tr>
<th>Wave characteristics</th>
<th>( H_0 (m) )</th>
<th>( T(s) )</th>
<th>( H_0 (m) )</th>
<th>( T(s) )</th>
<th>( H_0 (m) )</th>
<th>( T(s) )</th>
<th>( H_0 (m) )</th>
<th>( T(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \eta ) (cm)</td>
<td>( \lambda ) (cm)</td>
<td>( \eta ) (cm)</td>
<td>( \lambda ) (cm)</td>
<td>( \eta ) (cm)</td>
<td>( \lambda ) (cm)</td>
<td>( \eta ) (cm)</td>
<td>( \lambda ) (cm)</td>
</tr>
<tr>
<td>Experimental Model</td>
<td>0.65</td>
<td>6.92</td>
<td>0.63</td>
<td>8.22</td>
<td>0.24</td>
<td>6.8</td>
<td>0.41</td>
<td>8.22</td>
</tr>
<tr>
<td></td>
<td>0.72</td>
<td>7.77</td>
<td>0.72</td>
<td>8.39</td>
<td>0.61</td>
<td>6.54</td>
<td>0.33</td>
<td>8.39</td>
</tr>
<tr>
<td></td>
<td>0.61</td>
<td>6.54</td>
<td>0.33</td>
<td>7.71</td>
<td>0.24</td>
<td>6.8</td>
<td>0.41</td>
<td>8.22</td>
</tr>
<tr>
<td></td>
<td>0.41</td>
<td>6.8</td>
<td>0.41</td>
<td>8.22</td>
<td>0.24</td>
<td>6.8</td>
<td>0.41</td>
<td>8.22</td>
</tr>
</tbody>
</table>

![Figure 9. Bed Ripple Wave Length changes under wave with 11 cm height and 2.2 s period](image1.png)

![Figure 10. Bed Ripple Height changes under wave with 11 cm height and 2.2 s period](image2.png)
A measure of the relative error between measured and predicted values can be defined as

\[ \Delta = \exp \left\{ \frac{1}{n} \sum_{i=1}^{n} \left( \ln(y') - \ln(\hat{y}') \right)^2 \right\} \] (21)

Where \( \hat{y} \) is the measured value and \( y \) is the predicted value. This quantity is a multiplicative factor that indicates the possible variation about the predicted value. For example, if \( \Delta \) equals 1.34, the average error is equal to 34%. As figure 8 shows, Nielsen curves have best agreement with our data, further more we measured this relative error between our data and these curves.

Table 3 shows relative error for ripple dimensions, between experimental, numerical and predicted values. As shown in table 3, the relative error for ripple height is 1.04 and 1.26 for experimental and numerical models, respectively.

The models performed similarly at predicting ripple length and had relative errors of 1.02 and 1.07 for the experimental and numerical models, respectively. The models were better at predicting ripple steepness than at predicting ripple height or ripple length independently. The experimental model had a relative error of 1, whereas the numerical model had a relative error of 1.04 in predicting ripple steepness.

| Table 3. Relative error between Measured, Modeled and Nielsen [1981] predicted dimensions |
|---------------------------------|---------------------------------|------------------|------------------|
| Ripple height                   | 1.04                            | 1.26             |
| Ripple length                   | 1.02                            | 1.07             |
| Ripple steepness                | 1                               | 1.04             |

**Conclusion**

Different wave conditions for ripple formation on flat bed have been examined through wave-flume experiments. Data were analyzed using the mobility number and Shields parameter. Results showed that ripple height increased with orbital excursion, as indicated by Nielsen [1992] for wave generated ripples. According to Table 2, the height of the wave ripples (0.2-1 cm), and their length of 10 cm, gives them a characteristic steepness of 0.09 to 0.2. They are therefore likely to be ‘post-vortex’ wave ripples, as in Bagnold’s [1963] classification.

Results showed that the changes in wave periods only changes the pattern of ripples formed in bed. However, the change of parameters such as bed slope and sediment size can change the ripples pattern and their dimensions. So bed parameters including bed slope, sediment size and bottom friction coefficient, depending on their significance, are considered as the important factors affecting dimensions of bed ripples.

The comparison of the measured and modeled ripple dimensions with the predictions by the wave-ripple predictor of Nielsen [1981] indicates that both experimental and numerical methods have good agreement with Nielsen curves. Also, good agreement between result in some parts of the plot shows that empirical relationships work better in these sections. It means that in the start of flume where is deeper, results of empirical relationships are more close to experimental model but it deviates at intermediate depths, also where we become far from flume start where depth decreases, these results become close to each other. On the whole the experimental results are in more agreement with the previous empirical data.

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