“DOTS and Boxes” is a very popular game amongst all generations and the last pages of notebooks of most school-kids are filled with either ‘tic-tac-toe’ or ‘dots and boxes’. The game is played on a board of dots arranged in a rectangular array (certain number of rows and columns). Each player in turn draws a line to connect two nearest dots (vertical or horizontal) and tries to complete squares (called boxes). Completing a box is equivalent to capturing the box and after capturing a box, a chance is earned to draw one more line. The person capturing the maximum number of boxes in the end wins.

However, the game is not on an even ground and the chance to win the game needs certain scientific strategies, which are elaborated in this article.

**Playing Methodology**

The game involves a board of unit box, generated by 2x2 matrix of dots (4 nos). Since the box needs four lines for completion, and each player draws one line at a time, the game of one box is always won by the player going second. This is depicted in Figure 1.

The game between two players ‘A’ and ‘B’ is described and the turn number against each drawn line is also shown in Figure 1. The first player depicted here as ‘A’ connects any two dots (move A1). For the second player, two strategies exist for his first move (depicted by B1): either draw a line parallel to the line drawn by player ‘A’ (first sketch of Figure 1) or connect any dot already connected by player ‘A’ with the adjacent dot (second sketch of Figure 1). In the next turn, player ‘A’ draws (move A2) any two of the remaining lines and the box along with the game is claimed by player ‘B’ (move B2). This game does not need any strategy and the second player is a
The game of two boxes is played using seven dots arranged in an array of 2 rows and 3 columns (Figure 2). The total number of lines that is to be drawn for completing the game is seven. It seems that this board favours the first player, but after claiming the first box, the winner gets a chance to draw one extra line. Finally, this game is also mostly won by the second player if in one chance more than one box is claimed.

The game can be turned in the favour of any player for bigger boards. Before entering into the intricacies of evolving a winning strategy, some of the mathematical identities valid for a game with an array of ‘m’ horizontal and ‘n’ vertical dots are given below.

Total number of dots = m x n
Total number of horizontal lines = mx(n-1)
Total number of vertical lines = nx(m-1)
Total number of lines needed = 2mn-m-n
Total number of boxes = mn-m-n+1

For a bigger board of 10x9 dots, m=10 and n=9. This board has 80 horizontal lines, 81 vertical lines, 161 total lines and 72 boxes. The total number of moves is less than the total number of lines because after claiming one box, the box winning player gets a chance to draw another line. However, after claiming the last box, no extra chance is needed for the game. The actual number of moves may be reduced if in one chance more than one box is claimed.

Another extreme situation occurs when a series of all the unclaimed boxes are created and a single player gets all the boxes. Yet another variation occurs, when drawing a single line results in claiming of two boxes. So, the number of moves for a given array of dots may vary. If the total number of moves is even, chances of win for the second player exist. In the game, all odd moves are executed by the first player and all the even moves are executed by the second player.

For a 3x3 array of dots, the total number of lines is 12 and the total number of boxes is 4. For a game between two efficient players, before any boxes are claimed the situation of array is depicted in Figure 3. In sketch ‘P’ of Figure 3, six moves are already completed. The seventh move is to be executed by the first player, when he gives the lower single box to the second player (in move 8). The second player has to draw one more line in move 8, which results in claiming of 3 boxes in the 9th move of the first player. In this arrangement, the first player wins by 3:1 in total 9 moves.

In sketch ‘Q’ of Figure 3, four moves are over and the fifth move is to be executed by the first player. In this arrangement, each player in turn claims one box and concedes the next box. So, the game is finished equal as 2:2 in 9 moves. For sketch ‘R’ in Figure 3, seven moves are over and the next move is to be executed by the second player. This results in a win with 4:0 by the first player in 9 moves. For sketch ‘S’ of Figure 3, eight moves are over and in the next move the first player gives all 4 boxes to the second player. This game is won by the second player with 4:0 in 10 moves.

The summary of results for 3x3 array depicted in Figure 4 is tabulated as Table 1. It is clear that if the number of moves is odd, the first player wins. The second player can win only when a closed loop is created and the number of moves is made even. Creation of a loop may be a way of changing the number of moves. So, definitely some strategy is needed when the second player can win the game.

Obviously, the 3x3 array seems to be biased towards the first player. As the number of dots on the board increases, the total number of possible sketches may increase and accordingly a scientific strategy is to be implemented to win the game. The next section of the article explores some of the winning strategies for the game of “Dots and Boxes”. However, the real science behind the game is obvious for a board with at least 9 boxes.

### Winning Strategy

Both the players involved in the game are assumed sane and neither box with three sides is created in an unwanted way by any of the players nor any box with all the three sides drawn is unclaimed without any valid strategy for future gain, during the game. The best strategy is evolved when the board is divided into long chains (similar to sketch ‘R’ of Figure 3),
where placement of one line results in claiming of all the boxes of a chain by the next player. Chain the is name given to a series of more than 3 boxes, which can be claimed in a single move. If more number of long chains is created, a strategy of Double-Cross is implemented, where the last two boxes of a chain are unclaimed and sacrificed for the next player, so as to claim the next long chain (more number of boxes), later. A simple double-cross strategy is explained with Figure 5.

The top left sketch of Figure 5 is the arrangement of the board obtained after drawing 12 lines. It has one lone box (loony), one chain of 3 boxes and one chain of 5 boxes. As the board has a total of 9 boxes, the player who takes the longer chain (5 boxes), wins. The 13th move is executed by the first player, who has to sacrifice at least one box (bottom right box) to the second player. After claiming the box, the second player has to draw another line and the line drawn is the central right line. Now, it becomes the turn of the first player to capture some of the boxes.

For a simple game, the first player can claim three boxes, as shown in the top right sketch, conceding the remaining 5 boxes to the second player. This results in a win by the second player with a margin of 6:3. However, if the first player plays a double-cross by not claiming all 3 boxes and sacrificing the last two boxes to the first player, as shown in the bottom left sketch, the second player is forced to give the longer chain to the first player and the first player can win the game by the same margin (6:3). After a double-cross, two boxes are claimed by drawing one line.

As this is a board of even number of dots (16), without double-cross, the second player is always destined to win. However, if one double cross is executed, the first player can win. As a thumb rule, if the sum of the number of dots and number of double-crosses is odd, the first player wins else the second player wins.

Double-cross can also be used to enhance the winning margin. The loop may be considered as equivalent to two chains and this should be executed with caution by both the players in the game. Outcome of a game may vary using double-cross strategy in simple game. Various options (total 9) and probable outcomes for each option (boxes claimed by first player: boxes claimed by the second player) for a game with 4x4 dots is depicted as Figure 6. Various outcomes depicted in the figure indicate variations in moves by both the players. The other possibilities for each of the developed chains on board of 4x4 dots can also be worth exploring.

For larger boards, many options are possible, but the thumb rule of sum of dots and double-crosses remains valid. On a board with 10x10 dots, 81 boxes and 180 lines are drawn to complete the game. For a simple game, depicted by the left sketch of Figure 7, 9 chains are created, out of which double cross can be executed 8 times. As the sum of number of dots and number of double-crosses is even, the second player wins.

However, if the number of chain is reduced to 8 as shown in the right sketch of Figure 7, the number of double-crosses is reduced to 7. As the sum of number of dots and double-crosses is odd now, the first player becomes the winner. The main science behind winning the game is control on a number of chains, so that the opponent becomes the first to sacrifice boxes/chains.

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