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Role of VLF power line harmonic radiation in precipitating energetic electrons at high latitude

Ram Prakash*, D D Gupta & Manoj Kumar Singh

Department of Physics, Bipin Bihari (PG) College, Jhansi 284 001, UP, India

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A study is carried out on energetic electron precipitation at high latitude (L = 4.3) by VLF power line harmonic radiation. The life-time of energetic electrons interacting with coherent VLF line radiation has been found to be 5.56 days only. This indicates a significant precipitation of energetic electrons (energy ~ a few keV) at high latitudes (L=4.3). The average flux of precipitating electrons by VLF line radiation has been estimated to be 3.47×10^{-3} ergs cm^{-2} s^{-1} which is almost the same as caused due to coherent whistler-mode waves of 1 pT intensity at 5 kHz and is found to be consistent with energy flux deposited in the lower ionosphere at L ~ 2.4 caused by lightning induced precipitation [Inan *et al., J Geophys Res (USA),* 92 (1987) 3293].

Keywords: Energetic electron, VLF power line harmonic radiation, Electron precipitation

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1 Introduction

The pitch angle scattering of trapped energetic particles is one of the important consequences of wave-particle interaction phenomena occurring in the terrestrial magnetosphere. This results in the perturbation of the stable orbits of the energetic particles and their subsequent precipitation into the denser lower atmosphere. The whistler-mode waves play an important role in the loss of trapped energetic particles. They perturb the adiabatic invariant motion of the particles and wave-induced pitch angle and energy changes result in lowering the 'mirror' height of particles and their precipitation into the lower ionosphere. The energetic particles precipitated by the waves give rise to secondary ionization, conductivity enhancement, X-ray and optical emissions, heating and transient increase of cold particle population in the lower ionosphere.

The whistler-mode waves include a variety of waves such as: (i) lightning generated whistlers; (ii) spontaneous natural emissions like hiss and chorus; (iii) man made signals radiated out from VLF transmitters and large power grids; and (iv) triggered emissions. All of these waves contribute to the loss of trapped radiation belt particles. A considerable work has been done on pitch angle scattering of radiation belt particles by wide-band incoherent whistler-mode waves such as plasmaspheric (ELF) hiss and VLF hiss[1-4]. The coherent magnetospheric whistler-mode signals (having instantaneous band-width that is much smaller than the wave frequency) such as natural whistlers, discrete VLF chorus emissions, signals injected into the magnetosphere from the ground based transmitters, discrete emissions triggered both by whistlers and transmitter signals and power line harmonic radiation (PLHR) may also contribute substantially to the loss of the trapped energetic electrons.

There exists an evidence of harmonic radiation from the power distribution system entering the earth's magnetosphere and stimulating VLF emissions there[5]. The PLHR influences the particle population strongly in the magnetosphere and can initiate whistler precursors[6,7]. The satellite studies have revealed a permanent zone of enhanced VLF activity over the north east industrial regions of America[8-10]. Some theoretical studies also indicate the role that the weak coherent PLHR play in the growth phase of VLF emissions[11-13].

The spectrograms of broad-band ELF/VLF goniometer data based on ground measurements made at Halley bay, Antarctica (L=4.3) have shown the existence of discrete VLF power line harmonic radiation in a frequency range of 1-4 kHz (ref. 14). This radiation is almost identical in properties with PLHR[5], but there are some interesting differences also. Whereas the PLHR has a regular frequency spacing of 120 Hz (ref. 5), this line radiation has the frequency spacing that are widely distributed about mean values between 50 and 90 Hz, triggers emissions and often exhibits two hop amplitude modulation, which hint towards its magnetospheric origin[14]. The origin of magnetospheric emissions connected with PLHR is not clear. The theories of their origin may be controversial but the role of these emissions in energetic electron precipitation can not be neglected.

In the present paper, therefore, the role of this VLF power line harmonic radiation in the energetic electron precipitation at high latitude (L=4.3) has been examined by evaluating equatorial coherent diffusion coefficients and flux of the precipitating electrons.

2 Theoretical background and expressions used

A coherent whistler-mode wave interacts with energetic electrons in the equatorial plane for which the resonance condition is given by[15]

$$\omega + kv_R = \Omega_e \qquad \text{...(1)}$$

Where, ω, is the angular wave frequency; Ω_e, angular electron gyro-frequency; k, wave number; and v_R, the electron resonance velocity. The wave propagation is assumed to be field aligned and relativistic factors γ is taken to be 1 (non relativistic case). From this, the resonance energy of energetic electrons is found to be[16]

$$E_R = \frac{B_0^2 \Omega_e}{2\mu_0 n_0 \omega}\left(1 - \frac{\omega}{\Omega_e}\right)^3 \qquad \text{... (2)}$$

where, B_O, is the equatorial value of the earth's magnetic field; μ_0, the magnetic permeability of free space; and n_0, the equatorial electron density. E_R is related with the electron resonance velocity v_R, as

$$E_R = 250\ v_R^2 / c^2 \qquad \text{... (3)}$$

where, c, is the speed of light in vacuum.

The diffusion coefficient for coherent wave-particle interaction near the geomagnetic equator is defined as[17]

$$D^C \equiv \ < (\Delta\alpha)^2 > / T_r \qquad \text{... (4)}$$

where, Δα, is the net total pitch angle change for each particle; the angular brackets denote an average over the initial particles Larmor phase; and T_r, is the resonance time defined as[17]

$$T_r \sim L_R / v_R \qquad \text{... (5)}$$

with

$$L_R \simeq \left[(16/9)\pi v_R L^2 R_e^2 / \Omega_e\right]^{1/3} \qquad \text{.... (6)}$$

Here L_R is known as interaction/resonance length for resonant interaction[18] around a point close to the geomagnetic equator.

The precipitated flux J_P is related with the trapped energetic electron flux J_T as[19]

$$\frac{J_P}{J_T} = \frac{T_M}{T_L} \qquad \text{... (7)}$$

where, T_M and T_L, are the minimum life-time corresponding to strong diffusion and electron life-time, respectively. T_M is defined as[1]

$$T_M = 2\frac{T_e}{\alpha_0^2} \qquad \text{... (8)}$$

where, α_0, is the equatorial loss cone angle; and T_e, the electron escape time (roughly 1/4 of a bounce period). The life-time T_L is taken to be the inverse of diffusion coefficient.

3 Results and discussion

Figure 2 of Mathews & Yearby[14] shows the VLF line radiation wave amplitudes. For clear understanding of these signals, these amplitudes are re-plotted as a function of wave frequency in Fig. 1

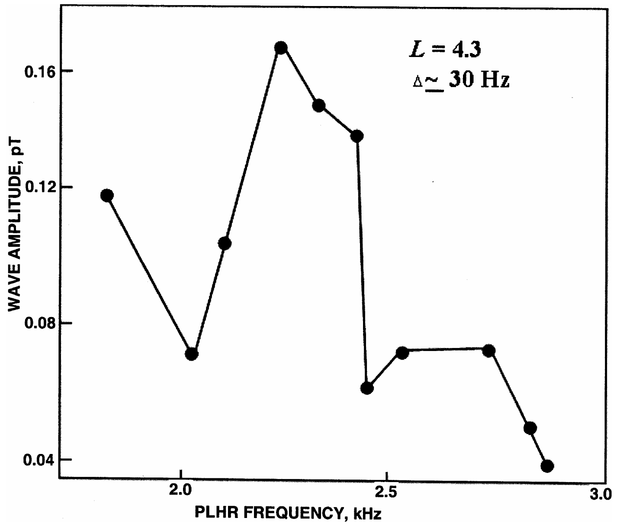

Fig. 1 — Pilot showing variation of VLF power line harmonic radiation amplitude B_W as a function of wave frequency at L=4.3

which clearly shows that the wave amplitude of the VLF line radiation lies between 0.04 and 0.16 pT. The VLF lines starting from 1.89 kHz have a band width of $\Delta f \simeq 30$ Hz Since $\Delta f << f$ (wave frequency) and hence the VLF power line hormonic radiations considered here are the coherent signals.

The calculations are done at L=4.3 for the different wave frequencies of 1.89-2.78 kHz which are close to third and fourth harmonics of 60 Hz. The VLF power line harmonic radiation considered in the present paper consists of several line emissions that do not always appear at exact harmonics of 50 or 60 Hz. They are also not spaced at exactly the power system frequency. In some cases, the lines shift in frequency. Similar characteristics have been shown by the PLHR reported by Helliwell *et al*[5]. The equatorial electron density (n_0) at L=4.3 is taken to be $2.56\text{x}10^{8}$ m^{-3} which roughly corresponds to a diffusive equilibrium model[20] of the ionosphere. The angular electron gyrofrequency of L=4.3 in the equatorial plane is found to be $6.9038\text{x}10^{4}$ rads^{-1} by the relation:

$$\frac{\Omega e}{2\pi}=\frac{873.6x10^{3}}{L^{3}}$$

Using the dipolar magnetic field model, the equatorial value of the earth's magnetic field (B_0) at L=4.3 is computed to be $3.9242\text{x}10^{-7}$T.

The resonance energies (E_R) of the electrons gyroresonantly interacting with the coherent VLF line radiation in the equatorial plane at L=4.3 are computed by using Eq. (2). The calculated values of E_R are found to be of the order of a few keV and are plotted as a function of wave frequency in Fig. 2. The value of E_R decreases with increasing value of resonant wave frequency of the coherent signal and its values vary from 2.5 to 4.94 keV. Using Eq. (3), the values of the resonance velocity v_R are found to be in the range (3-4.22) $\text{x}10^{7}$ ms^{-1}.

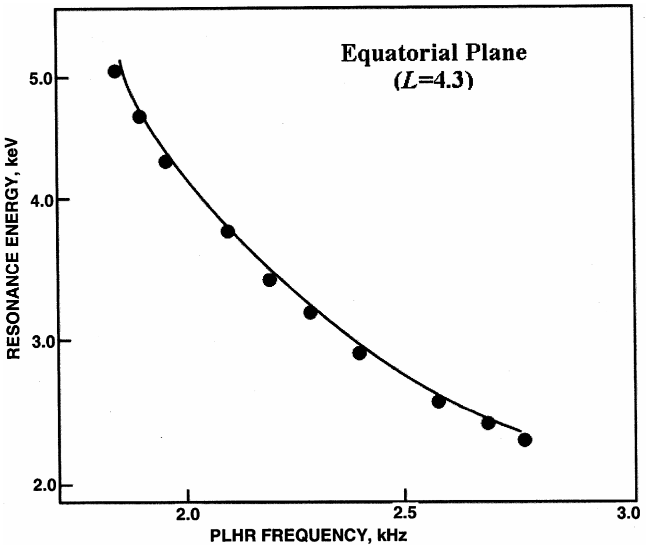

Fig. 2 — Plot showing variation of resonance energy of energetic electrons (gyro-resonantly interacting with the coherent VLF power line harmonic radiation) with wave frequency at L=4.3

Next, the resonance time (T_r) for the coherent wave-particle interaction is calculated at L=4.3 in the equatorial plane by employing Eqs (5) and (6). The calculated values of T_r presented in Table 1 show that the calculated value of T_r increases with the increasing value of signal frequency and lies in the range 32-41 ms.

Now, the coherent diffusion coefficient D^C is to be calculated using Eq. (4). For this, the value of pitch angle scattering ($\Delta\alpha$) is to be evaluated. In case of wave-particle interaction involving incoherent wide-band whistler-mode waves, $\Delta\alpha$ is inversely proportional to resonance velocity v_R (refs 17, 21). It is assumed that this is true in case of coherent wave particle interaction also. So,

$$\left[<(\Delta\alpha)^2>\right]^{1/2}\propto\frac{1}{v_R},\ \text{i.e.}<(\Delta\alpha)^2>\propto\frac{1}{v_R^2}$$

With this assumption, one can determine the value of $<(\Delta\alpha)^2>$ for equatorial coherent wave-particle interaction between energetic electrons and VLF line radiation at L=4.3. Unlike in the case of incoherent wave-particle interaction, one does not find an appropriate expression for finding out ($\Delta\alpha$) in the case of coherent wave-particle interaction. So, one relies on the values determined by other researchers. Using

Table 1 — Calculated values of resonance time (T_r) at L=4.3 for different frequency components of VLF power line harmonic radiation

Frequency, kHz	Resonance time (T_r), ms
1.89	32.44
2.00	33.46
2.07	34.11
2.17	35.04
2.25	35.78
2.31	36.35
2.43	37.49
2.50	40.16
2.67	39.80
2.74	40.49
2.78	40.70

the test particle simulation model developed by Inan *et al.*[22] for the gyroresonance interaction between energetic electrons and coherent signals, Inan[17] has evaluated the value of mean square pitch angle scattering $<(\Delta\alpha)>$ caused due to 5 kHz coherent signal having an equatorial wave magnetic field intensity of 1 pT at L=4. This value of $<(\Delta\alpha)>$ has been found[17] to be $\simeq$ 0.025 deg^2. However, this value can not be used here in the calculations as such and it must be modified considering the fact that $<(\Delta\alpha)^2> \propto \frac{1}{v_R^2}$. The above mentioned value i.e. $<(\Delta\alpha)^2> \simeq 0.025$ deg^2 is based on Fig. 3 of Inan[17]. The values of v_R given in Fig. 3 of Inan[17] are ~10^6 ms^{-1}, while the value of v_R obtained in present study are ~10^7 ms^{-1} which are higher by an order of magnitude of 10. So, the value of $< (\Delta\alpha)^2 >$ in case of the VLF line radiation considered in the present study is taken to be 0.025x10^{-2} deg^2 or 7.62x10^{-8} rad^2. By using this value of $< (\Delta\alpha)^2 >$ in Eq. (4) along with the calculated values of resonance time (T_r) (shown in Table 1), the coherent diffusion coefficients (D^C) are calculated for the wave-particle interaction involving the VLF line radiation and the results are presented in Fig. 3. Figure 3 depicts the variation of D^C with frequency of VLF line radiation. The value of D^C is found to decrease slightly with increasing wave frequency. The calculated values of D^C presented in Fig 3 lie in the range (1.87-2.35) x10^{-6} rad^2 s^{-1}. The average value of D^C is found to be D^C (av) = 2.08x10^{-6} rad^2 s^{-1}. From this, the average life-time of the electrons is estimated to be T_L = 5.56 days only. This small life-time indicates that significant precipitation (whether weak or strong) of a few keV electrons takes place by coherent VLF power line harmonic radiation at higher latitudes such as that corresponding to L=4.3.

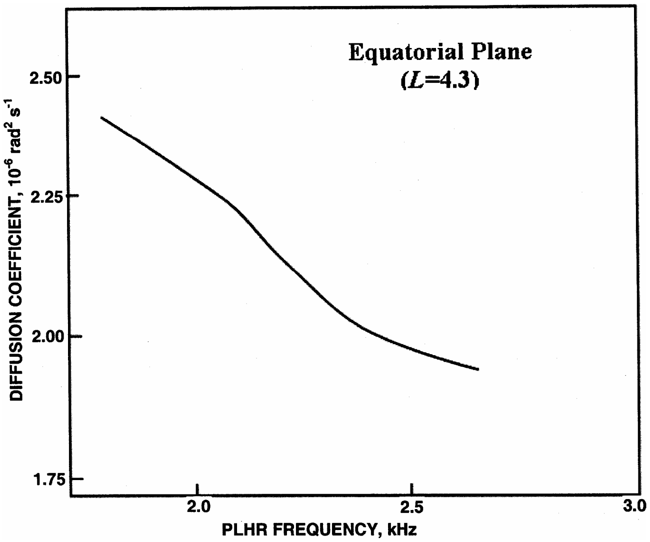

Fig. 3 — Plot depicting variation of coherent diffusion coefficient D^C with frequency of VLF line radiation at L=4.3

The loss angle, α_o at L=4.3 is 0.085 rad; and average resonance velocity of the electrons is 3.93 x 10^{-7} ms^{-1} (corresponding to the average energy of 4.30 keV). With these values, it is found that T_M = 192.93 s (Eq. 8). Thus, T_L/T_M= 2.49x10^3 which indicates that a significant weak diffusion is caused by the VLF line radiation at L=4.3. This finding is confirmed by calculating the value of diffusion strength parameter Z (ref. 1).

It is to be pointed out here that Kennel & Petschek[1] has given the concept of weak and strong diffusion in terms of a parameter $Z = \alpha_0 / \sqrt{DT_e}$ with the electron escape time $T_e = LR_e / v_R$. In the present case, the value of diffusion parameter Z is found out to be 60.71. So, one is in the weak diffusion limit (Z >> 1). Thus, the diffusion of electrons caused by PLHR at high latitude (L= 4.3) in the present case is weak but significant. Further, Eq. (7) gives J_P = 4.01x10^{-4} J_T. The precipitated energy flux is found to be proportional to the differential energy spedrum (Φ_0^E) of the trapped energetic particles[23]. Inan *et al.*[23] consider Φ_0^E = 10^8 el cm^{-2} s^{-1} sterad^{-1} as the differential energy spedrum for 1 keV electrons having 90° pitch angle. Lyons & Williams[24] report the flux levels of 10^6-10^8 el cm^{-2} s^{-1} sterad^{-1}. Although, the flux levels are highly variable with L-value, geomagnetic conditions and local time, such flux levels have also been observed by Dynamics Explorer satellite[23]. So, at L=4.3, a trapped flux of ~keV electrons is taken to be 1x10^8 el cm^{-2} s^1 sterad^{-1}. This gives a precipitated flux J_P = 5.04x10^5 el cm^{-2} s^{-1}. When converted into energy flux, taking the average energy of electrons to be 4.30 keV, the average flux of the precipitating electrons is estimated to be J_P=3.47x10^{-3} ergs cm^{-2} s^{-1}. For a typical trapped electron distribution, Inan *et al.*[23] have estimated the peak precipitated energy flux at L=4 to be 5x10^{-3} ergs cm^{-2} s^{-1} caused due to the coherent waves of 1 pT intensity at 5 kHz. These two values are nearly equal and of the same order. Further, the precipitated energy flux of 3.47x10^{-3} ergs cm^{-2} s^{-1} estimated in the present study as a result of coherent wave-particle interaction involving line radiation at L=4.3 is found to be consistent with the energy flux of ~10^{-4}-10^{-2} ergscm^{-2} s^{-1} deposited in the lower ionosphere at L ~ 2.4 caused by lightning induced precipitation[25] and approximately equal to the energy flux deposited in the low latitude precipitation zone for energy greater than 20 keV during magnetically disturbed periods as a result of wave-particle interactions involving ELF-VLF emissions of natural origin[26]. Thus, the VLF line radiation seems to contribute substantially to total daily global energy deposition of ~5x10^{19} erg (deg. latitude)$^{-1}$ (ref. 27) or 4x10^{20} ergs (ref. 28) via coherent first order gyro-resonance wave-particle interaction. Thus, it is concluded that like other whistler-mode ELF -VLF waves, the VLF power line harmonic radiation (PLHR) is also a strong and affective tool for the precipitation of the energetic (~ a few keV) electrons at high latitudes.

Rycroft[29] has pointed out that the power line harmonic radiation producing narrow-band whistler-mode radiation can cause precipitation of energetic electrons. Bullough[30] has suggested that the secular increase in thunderstorm activity over Canada could be partly due to increased power line radiation and associated charged particle precipitation.

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