Indian Journal of Radio & Space Physics
Vol. 35, February 2006, pp. 22-25

Probability of sideband generation at low latitudes

D P Singh
Department of Physics, F G M Government College Adampur (Hisar) 125 052
&
U P Singh
Department of Physics, S J M Institute of Engineering & Technology, Radaur (Yamunanagar) 135 133, India

Received 17 February 2005; revised 30 September 2005; accepted 30 November 2005

Possibility of sideband generation at low, middle and high latitudes is studied considering non-linear interaction between resonant electrons and very low frequency signals transmitted into the magnetosphere. It is shown that as L value increases, sideband spacing decreases and, at low L shells, frequency difference of 1500 Hz in interacting signals (i.e. from 2500 to 4000 Hz or from 4000 to 5500 Hz) produces insignificant spacing values. It is also shown that mean fractional spacing at low latitudes is more than maximum(observed) fractional spacing at L = 4, indicating that sideband generation at low latitudes has a remote possibility.

Keywords: Wave-particle interaction, Whistler waves, Wave propagation

PACS No.: 94.20.Rr, 52.35.Hr, 94.20.Bb

1 Introduction

Non-linear wave-particle interactions between whistler mode waves and energetic electrons in the magnetosphere have been a subject of many experimental as well as theoretical studies[1,2]. One clear evidence of the non-linear character of the magnetospheric wave amplification process is the fact that when a monochromatic wave is injected into the magnetosphere, the output wave often contains frequencies different from the transmitted frequency. In this case, under certain circumstances, a carrier wave (in whistler mode) transmitted into the magnetosphere is found to be accompanied by one or more quasi-constant frequency components (known as sidebands) close to the carrier frequency. These sidebands have typical spacing sidebands[1-4]. A lot of theories have been propounded for explaining this process[1-7] of sideband generation .

Recently Ikeda[3] examined the possibility of the sideband generation in whistler mode via a non-linear Doppler-shifted cyclotron interaction between untrapped electrons and the whistler mode carrier signal. The untrapped electrons resonate with the quasi-monochromatic whistler mode signal to generate sidebands as well as for broadening of the transmitted carrier frequency.

In this paper (i) the variation of spacing, i.e. gap width between central frequency and a sideband and (ii) possibility of sideband generation at low latitudes by adopting the theory put forth by Ikeda[3] are studied. It is observed that the spacing increases as McIlwain parameter (L, magnetospheric distances measured in unit of earth radius, 6372 km) decreases.

2 Method of calculation and ambient plasma density

In a recent paper, Ikeda[3] has shown that sideband spacing generated due to interaction between resonant electrons and transmitted/interacting whistler mode signal ($\omega_H > \omega$, ω_H being the angular cyclotron frequency of energetic electrons and ω, the wave angular frequency) can be expressed as

$$f_{SBS} = 0.0633\ n.\sqrt{(k.U_\perp.\omega_b)} \qquad \ldots (1)$$

where n is the order of sideband generation, k the wave number, $U_\perp$ the perpendicular speed of resonant electrons (such that $U_\perp = U_R \tan\alpha$, where U_R and α are the resonant velocity and pitch angle of the energetic electron, respectively) and ω_b the angular frequency of wave magnetic field. Wave number k can be calculated from the following formula

$$k = \omega.\mu/c \qquad \ldots (2)$$

where c is the speed of light and μ is known as refractive index of the medium, which can be computed from the following equation[8]

$$\mu = \sqrt{\{\omega_p^2/\omega\,(\omega_H - \omega)\}} \quad \ldots (3)$$

where ω_p is the plasma frequency of electrons. The resonant velocity of electrons (U_R) can be calculated from the equation

$$U_R = (\omega_H - \omega)/k \quad \ldots (4)$$

In the present study of sideband generation, equatorial half loss-cone pitch angle (α_0) is required, which can be calculated by adopting the following expression[8]

$$\sin^2\alpha_0 = E_m^3/(L^2\sqrt{(4L^2 - 3E_m L)} \quad \ldots (5)$$

where, $E_m = (R_E + H_m)/R_E$, R_E being earth radius and H_m the mirror height. The cold plasma densities (N_E) at $L = 1.1 - 5.0$ used in this paper are taken from Angerami and Thomas[9], and Angerami and Carpenter[10] corresponding to diffusive equilibrium model. These N_E values are given in Table 1. These ambient plasma density values have been used earlier by Singh *et al.* [11,12] and Singh[13,14] in studying interaction between electron cyclotron waves and resonant electrons in the magnetosphere.

3 Results and Discussion

Though the main objective of the present study is to determine the probability as to whether sideband generation phenomenon can occur at low latitudes, such a study is made at a range of L shells (between 1.1 and 5.0) so that a comparative study could be made comprising not only low latitudes (L = 1.1-1.7), but also mid (L = 2-3) and high latitudes (L = 4-5). The half loss-cone pitch angle (α_0) has been calculated by taking mirror height (H_m) to be 120 km above the earth's surface. Inan[8] and Singh[14] have shown that considering H_m values between 100 and 150 km brings no phenomenal change in α_0 values. The computed α_0 values along with cold plasma number density (N_E) at different L shells are given in Table 1, which shows that as L increases α_0 value decreases.

Since the aim is to study the gap (or spacing) between transmitted (central) frequency ($\omega/2\pi = f_{int}$) and sidebands, n = 1 only is considered; as for second order (n =2) sidebands, spacing will be double of the first order ones. Equation (1) clearly indicates that minimum spacing will be caused by low pitch angles, but as only those pitch angles for which $\alpha \geq \alpha_0$ will contribute towards wave growth, minimum spacing will be possible only when $\alpha = \alpha_0$. Here wave magnetic amplitude (B_w) is taken to be 1 pT, which is easily available in the considered L range of 1.1 - 5. Values of this minimum possible f_{SBS} computed for first order sideband generation are also given in Table 1 which vividly shows that:

(i) Whatever may be the value of interacting signal, spacing is highest at L = 1.1.
(ii) For a given f_{int}, as L value increases f_{SBS} decreases.
(iii) At a given L shell, f_{SBS} decreases as f_{int} increases.
(iv) It is notable that at a given shell of interaction and for any two consecutive frequencies, change in f_{SBS} value is constant, i.e. f_{SBS} (5.5 kHz) $- f_{SBS}$ (4 kHz) = Δf_{SBS} =f_{SBS} (4 kHz) $- f_{SBS}$ (2.5 kHz)

Table 1—Values of cold plasma density (N_E, in el.cm^{-3}), half loss-cone pitch angle (α_0, in radian) and sideband spacings (f_{SBS} in Hz, n = 1 and B_w = 1pT) which are computed at different transmitted frequencies for interacting resonant electrons of $\alpha = \alpha_0$

L	N_E	α_0	f_{SBS} for		
			f = 2.5 kHz	f = 4 kHz	f = 5.5 kHz
1.1	20000	1.007	67.64	67.58	67.52
1.3	4610	0.658	36.75	36.69	36.62
1.5	2710	0.488	24.53	24.46	24.39
1.7	1980	0.391	17.88	17.80	17.72
2.0	800	0.294	11.95	11.86	11.78
3.0	1000	0.1512	4.47	4.35	4.24
4.0	400	0.0955	2.17	2.02	1.85
5.0	182	0.0675	1.16	0.95	0.64

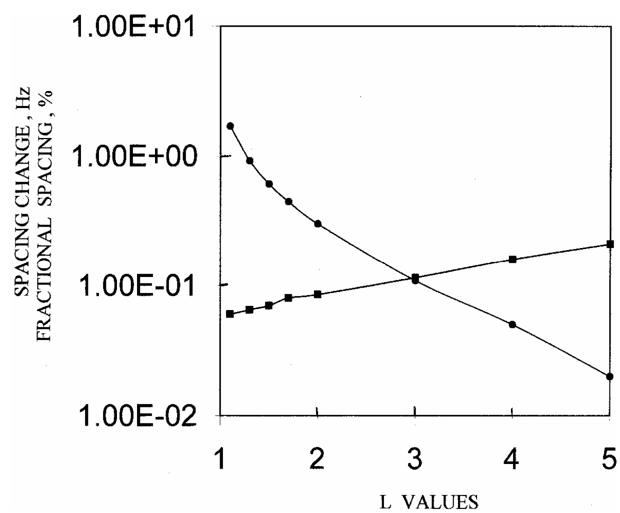

Fig. 1—Changes in sideband spacing minimum) [-■-] due to interacting signal frequency changes from 2500 to 4000(or 4000 to 5500 Hz), and minimum fractional side band spacing (-●-) at various L shells

Variation of Δf_{SBS} is depicted in Fig. 1, which shows that as L increases, Δf_{SBS} also increases. It suggests that at low latitudes, larger differences in interacting signal frequencies will be a necessary condition to observe sidebands around them, whereas at mid (above 45^0 geomagn. lat.) and high latitudes (geomagn. lat. of 60^0 and above) smaller differences in interacting signal frequencies may produce significant sidebands.

Now the computed f_{SBS} value is divided by the interacting frequency to compute minimum fractional spacing, which is comparedwith the experimental ones recorded at L = 4. The variation of this minimum fractional spacing (MFS) for representative signal of 4 kHz is also depicted in Fig. 1. It is evident from Fig. 1 that as L increases MFS value decreases, which suggests that sideband spacing will be larger at low latitudes and smaller at high latitudes. Park[2] has reported sideband spacings after analyzing data of VLF transmitter experiments conducted at Siple, Antarctica
(L = 4). He found that very low frequency (3-30 kHz) signals injected into the magnetosphere often generate sidebands in the spacing range of 2-100 Hz (though in his Fig. 5 he included only the data with spacing range 25-65 Hz) and were a consequence of non-linear wave-particle interaction. Because of above reason we consider data recorded at L= 4 as standard, to study the possibility of sideband generation at low latitudes. Park[2] found that minimum sideband spacing at L= 4 was 2 Hz. As is clear from Table 1, the present method of calculation too produces minimum sideband spacing of 2 Hz at L= 4. From Fig. 1 it is seen that in the L range of 1.1-2, MFS is in the range of 0.34-1.34%, which is quite unbelievable, as at L= 4 it is only 0.05%, indicating that sideband generation at low latitudes is an impossible process. Park[2], in his Fig. 5, has shown that average spacing at L= 4 is ~45 Hz, which is 22.5 times that of minimum spacing observed at this shell. We multiply all minimum f_{SBS} values (as presented in Table 1) by 22.5 to calculate average spacings at different L shells and show them in Table 2. This Table 2 also shows the mean (or average) fractional spacing. The experimental values of mean and maximum fractional spacing (as observed at L = 4) are 1.0% and 2.2% whereas even the mean spacing in the L range of 1.1-2.0 is greater than 2.2%, suggesting that sideband generation at low latitudes is a remote possibility.

Table 2—Possible mean values of f_{SBS} and fractional spacing computed at various L shells for interacting signal of 4 kHz

L	f_{SBS} (mean) Hz	Fractional spacing (mean) %
4.0	45	1.1
3.0	98	2.5
2.0	267	6.7
1.7	401	10.0
1.5	550	13.8
1.3	826	20.7
1.1	1521	38.0

Acknowledgements

One of the authors (DPS) is grateful to the University Grants Commission, New Delhi, for providing him financial assistance in the form of a minor research project as well as to Commissioner of Higher Education, Haryana. The authors are thankful to Dr M Ikeda of Mushasi University, Japan for providing literature. Thanks are also due to the principals of the respective colleges for providing research facilities to the authors.

References

1 Matsumoto H, *Non-linear whistler mode interaction and triggered emissions* in *the magnetosphere: A review in Wave Instabilities in Space Plasmas*, edited by P J Palmadesso and K Papadopoulos, (D Reidel, Dordrecht, The Netherlands) 1979, p 163.

2 Park C G, Generation of whistler mode sidebands in the magnetosphere, *J Geophys Res (USA)*, 86 (1981) 2286.

3 Ikeda M, Approximate derivation of self-exciting whistler mode sideband wave frequencies, *Indian J Radio & Space Phys*, 31 (2002) 121.

4 Rastani K,Inan U S & Helliwell R A, DE 1 observations of Siple transmitted signals and associated sidebands, *J Geophys Res (USA)*, 90 (1985) 4128.

5 Brinca A L, Whistler sideband growth due to non-linear wave-particle interaction, *J Geophys Res* (*USA*), 77 (1972) 3508.

6 Karpman V I, Istomin J A N & Shklyar D R, Nonlinear frequency shift and self-modulation of the quasi-monochromatic whistlers in the inhomogeneous plasma (magnetosphere), *Planet & Space Sci* (*UK*), 22 (1974) 859.

7 Chang D C D, Helliwell R A & Bell T F, Sideband mutual interactions in the magnetosphere, *J Geophys Res (USA)*, 85 (1980) 1703.

8 Inan U S, *Tech Report No 3414-3*, Stanford Univ, Stanford, California, USA, 1977.

9 Angerami J J & Thomas J O, *Tech Report No 3412-3*, Stanford Univ, Stanford, California, USA, 1963.

10 Angerami J J & Carpenter D L, Whistler studies of the plasmapause in the magnetosphere, 2 Equatorial density and total electron content near the knee in the magnetospheric ionization, *J Geophys Res* (*USA*), 71 (1966) 7110.

11 Singh D P, Singh U P & Singh R P, Intensity peaks in low latitude VLF emissions observed at Ariel satellites, *Planet & Space Sci* (*UK*), 40 (1992) 1425.

12 Singh D P, Singh U P & Singh R P, Limiting wave growth for electron cyclotron waves, *Earth Moon & Planets* (*The Netherlands*), 64 (1994) 145.

13 Singh B, On the ground observation of whistlers at low latitudes, *J Geophys Res* (*USA*), 81 (1976) 2429.

14 Singh D P, L-dependence of trapped electron diffusion by ELF waves, *Indian J Radio & Space Phys,* 21(1992) 250.