Correlation of the long-term cosmic ray intensity variations with sunspot numbers and tilt angle

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Based on the monthly data of sunspot numbers (SSN), tilt angle (TA) and cosmic ray intensity (CRI), a detailed correlative analysis has been performed to study the relationship of CRI (observed by the neutron monitor stations having different cut-off rigidity) with SSN and TA for the period 1976-2005, covering solar cycles 21, 22 and 23. It is an observed fact that SSN and TA are highly correlated with each other and cosmic ray intensity shows anti-correlation with them. The present analysis is found to support the earlier findings and holds good for solar cycles 21, 22 and current cycle 23. Further, the cross-correlation coefficients between CRI-SSN and between CRI-TA have been obtained considering the time-lag factor and it is found that time-lag is larger for odd solar cycles in comparison to even cycles. The odd-even behaviour in relation to time-lag is also evident from the hysteresis curve between CRI-SSN and between CRI-TA, where wider loops for odd solar cycles have been observed. Moreover, we find that correlation between CRI and SSN as well as between CRI and TA is better during negative polarity than the positive polarity of solar magnetic cycle. From the tilt angle observations, it has been noticed that the behaviour of cycle 23 in declining phase is different from that of cycle 21 and 22.

Keywords: Cosmic ray intensity, sunspot numbers, tilt angle

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1 Introduction

The long-term behaviour of cosmic rays in relation to solar activity has been extensively studied by many authors at different epochs. Now it is an established fact that galactic cosmic rays are inversely correlated with sunspot numbers, having their maximum intensity at the minimum of the sunspot cycle. The cosmic ray intensity (CRI) curve also appears to follow a 22-year magnetic cycle with alternate maxima being flat-topped and peaked as predicted by models of cosmic ray modulation. These models were based on the observed reversal of the Sun’s magnetic field polarity after every 11-year cycle and curvature and gradient drifts in the large-scale magnetic field of the heliosphere.

Recently, features of the interplanetary medium have been explained on the basis of heliospheric neutral current sheet, which separates the entire heliosphere into the two regions of opposite magnetic field polarity. In each hemisphere, the field is well approximated by a Parker Archimedian Spiral, where the field being outward in one hemisphere and inward in the other. The field direction in each hemisphere changes in every ~ 11-year sunspot cycle. At the solar minimum, the current sheet is nearly equatorial having magnetic field of northern hemisphere in one direction and the southern magnetic field having the opposite sign. The solar magnetic field structure becomes complex near the sunspot maxima with the increase in the inclination of the current sheet. The inclinations of the heliospheric neutral current sheet with respect to the equatorial plane of heliosphere are often named as tilt angle. The waviness of neutral current sheet, i.e. tilt angle has been used as a key index by various investigators to explain the long-term modulation of cosmic rays. The tilt angle ($\alpha$) is computed by averaging the maximum latitude through the neutral line in the north and south hemisphere during each Carrington rotation. The heliospheric neutral current sheet (HCS) and its waviness provides us basic physical mechanism to explain the long-term modulation of galactic cosmic rays.

In the present paper, an attempt has been made to study the correlative behaviour of CRI with SSN and TA. The detailed correlative analysis has been
performed considering the time-lag factor in the light of solar magnetic cycle. The results of time-lag analysis have been further verified by the different shape of the hysteresis loops for odd and even cycles as well as for the period of negative and positive polarity of the solar magnetic field.

2 Data and method of analysis
In the present work, waviness of heliospheric neutral current sheet (tilt angle) has been considered as a key parameter to study the modulation of cosmic rays in relation to the SSN and CRI for the period 1976-2005. To study the rigidity dependence of cosmic ray modulation, monthly mean values of neutron monitor stations of different cut-off rigidity (Oulu, \( R_c \sim 1 \) GV, Kiel, \( R_c \sim 3 \) GV and Huancayo, \( R_c \sim 13 \) GV) have been used, whereas, the values of tilt angle was obtained from the Wilcox solar observatory (WSO, classical model).

The correlation coefficient between CRI-SSN and between CRI-TA with time-lag has been calculated for the solar cycles 21, 22, and 23 using “minimizing correlation coefficient method”. In this method, initially CRI-SSN and CRI-TA series have been selected for the same period (zero time-lag) and then CRI series has been shifted by a step of one month (i.e. one month time-lag) to calculate the value of cross-correlation coefficient between both the series and so on. Similarly, the SSN and TA series has also been shifted by a step of one month to calculate the value of cross-correlation coefficient for different time-lags. As such, the time (number of shifted months) is obtained, where the anti-correlation coefficient is maximum, which is termed as the time-lag between both the series. Moreover, probable error for each value of correlation coefficient has been calculated by the formula:

\[ P.E. = 0.6745 \left(1 - r^2 \right) / \sqrt{N} \]

where \( r \) is the correlation coefficient and \( N \) the size of sample.

To obtain hysteresis curves, 30-months (1/4 of the average 11-years cycle) moving average of the % CRI, SSN and TA data series have been calculated\(^\text{17}\). \n
3 Results and discussion
The solar modulation of cosmic rays with regard to various solar activity parameters (such as SSN and TA) has been the topic of great interest for many authors\(^\text{18-20}\). The inverse correlation between TA and CRI along with 22-year variational patterns in TA has been reported for the past solar cycles. Here, an attempt has been made to extend the study for recent period to assess the relationship of SSN and TA to CRI considering low (Oulu \( R_c \sim 1 \) GV), middle (Kiel \( R_c \sim 3 \) GV) and high (Huancayo \( R_c \sim 13 \) GV) cut-off rigidity neutron monitor stations for the period 1976-2005, which covers solar cycle 21, 22, and 23. The authors have further examined the effect of positive and negative polarity of solar magnetic cycle on the degree of correlation between the said parameters.

To observe the variation in CRI at different cut-off rigidity stations with TA, the monthly mean value of CRI (in %) for Oulu (\( R_c \sim 1 \) GV), Kiel (\( R_c \sim 3 \) GV) and Huancayo (\( R_c \sim 13 \) GV) has been used from 1976 to 2005. The general trend of inverse correlation between TA and % CRI (normalized to 100% in May 1965, being the maximum count rate of neutron monitors) for all the three stations during the entire period of investigation is shown in Fig. 1. Taking into account the similar variational pattern of low to high cut-off rigidity stations, the monthly mean value of Kiel (\( R_c \sim 3 \) GV), a middle cut-off rigidity neutron monitor station has been considered for further study.

The variation of CRI (Kiel) and TA alongwith SSN from 1976 to 2005 is clearly seen in Fig. 2. While, the increase in TA was remarkably similar during the rising phase of the last three cycles (Fig. 2), HCS evolution seems to differ on the decline of even and odd-numbered solar cycles. Specifically, the TA appears to collapse towards low angles more rapidly during the decline of even-numbered cycles, such as during cycle 22 (peak in ~1990).

From Fig. 2, it is evident that the variations in SSN and TA are in close correspondence with one another, whereas, CRI is inversely correlated with both the parameters, having some time-lag during the entire period of investigation. Moreover, the degree of correlation has been obtained by calculating the cross-correlation coefficient between CRI and SSN as well as between CRI and TA. The results of correlative analysis (without and with time-lag) between said parameters are presented in Table 1. It is clearly observed that the time-lag between CRI-TA is larger than the CRI-SSN for odd solar cycles and smaller (and similar) for even cycles. The correlation coefficients between CRI-TA and between CRI-SSN with different time-lag and statistical error bars for solar cycles 21, 22 and 23 are also shown in Figs 3 and 4, respectively.
Fig. 1—Long-term variation of cosmic ray intensity (%) observed at Oulu, Kiel & Huancayo neutron monitors with tilt angle (degree) from 1976 to 2005.

Fig. 2—Monthly variation of cosmic ray count rates observed at Kiel neutron monitor with tilt angle (degree) and sunspot numbers from 1976 to 2005 [Rectangle boxes (at top) between positive and negative polarity shows the mixed polarity of the solar magnetic cycle.]
A distinction between characteristic behaviour of even and odd solar cycles, as well as their declining and ascending phases give an information towards the different modulation mechanism operating during even and odd cycles. As far as the solar activity is concerned, there are symmetric and asymmetric cycles, where generally the rise is faster with longer declining phase. Considering the cosmic ray

Table 1—Correlation coefficient between CRI-SSN and between CRI-TA without and with time-lag for the solar cycles 21 to 23

<table>
<thead>
<tr>
<th>Solar cycle (SC)</th>
<th>Correlation Coefficient (without time-lag)</th>
<th>Maximum anti-correlation coefficient (with time-lag)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CRI-SSN</td>
<td>CRI-TA</td>
</tr>
<tr>
<td>SC - 21</td>
<td>-0.539</td>
<td>-0.495</td>
</tr>
<tr>
<td>SC - 22</td>
<td>-0.882</td>
<td>-0.908</td>
</tr>
<tr>
<td>SC - 23</td>
<td>-0.772</td>
<td>-0.614</td>
</tr>
</tbody>
</table>

Fig. 3—Correlation coefficient factor between cosmic ray intensity (Kiel) and tilt angle with different time-lags for solar cycles 21, 22 and 23 [The statistical error bars for the each value of correlation coefficient are also shown.]

Fig. 4—Correlation coefficient factor between cosmic ray intensity (Kiel) and SSN with different time-lags for solar cycles 21, 22 and 23 [The statistical error bars for the each value of correlation coefficient are also shown.]
modulation during solar cycles, the shape of cosmic ray curves for the even cycles differs systematically from the shape of odd cycles. The odd cycle is characterized by a simple and relatively smooth increase to the maximum (about 7.5 years), whereas the even cycles on the average are characterized by two maxima. The first maximum is observed to be relatively rapid, after the previous minimum in CRI (3-4 years). The second and well developed maxima tend to occur at the same time in the cycle as the maxima of the odd cycles. The rapid recovery of CRI following even numbered cycles leads to initially flat-topped peak. The relatively sluggish rise in CRI during the onset of the succeeding odd-numbered cycles further ensures a relatively long duration with high CRI. Conversely, the slow recovery of CRI following odd-numbered solar cycles and better anti-correlation between SSN and CRI during the subsequent even-numbered cycles favours a high peak of CRI maximum. The differences in odd-even cycles has been explained in terms of propagation mechanism as during even cycles, convection plays the most dominant role, while diffusion dominates during the odd cycles.

The hysteresis curves between CRI and SSN as well as between CRI and TA have been plotted for solar cycles 21 and 22 (Figs 5 and 6). The broad hysteresis loops for the odd cycles have been observed, whereas, they are narrow in the case of even cycles. This interchange of thick and thin cycles seems to be related with the change in general solar magnetic field polarity. The broad loops

![Fig. 5](image_url)  
**Fig. 5**—Hysteresis curve between CRI (Kiel) and SSN for the solar cycles 21 and 22

![Fig. 6](image_url)  
**Fig. 6**—Shows the hysteresis curve between CRI (Kiel) and TA for the solar cycles 21 and 22
correspond to (+/-) reversals and narrow ones correspond to (-/+). 

Nagashima and Morishita\(^1\) has recognized that if the CRI near the Earth is higher under negative polarity and the relationship between solar indices and CRI is similar for different polarities, then the (+/-) transition would broaden the hysteresis loop and the (-/+). This transition would make it narrower.

Furthermore, the values of time-lag between CRI-SSN and CRI-TA for positive and negative polarity have been calculated, which are presented in Table 2. It has been found that the time-lag between CRI-SSN is greater for negative polarity and smaller for positive polarity, whereas, time-lag between CRI-TA is greater for positive polarity and smaller for negative polarity. To verify these results the authors have plotted the hysteresis curves between CRI-SSN and CRI-TA for positive and negative polarity of the solar magnetic cycle (Figs 7 and 8). The hysteresis loops between CRI-SSN are wider for negative polarity and narrow for positive polarity of the cycle (Fig. 7). These results have also been checked for the period 1960-1980 (one complete solar magnetic cycle). The hysteresis loops between CRI-TA are wider for positive polarity than that for negative polarity of the cycle (Fig. 8).

Moreover, polarity based scattered graphs between CRI-SSN and CRI-TA have been plotted for the period January 1982-August 1990 (\(qA < 0\), i.e. negative polarity) and April 1992-December 1998 (\(qA > 0\), i.e. positive polarity). It has been found that the correlation coefficient between CRI-SSN and between CRI-TA is ~ 0.805 and ~ 0.906, respectively, for \(qA < 0\), whereas, it is ~ 0.77 and ~ 0.512 (Table 2) respectively for \(qA > 0\) (Figs 9 and 10). It is evident that the correlation is better during the negative polarity than the positive polarity of the solar magnetic cycle.

The current sheet, which divides the heliosphere into northern and southern parts provides an additional cosmic ray transport in radial and latitudinal directions. In the drift mechanism of cosmic-ray modulation, positively charged cosmic rays preferentially enter the heliosphere from the

<table>
<thead>
<tr>
<th>Polarity of the solar cycle (SC)</th>
<th>Correlation coefficient (without time-lag)</th>
<th>Maximum anti-correlation coefficient (with time-lag)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRI-SSN Correlation coefficient</td>
<td>CRI-TA Correlation coefficient</td>
<td>CRI-SSN Max. anti-correlation coefficient</td>
</tr>
<tr>
<td>CRI-TA Max. anti-correlation coefficient</td>
<td>Time-lag, month</td>
<td></td>
</tr>
<tr>
<td>Negative polarity ((qA &lt; 0))</td>
<td>– 0.805</td>
<td>– 0.906</td>
</tr>
<tr>
<td>Positive polarity ((qA &gt; 0))</td>
<td>– 0.7705</td>
<td>– 0.512</td>
</tr>
</tbody>
</table>

It can be observed from the table that the correlation coefficient is higher for negative polarity than for positive polarity. This indicates that the cosmic rays are transported more efficiently during negative polarity.

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**Fig. 7**—Hysteresis curve between CRI (Kiel) and SSN for positive and negative polarity of the cycle

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**Fig. 8**—Hysteresis curve between CRI-TA and SSN for positive and negative polarity of the cycle
direction of the solar poles during $qA > 0$ cycles (corresponding to times when the polarity of the solar magnetic field is outward in the northern hemisphere) such as $\sim 1970$-$1980$ and $\sim 1990$-$2000$. During $qA < 0$ periods such as $\sim 1960$-$1970$ and $\sim 1980$-$1990$, when the solar field polarity is reversed, cosmic rays (positively charged) approach the Sun along the HCS. During $qA > 0$ period, it might be expected that incoming cosmic rays will be less affected by drift effects associated with an increase in the tilt angle at the beginning of a solar cycle (odd-numbered) or by diffusion associated with enhanced coronal mass ejection (CME) activity. These CMEs, which are thought to be a key element in diffusion/convection-based pictures of modulation, are characteristically confined to the Sun’s equatorial regions early in the solar cycle and appear at higher latitudes during the course of the cycle, as the streamer belt at the base of the HCS moves poleward. At the beginning of even-numbered cycles ($qA < 0$), when cosmic rays approach the Sun along the HCS, they will be more readily affected by changes in the tilt angle and
low-latitude CMEs. Thus, the difference in the responsiveness to solar activity changes at the onset of even- and odd-numbered solar cycles is consistent with a drift effect.

The results provide a new insight regarding the CRI variation associated with sunspot numbers and tilt-angle for recent solar cycles. The nature/degree of correlation of cosmic rays with solar activity parameters (SSN and TA) during different phases and polarity of solar cycle is of significance to understand the propagation mechanism of cosmic ray modulation.

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Fig. 10—Scattered graph between CRI-SSN and CRI-TA for the positive polarity (April 1992-December 1998, qA > 0) [The CRI data has been taken from the mid-latitude Kiel neutron monitor station.]


