Theoretical analysis on circular sector microstrip antennas

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A rigorous theoretical analysis of a circular sector microstrip antenna with sector angle $\alpha$ is carried out. General expressions for radiated fields by antenna are derived by applying cavity model based modal expansion technique and are used to study its radiation parameters. Similar to a circular disk antenna, better results for circular sector antenna are also obtained in TM$_{11}$ mode of excitation. The computed results of a sector antenna with $\alpha = 60^\circ$ are compared with simulation results and a difference of 2.5%, 38%, 7% and 1.9%, respectively in resonance frequency; input impedance, directivity and bandwidth is recorded.

Keywords: Microstrip antenna, Circular sector microstrip antenna, Cavity model and radiation patterns, Resonance frequency; Input impedance, Directivity, Bandwidth
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1 Introduction

Different workers have extensively analyzed circular patches of microstrip antennas in recent years under different conditions¹⁻³. However geometrical constraints become vital when these radiators are needed for conformal mounting on pre-existing structures. In that case utilization of other shapes, which are not circular, becomes more advantageous. Annular ring⁴, gap open ring⁵, annular sector⁶ and ring slot antennas⁷ have been investigated by few workers but very little effort to analyze circular sector antennas have been made. In this paper, a general circular sector antenna with sector angle $\alpha$ is investigated theoretically by considering infinite ground plane dimensions. Fields inside the cavity are evaluated and expressions for the far zone radiated fields by antenna are derived by using vector potential technique. The results of this radiator in dominant mode are computed and compared with the simulation results obtained with IE3D simulation software. A fairly good agreement between computed and simulated results validates the present technique of treating a circular sector microstrip antenna.

2 Theoretical analysis

A circular sector microstrip antenna with coordinate system is shown in Fig. 1. It consists of a planer circular sector of sector angle $\alpha$, radius $\alpha$ on a thin dielectric substrate of thickness $h$, relative substrate permittivity $\varepsilon_r$ and loss tangent $\tan \delta$. Out of different analytical techniques⁸, present geometry is
analyzed by applying cavity model based modal expansion technique, because it is simple in application and reliable results may be obtained. Since the substrate thickness \( h \ll \lambda_0 \), the fields within the substrate do not vary along the \( z \)-direction and component of current normal to the edge of antenna approaches zero at the edges. Under this condition, present structure supports only TM\(_{nm}\) modes. With this assumption, antenna is considered as a circular resonator with magnetic sidewalls bounded at its top and bottom electric walls. Many modal waves may get excited when a cable feeds such an antenna. A uniform current of effective width 2\( w \) centered on the feed axis at a distance \( d \) from the center of the patch is considered flowing from the ground plane to the patch.

Let \( \psi_{m \nu} \) be the eigen function of the homogeneous wave equation\(^9\) and \( k_{m \nu} \) is the eigen values of wave number in the dielectric substrate. Assuming eigen functions to be orthogonal, the solution of wave equation for \( E_z \) in the cavity with excitation current \( J_z \) in \( z \)-direction will be\(^9\)

\[
E_z = j \omega \mu_0 \sum_{m} \sum_{\nu} \frac{1}{k_z^2 - k_{m \nu}^2} \psi_{m \nu} \psi_{m \nu}^* \psi_{m \nu} \quad \text{(1)}
\]

with

\[
k = k_0 \sqrt{\varepsilon_r[1 - j \tan(\delta)]} \quad \text{(2)}
\]

If we consider that \( m, \nu \)th cavity mode for a circular sector microstrip antenna is\(^9\)

\[
\psi_{m \nu} = J_n(K_{m \nu} \rho) \cos(\phi') \quad \text{(3)}
\]

then

\[
\langle \psi_{m \nu} \psi_{m' \nu} \rangle = \int_0^{2\pi} \int_0^1 J_n(k_{m \nu} \rho) J_n(k_{m' \nu} \rho) \rho d\rho d\phi' \quad (4)
\]

\[
= \frac{h}{4} [J_n(k_{m \nu} a)]^2 \left[ a - \frac{\nu^2}{k_{m \nu}} \right] \left[ \alpha + \sin(2\nu \alpha) \right] \quad (5)
\]

Let the patch is excited in such a way that the input filamentary current at feed location \((d, \phi'')\) is

\[
\langle J_z \psi_{m \nu} \rangle = \int_{\phi'' - \pi/2}^{\phi'' + \pi/2} \int_0^h f_z(\psi) \frac{J_n(k_{m \nu} d)}{d} \cos(\nu \phi') d\psi'' dx \quad (6)
\]

\[
= 2 J_h \frac{\sin(\nu \psi)}{\nu} \cos(\nu \phi'') J_n(k_{m \nu} d) \quad \text{(7)}
\]

On substituting Eqs (4) and (5) in Eq. (1), the solution of electric field \( E_z \) which satisfies the boundary conditions, is given by

\[
E_z = - j \omega \mu_0 \sum_{m} \sum_{\nu} \frac{2 J_n(\nu \psi) \cos(\nu \phi'')}}{\nu} J_n(k_{m \nu} d) \quad (8)
\]

Here \( J_n(k_{m \nu} \rho) \) is the cylindrical function of first kind of Bessel’s function of order \( \nu \) and

\[
v = n \quad \text{if} \quad \alpha = 2\pi \quad \text{and} \quad \alpha = \frac{n \pi}{\alpha} \quad \text{otherwise} \quad (9)
\]

The resonance frequency of antenna is determined by

\[
f_r = \frac{\alpha}{2\pi c \sqrt{\varepsilon_0}} \quad (10)
\]

The dynamic dielectric constant \( \varepsilon_d \) used in Eq. (10) is calculated following an already existing method\(^{10-11}\) with a change that in the expressions of \( C_{0, \text{dyn}} \) and \( C_{\text{e,stat}} \), area of patch \((\pi a^2)\) is replaced by \( \frac{\alpha a^2}{2} \) and \( C_{e, \text{dyn}} \) is taken as

\[
C_{e, \text{dyn}} = \frac{1}{\alpha} \int_{0}^{\alpha} C_{e, \text{stat}} \cos(\nu \phi)^2 d\phi \quad (11)
\]

In this way, the resonance frequency of antenna now becomes a function of sector angle. The term \( \varepsilon_d \)
is already a function of \( \alpha \), in addition to patch radius and substrate thickness.

For all the geometries considered in this paper, feed location and flowing filamentary current \( (J_z) \) are taken as \((d, 0)\) and unity, respectively. The computed and simulated resonance frequencies of different sector antennas in \( \text{TM}_{nm} \) mode of excitation are shown in Table 1. Similar to a circular disk antenna, the resonance frequencies of all the geometries considered in this paper are minimum in \( \text{TM}_{11} \) mode of excitation. It is also found that simulated resonance frequency of each geometry, reasonably matches with the calculated resonance frequency of corresponding geometry. The resonance frequencies of a circular disk antenna and a semicircular disk antenna are identical in all modes of excitations. On keeping all parameters identical except sector angle \( \alpha \), it is found that resonance frequency of circular sector geometry decreases on increasing \( \alpha \). On an average, a difference of nearly 2.5% between calculated and simulated resonance frequencies of circular sector antennas in \( \text{TM}_{11} \) mode of excitation is recorded.

By applying equivalence principal and image theory, components of equivalent magnetic current sources \( M \) as shown in Fig. 1(b), i.e. \( M_\phi \), which represents outer \( M \) source associated with curved aperture and equivalent source associated linear apertures \( (M_{\rho 1} \text{ and } M_{\rho 2}) \). The \( M \)-values are applied to compute the radiation patterns in \( E (\phi = 0^\circ) \) plane and \( H (\phi = 90^\circ) \) plane by applying standard method.

The calculated \( E \)-plane patterns of two circular sector microstrip antennas \((\alpha = 60^\circ \text{ and } 90^\circ)\) are compared with a circular patch antenna in Fig. 2. Similar patterns in \( H \)-plane are obtained, which are omitted here. All the parameters except sector angle \( \alpha \)

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### Table 1—Resonance frequencies of different CSMA geometries in their dominant modes

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Geometry Type (Sector Angle)</th>
<th>Radius, mm</th>
<th>Mode ( \text{TM}_{nm} )</th>
<th>Calculated Resonance frequency, GHz</th>
<th>Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>60°</td>
<td>30</td>
<td>( \text{TM}_{30} )</td>
<td>4.171</td>
<td>4.12</td>
</tr>
<tr>
<td>2.</td>
<td>90°</td>
<td>30</td>
<td>( \text{TM}_{30} )</td>
<td>3.032</td>
<td>-</td>
</tr>
<tr>
<td>3.</td>
<td>180°</td>
<td>30</td>
<td>( \text{TM}_{31} )</td>
<td>2.885</td>
<td>2.92</td>
</tr>
<tr>
<td>4.</td>
<td>270°</td>
<td>30</td>
<td>( \text{TM}_{30} )</td>
<td>2.68</td>
<td>2.81</td>
</tr>
<tr>
<td>5.</td>
<td>355°</td>
<td>30</td>
<td>( \text{TM}_{30} )</td>
<td>2.64*</td>
<td>2.65*</td>
</tr>
<tr>
<td>6.</td>
<td>360°</td>
<td>20</td>
<td>( \text{TM}_{30} )</td>
<td>1.828</td>
<td>-</td>
</tr>
<tr>
<td>7.</td>
<td>360°</td>
<td>16.64</td>
<td>( \text{TM}_{11} )</td>
<td>2.61**</td>
<td>2.7**</td>
</tr>
</tbody>
</table>

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### Table 2—Total efficiency and bandwidth of circular disc in dominant mode

<table>
<thead>
<tr>
<th>Substrate permittivity, ( \varepsilon_r )</th>
<th>Reported ( (a), \text{cm} )</th>
<th>Calculated</th>
<th>Bandwidth, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>2.362</td>
<td>84.2</td>
<td>84.9</td>
</tr>
<tr>
<td>4.4</td>
<td>2.634</td>
<td>75.2</td>
<td>79.06</td>
</tr>
</tbody>
</table>

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... (11)

![Fig. 2—Calculated E-plane radiation patterns of CSMA geometries in their dominant mode of excitation](image)
Fig. 3—$E$-plane radiation patterns with three CDMAS geometries at $\alpha = 60^\circ$

Fig. 4—$H$-plane radiation patterns with three CDMAS geometries at $\alpha = 60^\circ$

with

$$X_i(f) = \frac{377 f h}{c} \log \left( \frac{c}{\pi f d_i \sqrt{\varepsilon_r}} \right) \quad \ldots (12)$$

Here, $Q_t$ the total quality factor is a function of sector angle $\alpha$ and includes the effect of radiation losses, dielectric losses and conductor losses.

The variation of normalized input impedance with frequency of a circular sector microstrip antenna with sector angle $\alpha = 60^\circ$ is compared with that obtained by simulation software in Fig. 5. Based upon these calculations, directive gain of antenna in $H$- and $E$-planes and return loss of antenna are computed and compared with simulation results in Figs 6 and 7, respectively.

Directive gain; quality factor, radiation efficiency and bandwidth of antenna are also determined for circular sector antennas in TM$_{11}$ mode of excitation and are compared with the corresponding simulated results in Table 2.

3 Results and discussion

The theoretical calculations of different radiation parameters of a general sector antenna are carried out in TM$_{nm}$ mode of excitation. The validation of computed results is done with the help of simulation software IE3D. The design parameters used in computation are mentioned in Table 1. All the design parameters except sector angle $\alpha$ for all the geometries are kept identical. The computed $E$-plane radiation patterns for different sector geometries shown in Fig. 2 indicate that on increasing sector angle, 3 dB beam width of antenna decreases or directivity of antenna decreases, which is also shown in Table 2 for different sector geometries. The computed and simulated $E$- and $H$-plane radiation patterns for a sector microstrip antenna with ($\alpha = 60^\circ$) are shown in Figs 3 and 4, respectively. These curves show a good agreement between simulated and computed results.
The computed and simulated input impedance of a circular sector antenna ($\alpha = 60^\circ$) in TM$_{11}$ mode of excitation is shown in Fig. 5 for the frequency range 3.87-4.47 GHz. On substituting $\alpha = 2\pi$ and $v = n$ in Eq. (9), fields inside the cavity of a circular disk antenna may be obtained, which in turn may be applied to obtain its input impedance. The directive gain of a circular sector antenna ($\alpha = 60^\circ$) in $H$- and $E$-planes are shown in Fig. 6. A small difference of 0.47 dB (5.8%) between calculated and simulated values in broadside direction is recorded. The computed return loss variation shown in Fig. 7 indicates that circular sector antenna ($\alpha = 60^\circ$) is resonating at 4.17 GHz. A difference of around 50 MHz (~ 1.2% variation) between computed and simulated resonance frequency is recorded, which can be tolerated, since computational analysis carried out and simulation software applied in this work are based on different theoretical models and they have their own working limitations. The variation of input impedance of a circular disk antenna with frequency with specified feed location is shown in Fig. 8. Computed and simulated input impedances of circular sector antennas at their respective resonance frequencies are shown in Table 3.
Computed directivity values of some sector antennas in $TM_{11}$ mode are also included in Table 3. It indicates that except for a circular disk antenna, directivity of circular sector antenna in general decreases on increasing $\alpha$ value. Same conclusion may be drawn for radiation efficiency and bandwidth of circular sector antennas. A difference of 7% and 1.9% between computed and simulated radiation efficiencies and bandwidth values is recorded for circular sector antenna with $\alpha = 60^\circ$.

The computed and simulated resonance frequencies, directivities, radiation efficiencies and bandwidth values of circular sector antenna with $\alpha = 60^\circ$ are in close agreement and a reasonably good matching between 50 ohm probe and antenna may also be achieved with mentioned feed point. The resonance frequency of different other geometries are also in close agreement with the simulated resonance frequencies (Table 1) though other results for these geometries are not validated. This suggests that the present technique may be applied to treat any circular sector antenna to a reasonably good accuracy.

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References