Numerical modelling of tidal circulation and tide-induced water level variation in Bombay harbour

V Abrol

Health Physics Division, Bhabha Atomic Research Centre, Trombay, Bombay 400 085, India

Received 30 August 1989; revised 16 March 1990

A nonlinear hydrodynamic numerical model has been applied to Bombay harbour waters subjected to tidal forcing at the mouth of the harbour. Buildup of water level and associated current pattern of the circulation have been studied. Modification of flow-field due to presence of Elephanta Island in the harbour is studied by excluding and including the Island in the computational grid for a typical tidal cycle. The study brings out the utility of numerical modelling technique in estimating tidal dynamics of semi-enclosed water bodies.

Prediction of tidal characteristics of water body of Bombay harbour is important for navigation, flushing properties, etc. The prediction of water movement is needed for water quality and monitoring studies. The numerical model described in this paper computes vertically integrated longitudinal and lateral velocity patterns as well as tidal elevation distribution. Time-history and spatial distribution of dynamical parameters at various epochs of the tidal forcing at the mouth of the harbour at Apollo Pier are estimated and presented graphically.

Mathematical Formulation

The basic equations for the tidal model are the shallow water equations. The coordinate system used has \( x \) and \( y \) horizontal coordinates increasing eastward and northward respectively. The equations of motion are

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - f v = - g \frac{\partial \eta}{\partial x} + \frac{k \rho u (u^2 + v^2)^{1/2}}{\rho}
\]

and

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu = - g \frac{\partial \eta}{\partial y} + \frac{k \rho v (u^2 + v^2)^{1/2}}{\rho}
\]

where \( u \) and \( v \) are corresponding depth averaged velocity components; \( g \), acceleration due to gravity; \( \tau_x, \tau_y \), wind stress component; \( k \), bottom friction coefficient taken as 0.0028; \( f \), Coriolis parameter \( (4.73 \times 10^{-7} \text{ sec}^{-1}) \); \( \rho \), water density \( (1.025 \times 10^3 \text{ g.m}^{-3}) \); \( \eta \), free surface elevation with respect to mean water level; and \( h \), depth of water.

The equation of mass continuity is

\[
\frac{\partial \eta}{\partial t} + \frac{\partial ((\eta + h)u)}{\partial x} + \frac{\partial ((\eta + h)v)}{\partial y} = 0
\]

For subsequent numerical treatment, Eqs (1) and (2) are expressed in flux form as

\[
\delta \eta/\delta t + \delta [(\eta + h)u]/\delta x + \delta [(\eta + h)v]/\delta y = 0
\]

and

\[
\delta [(\eta + h)u]/\delta t + \delta [(\eta + h)v]/\delta y - f(\eta + h)v
\]

\[
= - g(\eta + h)\delta \eta/\delta x + \tau_x/\rho - k u (u^2 + v^2)^{1/2}
\]

The boundary conditions applicable to above equations are specification of temporal evolution of
tidal elevation at the open boundary and discharge of water into the system along the other boundaries. On all side-walls the velocity component normal to the boundary is zero if there is no inflow.

Numerical techniques must be employed to obtain solutions to the coupled set of governing equations. A discrete sequence of gridpoints is defined by:

\[ x = x_i = (i-1) \Delta x, \quad i = 1, 2, \ldots \]
\[ y = y_j = (j-1) \Delta y, \quad j = 1, 2, \ldots \]

where \( \Delta x \) and \( \Delta y \) are the grid increments. The computations are performed on a staggered grid consisting of 3 distinct types of gridpoints. With \( i \) even the \( j \) odd, the gridpoint is a \( \eta \) point at which an elevation is computed. With \( i \) odd and \( j \) odd, the gridpoint is a \( u \) point at which \( u \) is computed. With both \( i \) and \( j \) even, the gridpoint is a \( v \) point at which \( v \) is computed. The arrangement of gridmesh is shown in Fig. 1.

Topographic boundaries of the system are chosen so that a north-south boundary coincides with \( u \) points, and an east-west boundary coincides with \( v \) points. This choice of boundaries easily achieves the boundary conditions of no normal flow through the lateral boundaries. Open-sea boundary consists of \( \eta \) points and \( u \) points. On the open-sea boundary, elevation is prescribed as the \( \eta \) points by a sinusoidal curve fitted to a typical tidal cycle taken from Indian tide tables. In the cycle used here in the model, 3 typical values of low, high and low seawater level elevations above mean sea level (MSL) were taken. A sinusoidal curve of the form \( \eta = a_1 \cos \left( \frac{\pi t}{t_1} \right) + h_1 \) is fitted to first low and high water level elevation above MSL with \( t_1 \) being the time interval in seconds between the low and high tide. Similarly a second equation \( \eta = c_1 \cos \left[ \pi (t-t_1)/t_2 \right] + d_1 \) is fitted to the already used high tide value and the subsequent low tide value above MSL with \( t_2 \) equal to the sum of \( t_1 \) and the time interval between the high and low tide occurrences. The \( c_1 \) and \( d_1 \) can be calculated from the algebraic equations. In this paper, a synthetic tidal cycle with \( t_1 \) and \( t_2 \) of 6 h and 11.5 h has been utilized to examine the numerical model. In future studies, the specific response of the semi-enclosed water body to semi-diurnal \( \left( M_2 \right) \) and diurnal \( \left( K_1 \right) \) tidal constituents would be examined.

Open sea boundary condition is

\[ \eta = -2.55 \cos \left( \frac{\pi t}{t_1} \right) + 0.25; \quad t \leq 360 \times 60 = t_1 \]

\[ = 1.95 \cos \left( \frac{\pi (t-t_1) / t_2}{t_2} \right) + 0.85; \]

\[ t_1 < t \leq 330 \times 60 + t_1 = t_2 \]

The discretized version of the equations of the model is numerically integrated ahead in time, from an initial state of rest, with the forcing (Eq. 6) until the transient response is dissipated by the dissipative forces in the system. We allow the model to reach a dynamic steady state situation by repeating the tidal input forcing function sufficient number of times at the open sea boundary (in the present study 8 cycles).

Without considering the total amplification matrix, a linear analysis of finite difference equations shows that 2 numerical stability criteria must be met. These conditions are the classical stability criterion (CFL condition)

\[ \Delta t_c \leq \frac{\left( gD_{\text{max}} \right)^{1/2}}{1/\Delta x + 1/\Delta y} \]

where \( D_{\text{max}} \) is the maximum water depth encountered in the model and a rotational criterion

\[ \Delta t_c \leq 1/f. \]

The critical time step, \( \Delta t_c \), must be larger than the time step employed. For present study, CFL condition is the most stringent.

**Application of Model**

Bombay harbour is approximately 23 km in length and 10 km in width and the depth ranges from 1.75 to 10 m. The complicated geometry of the system is easily incorporated in the numerical model. Fig. 2 delineates the location map of Bombay harbour with the computational model boundaries.
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Fig. 2—Location map of Bombay harbour with the computational model boundaries.

Fig. 3—Bathymetry of computational area.

Fig. 3 shows bathymetry of the model's computational area. A spatial step of 760 m and a time step of 10 sec were employed. Terms pertaining to wind forcing were excluded to study the effect of tidal forcing only.

In numerical simulations, two systems were studied (i) including Elephanta island in the model and (ii) excluding the island. The comparison brought the effect of presence of Elephanta island on the tidal circulation in the harbour. Figs 4 to 9 show the depth averaged current pattern in the harbour at various epochs of tidal cycle subsequent to attainment of dynamic equilibrium. As part of the computational algorithm, at each hour magnitudes of depth-averaged current velocity and of water elevation above MSL are available.

There is ample evidence from numerical simulation that the presence of Elephanta island induces eddy formation in its wake as tidal circulation sets in. Figs 4a, 6a and 9a show eddies and remnants of eddying vertically averaged flow in various reaches of the water body. This feature has been observed in one study of eddy formation in unsteady flow in continental shelf environment. Presence of Elephanta island causes current field to be stronger in the wake of island as it is evident from comparing Figs 7a to 9b. Figs 6a and 9a clearly bring out the presence of 2 flow fields at times shown therein. As both cases of presence/absence of island are presented, Figs 4a to 9b show the computed features of the depth-averaged flow field.

Amplification of water level rise was studied as the tidal long wave propagates towards the upper reaches of the harbour. There was significant time lag between occurrence of a specific water elevation at the mouth and its effect at the other end. Quantitative estimates of time lag are generated by the numerical model. This type of information is often of interest if the forcing function at the open sea boundary is surge time-history due to a landfalling cyclone. By running the model with this input, peak water levels in the interior can be estimated which would be useful in determining safe grade level of coastal installations from coastal flooding viewpoint. Also steps to be taken for flooding hazard mitigation can be suggested.

At north of Apollo Pier, measurements of time-history of water level variations and of current velocities are not available and thus verification studies require elaborate observational network. It is noteworthy that the utility of such numerical models for predicting the tidal characteristics has been shown at other semi-enclosed water-body sites (including complex estuary) where such models have been validated against observed data on currents and tidal elevations.

As the numerical model generates the time history of current and water elevations, this generated information can be used as an input to other studies like pollution traversal subsequent to release of effluent in the water body at any place.

Besides, by including wind forcing terms acting on the free surface, interaction of tidal circulation with meteorological field can be studied provided the history of wind-field during the tidal cycle is
Fig. 4-6 - Simulated depth averaged velocity vectors, [time and vector magnitude shown, time in h and grid interval in cm sec⁻¹]
Fig. 7-9 – Simulated depth averaged velocity vectors, [time and vector magnitude shown, time in h and grid interval in cm/sec⁻¹]
available as input. Computed current field data thus modified can assist in estimating the movement and transport of any spill in the water body.

Acknowledgement

Thanks are due to Mr S D Soman, Director, Health and Safety group, and Dr S V Lawande, Theoretical Physics Division, for their encouragement.

References