Radiation Efficiency of Some Satellite Aerials in an Ionized Medium

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Using linearized hydrodynamic theory and appropriate current distribution, radiation efficiencies of a dipole, a turnstile, a corner driven square loop and an Alford aerial immersed in an isotropic ionized medium are compared at various plasma frequencies. It is found that the radiation efficiency of all the aerials except that of the Alford aerial decreases rapidly with increase in plasma frequency. The efficiency of the Alford aerial is very high and remains almost constant at all plasma frequencies. It is concluded that the Alford aerial is a better aerial for working in the ionized medium and its deployment on Indian satellites for ionospheric research purposes is suggested.

1. Introduction

A dipole, a loop and a turnstile aerial are amongst the most commonly used aerials on satellites and rockets for studying top-side ionospheric properties, and in establishing communication links. The radiation properties of these aerials in vacuum have been widely investigated and described in literature. However, on satellites and rockets, these aerials encounter ionized medium and interact with it.

It is now well established that when an aerial is immersed in a plasma, its radiation properties are considerably altered. In addition to a non-radiating surface wave, an aerial in plasma generates longitudinal plasma (LP) waves as well as the usual transverse electromagnetic (TEM) waves. In the absence of an external magnetic field, the LP and the TEM waves can be decoupled into two independent modes. The total radiated power of an aerial in plasma medium consists of the power radiated in the TEM and the LP mode. The latter dissipates in the medium and decreases the efficiency of the aerial system.

The purpose of this paper is to compare the radiated powers in the TEM and the LP modes of some of the commonly used satellite aerials immersed in an isotropic, homogeneous plasma medium. In our investigation, the effects of sheath and motion of the heavy ions are neglected. And, the diameters of the aerial wires are assumed to be much less than their lengths.

2. Theory

The radiation efficiency ($\eta$) of a matched aerial in vacuum is given by

$$\eta = \frac{\text{power radiated}}{\text{total power input}} = \frac{R_o}{R_o + R_L} \quad (1)$$

where $R_o$ is the free space radiation resistance and $R_L$ is the loss resistance. In an isotropic plasma medium the radiation efficiency of the aerial can be given by

$$\eta = \frac{R_o}{R_o + R_p + R_L} \quad (2)$$

where $R_o$ and $R_p$ are the TEM and the LP components of the radiation resistance. Thus, for constant $R_o$, an increase in $R_p$ decreases the efficiency of the aerial.

The radiation resistance of the TEM mode, $R_o$ and that of the LP mode, $R_p$ are defined as

$$R_o = \frac{1}{Z_0} \lim_{r \to \infty} \int_0^r A \frac{E_0 \sqrt{\beta^2 + \gamma^2}}{Z_0} \sin \theta \, d\phi \quad (3)$$

$$R_p = \frac{1}{Z_0} \lim_{r \to \infty} \int_0^r A \frac{V_0^2}{Z_0} \frac{1}{\sqrt{\beta^2 + \gamma^2}} \sin \theta \, d\phi \quad (4)$$

where

$$A = 1 - \frac{\omega_p^2}{\omega_0^2} \quad (5)$$

and

$$\omega_p^2 = \frac{m_e}{\epsilon_0 m} \quad (6)$$

is the plasma frequency. $v_0$ is the rms thermal velocity of the electron given by

$$v_0^2 = \frac{3kT}{m} \quad (6)$$

where $k$ is the Boltzmann constant and $m$, the mass of an electron. The total radiation resistance $R_{total}$ is expressed as

$$R_{total} = R_o + R_p \quad (7)$$

The analytical expressions for the vector potentials $A_A$ and $A_4$ and the plasma perturbation $\phi$ are obtained using hydrodynamic theory for small perturbations and linearized continuity and force equations. These equations for the TEM mode are

$$V \times E_o = -j \omega v_0 H$$

$$V \times H = J - c_0 n_o V_e + j \omega n_o E_o$$

$$V \cdot H = 0$$

$$j \omega m V_e = -c E_o$$

and

$$V \cdot E_o = \rho / \epsilon_0 A^4$$

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and for the LP mode

\[ \mathbf{V} \times \mathbf{E}_p = 0 \]

\[ j\omega_0 \mathbf{E}_p - \varepsilon_0 \mathbf{v}_0 \mathbf{V}_p = 0 \]

\[ j\omega_0 \mathbf{n}_0 \mathbf{V}_p + j\omega_0 \mathbf{E}_p + \mu_0 \mathbf{n}_0 \mathbf{V}_p = 0 \]

The vector potential \( \mathbf{A} \) and the plasma pressure perturbation \( \mathbf{p} \) are obtained from Eqs. (8) and (9) as

\[ \mathbf{A} = \frac{\mu_0}{4\pi \varepsilon_0} \int_0^\infty I_x \exp(-j\beta_x z) \, dz \]  \hspace{1cm} (10)

\[ \mathbf{p} = \frac{j\varepsilon_0 \varepsilon_0}{4\pi \varepsilon_0 \varepsilon_0} \int_0^\infty I_x \exp(-j\beta_p z) \, dz \]  \hspace{1cm} (11)

where \( \beta_x \) and \( \beta_p \) are the propagation constants of the TEM and the LP modes respectively and are given by

\[ \beta_x = \frac{\omega}{c} \frac{\mathbf{a}}{\mathbf{v}_0} \frac{\mathbf{A}}{\mathbf{v}_0} \]  \hspace{1cm} (12)

\[ \beta_p = \frac{\omega}{c} \frac{\mathbf{A}}{\mathbf{v}_0} \frac{\mathbf{A}}{\mathbf{v}_0} \]  \hspace{1cm} (13)

The values of far zone TEM fields \( \mathbf{E}_0 \) and \( \mathbf{E}_\phi \) and that of LP field \( \mathbf{E}_r \) are obtained using following relations:

\[ \mathbf{E}_0 = -j\omega_0 \mathbf{A}_0 \]  \hspace{1cm} (14)

\[ \mathbf{E}_\phi = -j\omega_0 \mathbf{A}_\phi \]  \hspace{1cm} (15)

\[ \mathbf{E}_r = \frac{\mathbf{E}_0}{\mathbf{v}_0} \frac{\mathbf{E}_0}{\mathbf{v}_0} \]  \hspace{1cm} (16)

The geometries of four different types of aerials under investigation are shown in Fig. 1 [(a)-(d)]. The current distribution on the aerials is as follows:

1. Alford loop \( I_x = I_m \cos \beta(1/2-x) \) \hspace{1cm} (1a)
2. Corner driven loop \( I_x = I_m \exp(-j\beta S) \) \hspace{1cm} (1b)
3. Turnstile \( I_x = I_m \sin \beta(1-x) \) \hspace{1cm} (1c)
4. Dipole \( I_x = I_m \sin \beta(1+y) \) \hspace{1cm} (1d)

The expressions for \( \mathbf{R}_e \) and \( \mathbf{R}_p \) are obtained after substituting appropriate current distribution in Eqs. (10) and (11) and using Eqs. (13), (14) and (15). These can be written as

(1) For the Alford loop aerial

\[ \mathbf{R}_e = \frac{30a^2}{\pi \mathbf{v}_0} \int_0^\infty \int_0^2 \left[ \mathbf{E}_0^2 \sin^2 \left( \frac{\theta \mathbf{A}_0}{2} \right) + \mathbf{E}_\phi^2 \sin^2 \left( \frac{\theta \mathbf{A}_\phi}{2} \right) \right] \, d\theta \, d\phi \]  \hspace{1cm} (17)

where

\[ \mathbf{a} = \frac{\mathbf{a}}{\mathbf{v}_0}, \quad \mathbf{a}_0 = 1 - \sin^2 \theta \cos^2 \phi \]

\[ \mathbf{u}_1 = a \sin \theta \cos \phi, \quad \mathbf{u}_2 = -a \sin \theta \sin \phi \]

\[ \mathbf{u}_3 = \mathbf{v}_0, \quad \mathbf{u}_4 = c \]

\[ \mathbf{x} = [\delta_1 \sin (\theta \mathbf{A}_1) + \delta_2 \sin (\theta \mathbf{A}_2)]/2 \]

\[ \mathbf{y} = [\delta_3 \sin (\theta \mathbf{A}_3) + \delta_4 \sin (\theta \mathbf{A}_4)]/2 \]

\[ \mathbf{z} = [\delta_5 \sin (\theta \mathbf{A}_5) + \delta_6 \sin (\theta \mathbf{A}_6)]/2 \]

\[ \mathbf{\psi}_1 = -\mathbf{\psi}_2 \]

\[ \mathbf{\psi}_3 = \mathbf{\psi}_4 \]

\[ \mathbf{\psi}_5 = \mathbf{\psi}_6 \]

\[ \mathbf{\psi}_7 = \mathbf{\psi}_8 \]

\[ \mathbf{\psi}_9 = \mathbf{\psi}_10 \]

For a square loop aerial \( \delta = \pi/4 \).

(2) For corner driven square loop aerial

\[ \mathbf{R}_e = \frac{480a^2 \sin^2 \delta}{\pi \mathbf{v}_0} \int_0^\infty \int_0^2 \left( \left[ 1 - \Delta_1 \Delta_2 \right] \sin^2 \left( \theta \mathbf{A}_1 \right) \right) \, d\theta \, d\phi \]  \hspace{1cm} (21)

where

\[ \mathbf{\delta} = \mathbf{\delta} \left( 1 - a^2 \sin^2 \theta \sin^2 \phi \right) \left( 1 - a^2 \sin^2 \theta \sin^2 \phi \right) \]

\[ \mathbf{\Delta}_1 = (1 - a \cos \theta_1) \left( 1 - a \cos \theta_1 \right) \]

\[ \mathbf{\Delta}_2 = (1 - a \cos \theta_2) \left( 1 - a \cos \theta_2 \right) \]

\[ \mathbf{\psi}_1 = -\mathbf{\psi}_2 \]

\[ \mathbf{\psi}_3 = \mathbf{\psi}_4 \]

\[ \mathbf{\psi}_5 = \mathbf{\psi}_6 \]

\[ \mathbf{\psi}_7 = \mathbf{\psi}_8 \]

\[ \mathbf{\psi}_9 = \mathbf{\psi}_10 \]

For a turnstile aerial

\[ \mathbf{R}_e = \frac{30a^2}{\pi \mathbf{v}_0} \int_0^\infty \int_0^{2\pi} \mathbf{E}_0^2 \sin^2 \theta \, d\theta \, d\phi \]  \hspace{1cm} (24)

\[ \mathbf{R}_p = \frac{30(1-A^2) c}{\pi \mathbf{v}_0} \int_0^\infty \int_0^{2\pi} \mathbf{E}_0^2 \sin^2 \theta \, d\theta \, d\phi \]  \hspace{1cm} (25)

\[ \mathbf{T}_3 \cos^2 \theta + \mathbf{T}_4 \sin^2 \theta \]

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where

\[ T_1 = \cos(\beta \psi \sin \theta \cos \phi) - \cos \beta \psi \]

\[ T_2 = \cos(\beta \psi \sin \theta \sin \phi) - \cos \beta \psi \]

\[ T_3 = \cos(\beta \psi \sin \theta \cos \phi) - \cos \beta \psi \]

\[ T_4 = \cos(\beta \psi \sin \theta \sin \phi) - \cos \beta \psi \]

\[ \cos \beta \psi \sin \phi + \cos \beta \psi \sin \phi \] ... (26)

and \( u_1, u_2, u_3, u_4, u_5 \) and \( u_6 \) are given by Eqs. (18).

For displaced dipole aerial\(^\text{16}\),

\[ R_\| = \frac{30}{\pi A} \int_0^{2\pi} \int_0^\pi \left( a \sin \theta \right) \sin \theta \times \left[ \cos(\beta N_1) - \cos \beta \psi \cos(\beta M_1) \right] \sin \theta \, d\theta \, d\phi \] ... (27)

\[ R_\perp = \frac{30}{\pi A} (1-A^2) \int_0^{2\pi} \int_0^\pi \left( b \cos \theta \right) \sin \theta \times \left[ \cos(\beta N_2) - \cos \beta \psi \cos(\beta M_2) + \sin \beta \psi \sin(\beta M_2) \right] \sin \theta \, d\theta \, d\phi \] ... (28)

where

\[ M_1 = a d \sin \theta \sin \phi, \quad M_2 = \frac{c}{v_0} \]

\[ N_1 = a(\cos \theta + d \sin \theta \sin \phi), \quad N_2 = \frac{c}{v_0} \]

and \( d \) is the small transverse displacement between the feed points of the aerial caused due to spinning of the satellite. In the above expression the value of the propagation constant \( \beta \) is taken to be general but it is now confirmed that \( \beta \) is of the order of \( \beta_0 \) (ref. 11) and this value is used in our computations.

3. Results and Discussion

The double integrals occurring in Eqs. (16), (17), (21), (22), (24), (25), (27) and (28) have been evaluated using a general computer programme developed by the author\(^\text{12}\). The programme evaluates outer integral to an accuracy of 0.5% and inner integral within 1%, giving overall accuracy within 2%. The values of \( R_\|, R_\perp \) and \( R_T \) for \( \beta = \beta_0, \rho / \gamma \) = 0.25 for different plasma frequencies are listed in Table 1.

From Table 1 it is obvious that the variation in \( R_\| \) and \( R_\perp \) with \( \Omega = \omega / \omega_0 \) is similar for a dipole and for a turnstile aerial except that the magnitude of the former is half that of the latter. Also, the magnitude and variation of the values of \( R_T \) with \( \Omega \) for all the ariels is of the same order, while that of \( R_T \) differs greatly.

Using Eq. (2) and assuming \( R_L \) to be negligible for all the ariels, variation of \( \eta \) with \( \Omega \) is shown in Fig. 2. It is interesting to note that while the
efficiency of the Alford loop with $\Omega$ is almost constant, the efficiency of the other three aerials decreases rapidly. This shows that the Alford aerial is a much better aerial than the rest for satellite working in the plasma medium.

The power radiated in the TEM mode $P_e$ is given by

$$P_e = \frac{1}{2} I_0^2 R_e$$

and variation of $R_e/R_0$ is plotted against $\Omega$ in Fig. 3 which shows that the variation of this factor is similar for all the aerials though the magnitude is a little higher for a corner driven square loop aerial.

Fig. 4 shows the relative efficiency $\eta_r$ of these aerials as compared to that of a dipole

$$\eta_r = \eta_{\text{aerial}}/\eta_{\text{dipole}}$$

The striking fact is that $\eta_r$ for an Alford aerial is 10 times more than the rest and increases sharply with $\Omega$.

Fig. 5, in which $R_e/\eta_{\text{dipole}}$ dipole versus $\Omega$ is plotted, shows that though the electromagnetic power $P_e$ is more for a corner driven square loop than for an Alford loop, this advantage is offset by lower radiation efficiency and high value of $R_p$, which indicates that there is dissipation of plasma mode power in the medium ($P_e = \frac{1}{2} I_0^2 R_p$) as shown in Table 1.
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4. Conclusions

The following are the conclusions derived from the present study:
(i) The useful radiated power $P_e$ of all the aerials under study decreases with increase in plasma frequency; (ii) the dissipation of power in the plasma mode ($P_p$) increases rapidly with $\Omega$ for all the aerials except for the Alford loop aerial in which case it is very small and remains almost constant for all plasma frequencies $< \omega$; and (iii) the efficiency of the Alford aerial is more than 90% for all the plasma frequencies under investigation, while efficiency of the other classes of aerials falls rapidly with increasing plasma frequency.

It can, therefore, be concluded that because of lower plasma resistance $R_p$ and higher efficiency, the Alford aerial is the best of all the aerials under study. It is pertinent to mention that for Aerial 3 type satellites which have four extended outer booms, the Alford aerial can be fastened with ease. It is suggested that for high efficiency, one such aerial be deployed on one of the prospective Indian satellites for ionospheric and other allied measurements.

Nomenclature

- $E$ = electric field vector
- $e$ = electron charge
- $H$ = magnetic field vector
- $I$ = electric current
- $b$ = Boltzmann's constant
- $l$ = length of one element of aerial
- $m_e$ = ambient electron population density
- $n_e$ = perturbation of electron population density
- $P$ = radiated power
- $R$ = radiation resistance
- $r, \theta, \phi$ = spherical polar coordinates
- $T$ = absolute temperature
- $V$ = electron velocity vector
- $v_t$ = rms thermal velocity of electrons
- $\beta$ = general propagation constant of the current distribution on the aerial
- $\beta_e$ = propagation constant for the EM mode in plasma
- $\beta_p$ = propagation constant of P mode
- $\epsilon_0$ = permittivity of free space
- $\lambda_0$ = wavelength of EM wave in free space
- $\mu_0$ = permeability of free space
- $\rho$ = charge density
- $\Omega$ = angular source frequency
- $\omega_p$ = angular plasma frequency
- $\Theta$ = direction cosine

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