Magnetic Field within the Return Stroke Channel of Lightning

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An expression for the magnetic field inside the return stroke discharge channel has been derived from a knowledge of the current observed in it. It increases from zero at the centre with increase in the radial distance and becomes maximum at the periphery. The computed maximum magnetic field is about 3.8 kg for a channel radius of 1 cm, which is perhaps the maximum value ever possible in nature.

1. Introduction

Magnetic field measurements due to lightning discharges were often utilized in inferring indirectly the various characteristics of the discharge channel like discharge current, number of strokes per flash, the break-down processes, etc. Uman and McLain derived an expression which enables us to evaluate a current in the return stroke of a lightning discharge from the measurement of the magnetic flux density at large distances from the discharge channel. Rai and Bhattacharya derived an expression for the magnetic flux density very close to the return stroke channel of a lightning discharge. However, to the authors' knowledge, there are neither experimental data nor theoretical attempts to determine the magnetic field within the discharge channel of the lightning.

Electromagnetic radiation from the lightning discharges extends right from very low frequency (vlf) to microwave region. In many of the radiation processes like synchrotron, cyclotron, Cerenkov, etc. magnetic field in the medium plays an important role. In fact Kosarev et al. have suspected that the electron interaction with the magnetic field produced by the lightning current may be the possible source of microwave radiation from lightning. Hence, it is pertinent to study the magnetic field within the return stroke channel of lightning, where the active medium is present.

In the present investigation the authors have computed magnetic field inside the discharge channel with the knowledge of the current flowing in it.

2. Derivation of an Expression for the Magnetic Field Formation

The well-known cloud-to-ground lightning discharge is initiated by a faintly luminous negative stepped leader moving groundwards in steps of about 50 m in length and 50 μsec time pause between the steps. As the stepped leader approaches ground at an altitude of 20 to 70 m, a positive return stroke shoots up, the tip of which essentially moves in the preionized channel left by the previous leader. Negative charges in the immediate neighbourhood and in front of the channel-tip move towards the tip, thereby constituting lightning current, which is conveyed to the ground through the fully-ionized return stroke plasma. Behind the tip, the plasma attains thermodynamic equilibrium within 10^-8 sec and ion temperature becomes equal to the electron temperature. Electrons move due to their thermal velocity (about 10^6 m/sec) in all directions and ions being too heavy, are assumed to be at rest. The random current density of the plasma materials is essentially zero, because in such an isotropic distribution the number of particles crossing a given area from one side is equal to the number of particles crossing from the opposite side. Hence, the magnetic field due to plasma material is neglected.

The current in the return stroke of lightning is given by the well known expression of Bruce and Golde:

\[ I_t = I_0 (e^{-\alpha t} - e^{-\beta t}) \]  

where \( t \) is the time in seconds reckoned from the start of the return stroke from the ground and \( I_t \), the current at a time \( t \). Srivastava and Tantry showed that the following set of values for the current flowing through the channel to the ground, account well for the experimental observations of the field strength of atmospherics received at a distant point, where the radiation field is only dominant:

\[ I_0 = 22 \text{ k Amp} \]
\[ \alpha = 1.4 \times 10^4 \text{ sec}^{-1} \]
\[ \beta = 5 \times 10^4 \text{ sec}^{-1} \]

Since it is well known that the return stroke discharge channel is a fully-ionized plasma, it can be assumed to be a good conductor. The discharge channel is usually circular in cross-section of radius \( R_0 \)
which is the same throughout the discharge channel\textsuperscript{[10]}. For such a model the permeability of the active channel $\mu$ is essentially that for free space $\mu_0$. The magnetic field due to a current carrying conductor can be computed by the Ampere's law and is given by\textsuperscript{[11]}:

\begin{equation}
B (R, t) = \frac{\mu_0}{2\pi R} \int_0^R J (r, t) 2\pi r \, dr
\end{equation}

where $J (r, t)$ is the current density at a distance $r$ which is measured from the centre of the discharge channel.

The direction of $B (R, t)$ is tangential to the circle of radius $R$ drawn about the centre of the channel. Assuming the current distribution to be uniform throughout the cross-section, the validity of which will be discussed later, the current density at any instant $t$ is given by:

\begin{equation}
J (r, t) = \frac{I_0}{\pi R_0} (e^{-\alpha t} - e^{-\beta t})
\end{equation}

From Eqs. (2) and (3) we get

\begin{equation}
B (R, t) = \frac{\mu_0 I_0}{\pi R R_0^3} (e^{-\alpha t} - e^{-\beta t}) \int_0^R r \, dr
\end{equation}

or

\begin{equation}
B (R, t) = \frac{\mu_0 I_0 (R/R_0)}{2\pi R_0} (e^{-\alpha t} - e^{-\beta t})
\end{equation}

3. Pinch Effect

The Lorentz force $F = q(E + V \times B)$ is found to have a component $V \times B$ towards the centre of the channel, which alters the motion of charge carriers in such a way that they are forced or pinched together. As the total carrier charge must be conserved, the greater density of charge due to the carriers, when they are in motion can only be achieved by crowding towards the centre of the channel. If this magnetic force is sufficient to overcome the effects of the kinetic pressure of the plasma particles, the cylinder of plasma particles will contract. For the quasi-equilibrium pinch in the return stroke, where the radius of the plasma cylinder remains constant\textsuperscript{[10]}) throughout the discharge process, we find the Bennet distribution\textsuperscript{[12]} from the equilibrium conditions of the kinetic pressure of particles and the magnetic force, viz.

\begin{equation}
n = \frac{n_0}{(1 + n_0 br^2)}
\end{equation}

where

\begin{equation}
b = \frac{\mu_0 e^2 v^2}{8K (T_e + T_i)}
\end{equation}

$v$ is the velocity of electrons, $T_e$ and $T_i$ are the electron and ion temperatures which are essentially equal because the channel is in local thermodynamic equilibrium (LTE), $K$ is the Boltzmann constant, $n_0$ is the maximum electron number density, $n$ is the electron number density at $r$.

The distribution falls off very sharply with $r$, and for all practical purposes, the particle density falls to zero at a distance $r_p$, which is obviously the radius of the channel. The plasma is concentrated around the axis, where the distribution is not very sharp and the particles may be assumed to be uniformly distributed along the cross-section of the channel.

The electron density of the fully-ionized return stroke is $10^{16}$ electrons/m$^3$. For the thermal velocity of electrons at the channel temperature of 30,000$^\circ$K, the radius $r_p$ comes out to be approximately 1 cm, which is in excellent agreement with the theoretical and experimental predictions of various workers\textsuperscript{[13]}.

4. Results and Discussion

The empirical expression in Eq. (1) is given for the current variation at the base of the return stroke. It is generally assumed that the return stroke discharge channel is such a good conducting medium that the heavy current of the order of several kiloamperes at the moving tip is immediately conveyed to the ground as in metallic conductors; and hence throughout the return stroke channel the current is assumed to be constant at an instant $t$, i.e. the same current flows along the whole length of the channel at a given time. The current at the base is essentially time varying and peaks at about 10 $\mu$ sec. Hence, the assumption for return stroke to be good conductor is valid.

Further, our assumption of uniform current distribution throughout the horizontal cross-section requires the uniformity of charge carriers. Though the exact nature of charge distribution across the cross-section of the return stroke discharge channel is not known, it is usually assumed that the temperature and charge distribution in a stroke at a given time near the peak temperature are approximately constant across the cross-section of the stroke\textsuperscript{[14]}.
Fig. 1—Magnetic field variation with time for \( R/R_0 = 0.5 \) and 0.25

The values of the magnetic flux densities for \( R/R_0 = 0.5 \) and 0.25 have been calculated and shown in Fig. 1. At about 10 \( \mu \)sec after the initiation of the return stroke process, the magnetic field becomes maximum and then decreases continuously which is in accordance with the current variation at the base of the return stroke. The maximum value of the magnetic flux density for \( R/R_0 = 1 \) (not shown in Fig. 1) is about 3.8 kG at the periphery for a channel of radius 1 cm. If the radius of the channel \( R_0 \) in Eq. (4) above varies, the maximum flux density will also vary and the discharge channel of larger cross-section will yield a lower value of the magnetic flux density, and vice versa, provided all the other quantities remain the same as before. The magnetic field is found to be maximum at the periphery and zero at the centre of the channel; it also decreases as the observing point moves away from the discharge channel.

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