Extension of the ‘Two-Colour Method’ to Evaluate the Band Intensities in the Night Airglow

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In the filter photometry of the night airglow two filters are used, one centred on an 'emission line' and the other excluding the line, estimating the 'background'. The extension of this 'two colour method' for reduction of OH molecular band intensities is described. The errors involved due to the approximations are discussed. It is pointed out that one of the most important but uncertain factors in evaluating absolute intensities is the estimation of the ratio of the background intensities in the two wavelength regions transmitted by 'emission' and 'background' filters.

Introduction

In filter photometry the 'two colour method' of Roach and Meinel is used for reducing the emission line intensities from night airglow data to absolute units (rayleighs). In this method, two filters are used, one centred at or very near the emission wavelength and the other at a wavelength where there is no airglow emission but is as near as possible to the emission wavelength. We shall call the first filter as the line filter (I-filter) and the second, as the background filter (b-filter). To determine the intensity of the emission line, the contribution due to the total background intensity transmitted by the I-filter is determined approximately and subtracted from the total response of the I-filter. For reducing the intensities to absolute units, a secondary standard source such as a radio-activated 14C phosphor is used, the radiance of which with wavelength is known. The diffuse light source is expected to fill the full aperture of the photometer.

In this communication we first give the equations for calculating the intensities of emission lines in accordance with Smith and Alexander; then the equations are modified to calculate the intensity of an OH band. Useful approximations in actual use of the formula and errors involved due to the same are discussed at the end.

Line Intensity

Let dS and dC denote the deflections (or responses) of the photometer due to the night sky and 14C light source through the I-filter; also dS and dC denote the same quantities through the b-filter. All the night sky deflections to be used in the following are supposed to be already corrected for atmospheric scattering and absorption. Then for the I-filter the response is

\[ k \, dS = S \, R \, d\lambda \]

and for the b-filter

\[ k \, dB = S \, N \, d\lambda \]

where k is the geometrical constant of the photometer (i.e. energy per unit deflection); S, R and N are the photomultiplier response, the transmission of the filter and the radiance of the 14C source (in raleighs/Å) respectively. (Values of N from W cm² Å⁻¹ ster⁻¹ can be converted to raleighs/Å by multiplying by \(6.3227 \times 10^{17}\).)

Integrals in Eqs. (1) - (4) are carried out over the transmissions of I- and b-filters (ideally they should be from 0 to \(\infty\)). Assuming B and B' constant in narrow spectral regions, we have

\[ \frac{dS}{dC} = \frac{S \, N \, T \, d\lambda}{S \, N \, T \, d\lambda} = D \]

and

\[ \frac{dB}{D} = \frac{B \, f \, S \, N \, T \, d\lambda}{B \, f \, S \, N \, T \, d\lambda} = D' \]

Therefore

\[ B' = D' \frac{B \, f \, S \, N \, d\lambda}{B \, f \, S \, d\lambda} \]

Now B can be calculated if K is known.
With approximations (to be discussed at the end) the equation has the familiar form

\[
R_\lambda = (D_1 - K_1 D_6 N_{\lambda b}(N_{\lambda d}) N_{\lambda d}) Q \quad (10)
\]

where

\[ Q = \int T_{\lambda} \, d\lambda / T_\lambda \]

In Eq. (10) except \( D_1 \) and \( D_6 \) which may change for every observation, all other quantities are constant for a pair of \( l \)- and \( b \)-filters.

### Band Intensity

This technique can now be extended to a band such as OH (7-2) band. The OH rotational vibrational bands in night airglow and in the near infrared region have well separated \( P_1 \) and \( P_2 \) branches and rather crowded \( Q \) and \( R \) branches. Let the relative intensities of the \( P \) branch be

\[
\alpha P_1(1), \alpha P_1(2), \alpha P_2(2), \alpha P_4(3), \alpha P_5(4), \ldots.
\]

where \( \alpha \)'s denote relative intensities of the components of \( P_1 \) and \( P_2 \) branches as given by Chamberlain. Also let \( \alpha_Q \) and \( \alpha_R \) denote the total relative intensities of the \( Q \) and \( R \) branches.

Similarly let the filter transmission \( T_{\lambda} \) and the photomultiplier response \( S_{\lambda} \) at wavelengths for the above branches be denoted by

\[
T_{P_1(2)}, T_{P_1(3)}, T_{P_1(4)}, \ldots \quad T_{P_2(2)}, T_{P_2(3)}, \ldots
\]

and

\[
S_{P_1(2)}, S_{P_1(3)}, S_{P_1(4)}, \ldots \quad S_{P_2(2)}, S_{P_2(3)}, \ldots
\]

Thus, the deflection due to a OH band filter will be

\[
R_{OH} = \sum_{P_1}^8 \alpha P_1(2) S_{P_1(2)} T_{P_1(2)} + \alpha P_1(3) S_{P_1(3)} T_{P_1(3)} + \alpha P_2(2) S_{P_2(2)} + \alpha P_2(3) S_{P_2(3)} T_{P_2(2)} + \alpha Q S_Q T_Q + \alpha R S_R T_R
\]

where \( R_{OH} \) is the total intensity of the OH band in question. As \( P_1 \) and \( P_2 \) branches have negligible intensities after 8th rotational line, we consider only 8 components of \( P_1 \) and \( P_2 \). Therefore, the above expression takes the form

\[
R_{OH} = \sum_{P_1}^8 \alpha P_1(2) S_{P_1(2)} T_{P_1(2)} + \alpha P_1(3) S_{P_1(3)} T_{P_1(3)} + \alpha P_2(2) S_{P_2(2)} + \alpha P_2(3) S_{P_2(3)} T_{P_2(2)} + \alpha Q S_Q T_Q + \alpha R S_R T_R
\]

Along with the genuine OH radiation, the filter will also transmit background radiation. Let it be \( B \) rayleighs/\( \lambda \). Again, assume that \( B \) is constant over the spectral range of a single OH band under measurement. Then the photometer deflection (response) through the OH filter to the night sky will be

\[
k dS_{OH} = R_{OH} \left[ \sum_{P_1}^8 \alpha P_1 S_{P_1} T_{P_1} + \alpha Q S_Q T_Q + \alpha R S_R T_R \right]
\]

where \( k \) is the geometrical constant of the instrument. Let A denote the square bracket in the first term. Then,

\[
k dS_{OH} = R_{OH} \left[ \sum_{P_1}^8 \alpha P_1 S_{P_1} T_{P_1} + \alpha Q S_Q T_Q + \alpha R S_R T_R \right] + B \int \frac{dS}{d\lambda}
\]

The photometer response for the OH filter with \( ^{14}C \) light source will be

\[
k dC = \int \frac{dS}{d\lambda}
\]

If \( D_{OH} = \frac{dS}{d\lambda} \), then from Eqs. (11) and (12)

\[
R_{OH} = \int \frac{dS}{d\lambda}
\]

To evaluate \( B \), background intensity in rayleighs/\( \lambda \) transmitted through the OH filter, another filter in the neighbourhood of the OH band under measurement is used which does not include any airglow emission [say 6080 or 7115 \( \lambda \) filter for OH (7-2) band].

Let the deflection due to this filter from sky be \( dS_b \) and that due to \( ^{14}C \) source be \( dS_b \). Then the response of the photometer with \( ^{14}C \) source through the background filter is given by Eq. (4).

If the total night sky continuum intensity is \( B' \) rayleighs/\( \lambda \) in this spectral region and is assumed to be constant in the region transmitted by the \( b \)-filter, then we have

\[
k dS_b = B' \int \frac{dS}{d\lambda}
\]

Therefore, from Eqs. (4) and (14)
B' = D_b \frac{\int_b S_\lambda N_\lambda T_\lambda \, d\lambda}{\int_b S_\lambda T_\lambda \, d\lambda} \quad \ldots (15)

where

D_b = \frac{dS_b}{dC_b} \quad \ldots (16)

Thus when B' is known B can be calculated provided the ratio of background intensities at the two peak transmission wavelengths of the filters is known. If this ratio is K_{OH}, then

B = K_{OH} B' \quad \ldots (17)

Therefore, the intensity of the OH band is given by

R_{OH} = \frac{D_{OH}}{A_{OH}} \int \frac{S_\lambda N_\lambda T_\lambda \, d\lambda}{A_{OH} \int S_\lambda T_\lambda \, d\lambda} \quad \ldots (18)

Substituting for B from Eqs. (15) and (17), we get

R_{OH} = \frac{D_{OH}}{A_{OH}} \int \frac{S_\lambda N_\lambda T_\lambda \, d\lambda}{A_{OH} \int S_\lambda T_\lambda \, d\lambda} \quad \ldots (19)

where R_{OH} is in rayleighs.

Useful Approximations and Errors Involved

Considering Eqs. (9) and (19), which give intensities of an emission line and a band respectively with the two-filter method, it can be observed that:

1. In deriving these equations, it is already assumed that B and B', the background emission over the spectral regions transmitted through the two filters, are individually constant. When the l-filter and b-filter have moderate widths (\leq 50 \text{ Å}) this may not be a bad assumption. However, it cannot be generalized for all spectral regions and should be used with caution.

2. The ratio B/B' = K must also be known accurately for the reduction. Values of K are assumed from some individual night sky spectra. The background radiation in the night sky spectrum includes the three components, viz. (i) the extraterrestrial light, (ii) the zodiacal light, and (iii) the genuine airglow continuum. If one can further assume that the average spectral response of the first two sources is similar to that of the sun, B/B' for these two components may remain the same. Regarding the variation of the third component, however, practically nothing is known. From the 5350 \text{ Å} observations made at Mt. Abu, India, since 1964 with a fixed photometer looking to polestar (where starlight is constant), it is noted that the background intensity has variations even after accounting properly for extinction and scattering. The situation may be worse in the infrared region. Hence assuming a certain value of K seems to be
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Table 1—Characteristics of the Filters Used with Different Photometers

<table>
<thead>
<tr>
<th>Filter No.</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>XI</th>
<th>XII</th>
<th>XIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_p )</td>
<td>6300</td>
<td>5894</td>
<td>5750</td>
<td>5586</td>
<td>5300</td>
<td>6330</td>
<td>6080</td>
<td>6880</td>
<td>7270</td>
<td>6340</td>
<td>5880</td>
<td>5580</td>
</tr>
<tr>
<td>( \lambda_e )</td>
<td>6300</td>
<td>5893</td>
<td>5757</td>
<td>5587</td>
<td>5300</td>
<td>6330</td>
<td>6080</td>
<td>6880</td>
<td>7270</td>
<td>6340</td>
<td>5880</td>
<td>5580</td>
</tr>
<tr>
<td>( T_{max} )</td>
<td>69</td>
<td>79</td>
<td>74</td>
<td>78</td>
<td>14</td>
<td>71</td>
<td>77</td>
<td>42</td>
<td>32</td>
<td>40</td>
<td>43</td>
<td>24</td>
</tr>
<tr>
<td>H.W.</td>
<td>47</td>
<td>47</td>
<td>41</td>
<td>43</td>
<td>130</td>
<td>130</td>
<td>105</td>
<td>48</td>
<td>120</td>
<td>100</td>
<td>135</td>
<td>97</td>
</tr>
<tr>
<td>( S_A )</td>
<td>41.3</td>
<td>53.1</td>
<td>57.4</td>
<td>62.8</td>
<td>70.5</td>
<td>40.4</td>
<td>47.4</td>
<td>23.2</td>
<td>12.75</td>
<td>17.0</td>
<td>32.0</td>
<td>42.0</td>
</tr>
<tr>
<td>( N_A )</td>
<td>5.133</td>
<td>15.60</td>
<td>17.97</td>
<td>19.37</td>
<td>15.81</td>
<td>4.710</td>
<td>10.36</td>
<td>1.412</td>
<td>0.9059</td>
<td>4.57</td>
<td>19.51</td>
<td>19.37</td>
</tr>
</tbody>
</table>

\( \lambda_p \) is the peak transmission wavelength of the filter in Å; \( \lambda_e \) is emission wavelength in Å;
\( T_{max} \) is maximum % transmission of the filter; H.W. is half-width of the filter;
\( S_A \) is relative response of the photomultiplier; \( N_A \) is radiance of the \(^{14}\text{C} \) source in units of \( 10^{-11} \text{ W cm}^{-2} \text{ ster}^{-1} \text{ Å}^{-1} \).

Table 2—Response of the Photometer (in Arbitrary Units) with Successive Approximation

<table>
<thead>
<tr>
<th>Filter Wavelength, Å</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>XI</th>
<th>XII</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_A ) ( N_A ) ( T_A ) ( d\lambda )</td>
<td>7.839</td>
<td>3.099</td>
<td>2.935</td>
<td>4.101</td>
<td>2.622</td>
<td>1.443</td>
<td>2.019</td>
<td>2.413</td>
<td>5.004</td>
<td>5.856</td>
<td>3.281</td>
<td>1.438</td>
</tr>
<tr>
<td>( N_A ) ( S_A ) ( T_A ) ( d\lambda )</td>
<td>8.009</td>
<td>3.094</td>
<td>2.932</td>
<td>4.176</td>
<td>2.733</td>
<td>1.442</td>
<td>2.023</td>
<td>2.662</td>
<td>4.950</td>
<td>5.528</td>
<td>3.244</td>
<td>2.728</td>
</tr>
<tr>
<td>( S_A ) ( N_A ) ( S_A ) ( T_A ) ( d\lambda )</td>
<td>7.865</td>
<td>3.096</td>
<td>2.921</td>
<td>4.110</td>
<td>2.649</td>
<td>1.439</td>
<td>2.019</td>
<td>2.403</td>
<td>4.973</td>
<td>5.647</td>
<td>3.261</td>
<td>2.600</td>
</tr>
<tr>
<td>( S_A ) ( N_A ) ( N_A ) ( T_A ) ( d\lambda )</td>
<td>8.032</td>
<td>3.094</td>
<td>2.931</td>
<td>4.117</td>
<td>2.728</td>
<td>1.445</td>
<td>2.026</td>
<td>2.353</td>
<td>5.019</td>
<td>5.773</td>
<td>3.359</td>
<td>2.672</td>
</tr>
<tr>
<td>( S_A ) ( N_A ) ( T_A )</td>
<td>8.065</td>
<td>7.96</td>
<td>3.508</td>
<td>4.624</td>
<td>1.450</td>
<td>1.857</td>
<td>2.263</td>
<td>1.965</td>
<td>1.155</td>
<td>10.489</td>
<td>4.938</td>
<td>6.508</td>
</tr>
</tbody>
</table>

(at 1/2 \( T_{max} \))

Table 1 presents data of the filters and of the \(^{14}\text{C} \) light source which we have used. Fig. 3 gives the responses of \( S_{10} \) and \( S_{20} \) type photomultiplier tubes and the \(^{14}\text{C} \) light source used. Table 2 shows the results of calculations of the responses of the photometers with successive approximations using Eqs. (20)-(24) for filters listed in Table 1. The results lead to the conclusions:

(i) Values of the responses using Eqs. (20)-(24) do not differ by more than 2% when the filter width < 50 Å.
(ii) When the filters are rather broad the values calculated from above equations differ by 5-10%.
(iii) Results obtained by Eqs. (20) and (24) (the worst approximation) differ by 10-20% for narrow filters and more than 20% for broader filters.

It may, therefore, be concluded that it is only necessary to evaluate \( J \) \( T_A \) \( d\lambda \) while \( S_A \) and \( N_A \) can be considered constant within the narrow spectral range. \( J \) \( T_A \) \( d\lambda \) can be evaluated by detailed calibration of the filter with a good spectrophotometer.

It must also be remembered that the filters should not have secondary transmissions to which the

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**Fig. 3—Responses of \( S_{10} \) and \( S_{20} \) type photomultiplier tubes and the \(^{14}\text{C} \) light source**

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photomultiplier is sensitive (In modern multilayer filters, care is taken to suppress all secondary transmissions). Also, we have not considered the calibration of the secondary standard of light (\(^{14}\)C light), which is one of the very important problems, and its variation with temperature and time. The source should be used preferably at one temperature and should be calibrated against a standard source at least once a year. One should also take note of the effects of (i) time, (ii) temperature, and (iii) inclination of the filter to the incident light, on the transmission characteristics of the filter.

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References