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Extension of the 'Two-Colour Method' to Evaluate the Band Intensities in the Night Airglow

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In the filter photometry of the night airglow two filters are used, one centred on an 'emission line' and the other excluding the line, estimating the 'background'. The extension of this 'two colour method' for reduction of OH molecular band intensities is described. The errors involved due to the approximations are discussed. It is pointed out that one of the most important but uncertain factors in evaluating absolute intensities is the estimation of the ratio of the background intensities in the two wavelength regions transmitted by 'emission' and 'background' filters.

Introduction

IN filter photometry the 'two colour method' of Roach and Meinel[1] is used for reducing the emission line intensities from night airglow data to absolute units (rayleighs). In this method, two filters are used, one centred at or very near the emission wavelength and the other at a wavelength where there is no airglow emission but is as near as possible to the emission wavelength. We shall call the first filter as the line filter (l-filter) and the second, as the background filter (b-filter). To determine the intensity of the emission line, the contribution due to the total background intensity transmitted by the l-filter is determined approximately and subtracted from the total response of the l-filter. For reducing the intensities to absolute units, a secondary standard source such as a radioactivated ^{14}C phosphor is used, the radiance of which with wavelength is known. The diffuse light source is expected to fill the full aperture of the photometer.

In this communication we first give the equations for calculating the intensities of emission lines in accordance with Smith and Alexander[2], and Kulkarni and Sanders[3]; then the equations are modified to calculate the intensity of an OH band. Useful approximations in actual use of the formula and errors involved due to the same are discussed at the end.

Line Intensity

Let dS_l and dC_l denote the deflections (or responses) of the photometer due to the night sky and ^{14}C light source through the l-filter; also dS_b and dC_b denote the same quantities through the b-filter. All the night sky deflections to be used in the following are supposed to be already corrected for atmospheric scattering and absorption[4]. Then for the l-filter the response is

$$k\ dS_l = S_\Lambda T_\Lambda R_\Lambda + \int_l S_\lambda T_\lambda B\ d\lambda \qquad \ldots(1)$$

$$k\ dC_l = \int_l S_\lambda N_\lambda T_\lambda\ d\lambda \qquad \ldots(2)$$

and for the b-filter

$$k\ dS_b = \int_b S_\lambda T_\lambda B'\ d\lambda \qquad \ldots(3)$$

$$k\ dC_b = \int_b S_\lambda N_\lambda T_\lambda\ d\lambda \qquad \ldots(4)$$

where k is the geometrical constant of the photometer (i.e. energy per unit deflection); S_λ, T_λ and N_λ are the photomultiplier response, the transmission of the filter and the radiance of the ^{14}C source (in raleighs/Å) respectively. (Values of N_λ from W cm^{-2} Å^{-1} ster^{-1} can be converted to raleighs/Å by multiplying by $6.3227 \times 10^7 \times \lambda$ where λ is the wavelength in Å). Subscripts λ and Λ denote any wavelength and the emission wavelength respectively. R_Λ is the intensity of the emission line, B and B′ are the background intensities in raleighs/Å in the emssion line and background spectral regions. Integrals in Eqs. (1) - (4) are carried out over the transmissions of l- and b-filters (ideally they should be from 0 to ∞). Assuming B and B′ constant in narrow spectral regions (≈ 50 Å), we have

$$\frac{dS_l}{dC_l} = \frac{S_\Lambda T_\Lambda R_\Lambda + B \int_l S_\lambda T_\lambda\ d\lambda}{\int_l S_\lambda N_\lambda T_\lambda\ d\lambda} = D_l \qquad \ldots(5)$$

and

$$\frac{dS_b}{dC_b} = \frac{B' \int_b S_\lambda T_\lambda\ d\lambda}{\int_b S_\lambda N_\lambda T_\lambda\ d\lambda} = D_b \qquad \ldots(6)$$

Therefore

$$B' = D_b \int_b S_\lambda T_\lambda N_\lambda\ d\lambda \Big/ \int_b S_\lambda T_\lambda\ d\lambda \qquad \ldots(7)$$

Now B can be calculated if K_l is known where $K_l = B/B'$ ...(8)

Thus from Eqs. (5) - (8) we obtain

$$R_\Lambda = D_l \frac{\int_l S_\lambda N_\lambda T_\lambda \, d\lambda}{S_\Lambda T_\Lambda} - K_l D_b \frac{\int_b S_\lambda N_\lambda T_\lambda \, d\lambda}{\int_b S_\lambda T_\lambda \, d\lambda} \times \frac{\int_l S_\lambda T_\lambda \, d\lambda}{S_\Lambda T_\Lambda} \qquad ...(9)$$

With approximations (to be discussed at the end) the equation has the familiar form

$$R_\Lambda = (D_l - K_l D_b N_{\lambda b}/N_{\lambda l}) N_{\lambda l} \; Q \qquad ...(10)$$

where

$$Q = \int_l T_\lambda \, d\lambda / T_\Lambda$$

In Eq. (10) except D_l and D_b which may change for every observation, all other quantities are constant for a pair of l- and b-filters.

Band Intensity

This technique can now be extended to a band such as OH (7-2) band. The OH rotational vibrational bands in night airglow and in the near infrared region have well separated P_1 and P_2 branches and rather crowded Q and R branches[5]. Let the relative intensities of the P branch be

$$^{\alpha}P_1(2),\ ^{\alpha}P_1(3),\ ^{\alpha}P_1(4), \ldots\ ^{\alpha}P_2(2),\ ^{\alpha}P_2(3),\ ^{\alpha}P_2(4) \ldots\ldots$$

where α's denote relative intensities of the components of P_1 and P_2 branches as given by Chamberlain[5]. Also let α_Q and α_R denote the total relative intensities of the Q and R branches.

Similarly let the filter transmission (T_λ) and the photomultiplier response (S_λ) at wavelengths for the above branches be denoted by

$$T_{P_1(2)},\ T_{P_1(3)},\ T_{P_1(4)}, \ldots\ T_{P_2(2)},\ T_{P_2(3)}, \ldots\ldots \ldots T_Q \text{ and } T_R,$$

and

$$S_{P_1(2)},\ S_{P_1(3)},\ S_{P_1(4)} \ldots\ S_{P_2(2)},\ S_{P_2(3)} \ldots\ldots S_Q \text{ and } S_R.$$

Thus, the deflection due to a OH band fiiter will be

$$R_{OH}\Big(^{\alpha}P_1(2) \cdot S_{P_1(2)} \cdot T_{P_1(2)} + {}^{\alpha}P_1(3) \cdot S_{P_1(3)} \cdot T_{P_1(3)} \ldots\ldots + {}^{\alpha}P_2(2) \cdot S_{P_2(2)}\, T_{P_2(2)} + {}^{\alpha}P_2(3)\, S_{P_2(3)} \cdot T_{P_2(3)} \ldots + {}^{\alpha}Q \cdot S_Q \cdot T_Q + {}^{\alpha}R \cdot S_R \cdot T_R\Big)$$

where R_{OH} is the total intensity of the OH band in question. As P_1 and P_2 branches have negligible intensities after 8th rotational line, we consider only 8 components of P_1 and P_2. Therefore, the above expression takes the form

$$\sum_{P \,\begin{smallmatrix} P_1=2 \\ P_2=2 \end{smallmatrix}}^{8} R_{OH}\,(\alpha_P \cdot S_P \cdot T_P) + R_{OH}\ \alpha_Q \cdot S_Q \cdot T_Q + R_{OH}\ \alpha_R \cdot S_R \cdot T_R$$

Along with the genuine OH radiation, the filter will also transmit background radiation. Let it be B rayleighs/Å. Again, assume that B is constant over the spectral range of a single OH band under measurement. Then the photometer deflection (response) through the OH filter to the night sky will be

$$k\, dS_{OH} = R_{OH}\left[\sum_{P \,\begin{smallmatrix} P_1=2 \\ P_2=2 \end{smallmatrix}}^{8}\Big(\alpha_P \cdot S_P \cdot T_P + \alpha_Q \cdot S_Q \cdot T_Q + \alpha_R \cdot S_R \cdot T_R\Big)\right] + B \int_{OH} S_\lambda T_\lambda \, d\lambda$$

where k is the geometrical constant of the instrument. Let A denote the square bracket in the first term. Then,

$$k\, dS_{OH} = R_{OH}\ A + B \int_{OH} S_\lambda T_\lambda \, d\lambda \qquad ...(11)$$

The photometer response for the OH filter with ^{14}C light source will be

$$k\, dC_{OH} = \int_{OH} S_\lambda T_\lambda N_\lambda \, d\lambda \qquad ...(12)$$

If $D_{OH} = dS_{OH} / dC_{OH}$ then from Eqs. (11) and (12)

$$R_{OH} = \frac{D_{OH}}{A} \int_{OH} S_\lambda N_\lambda T_\lambda \, d\lambda - \frac{B}{A} \int_{OH} S_\lambda T_\lambda \, d\lambda \qquad ...(13)$$

To evaluate B, background intensity in rayleighs/Å transmitted through the OH filter, another filter in the neighbourhood of the OH band under measurement is used which does not include any airglow emission [say 6080 or 7115 Å filter for OH (7-2) band].

Let the deflection due to this filter from sky be dS_b and that due to ^{14}C source be dC_b. Then the response of the photometer with ^{14}C source through the background filter is given by Eq. (4).

If the total night sky continuum intensity is B' rayleighs/Å in this spectral region and is assumed to be constant in the region transmitted by the b-filter, then we have

$$k\, dS_b = B' \int_b S_\lambda T_\lambda \, d\lambda \qquad ...(14)$$

Therefore, from Eqs. (4) and (14)

$$B' = D_b \cdot \frac{\int_b S_\lambda N_\lambda T_\lambda \, d\lambda}{\int_b S_\lambda T_\lambda \, d\lambda} \quad \text{..(15)}$$

where

$$D_b = dS_b / dC_b \quad \text{...(16)}$$

Thus when B′ is known B can be calculated provided the ratio of background intensities at the two peak transmission wavelengths of the filters is known. If this ratio is K_{OH}, then

$$B = K_{OH} \, B' \quad \text{...(17)}$$

Therefore, the intensity of the OH band is given by

$$R_{OH} = \frac{D_{OH}}{A} \int_{OH} S_\lambda N_\lambda T_\lambda \, d\lambda - \frac{B}{A} \int_{OH} S_\lambda T_\lambda \, d\lambda \quad \text{...(18)}$$

Substituting for B from Eqs. (15) and (17), we get

$$R_{OH} = \frac{D_{OH}}{A} \int_{OH} S_\lambda N_\lambda T_\lambda \, d\lambda - K_{OH} \frac{D_b \int_b S_\lambda N_\lambda T_\lambda \, d\lambda}{A . \int_b S_\lambda T_\lambda \, d\lambda} \times \int_{OH} S_\lambda T_\lambda \, d\lambda \quad \text{...(19)}$$

where R_{OH} is in rayleighs.

Useful Approximations and Errors Involved

Considering Eqs. (9) and (19) which give intensities of an emission line and a band respectively with the two-filter method, it can be observed that:

(1) In deriving these equations, it is already assumed that B and B′, the background emission over the spectral regions transmitted through the two filters, are individually constant. When the l-filter and b-filter have moderate widths ($\leqslant$ 50 Å) this may not be a bad assumption. However, it cannot be generlized for all spectral regions and should be used with caution.

(2) The ratio B/B′ = K must also be known accurately for the reduction. Values of K are assumed from some individual night sky spectra [6,7]. The background radiation in the night sky spectrum includes the three components, viz. (i) the extraterrestrial light, (ii) the zodiacal light, and (iii) the genuine airglow continuum. If one can further assume that the average spectral response of the first two sources is similar to that of the sun, B/B′ for these two components may remain the same. Regarding the variation of the third component, however, practically nothing is known. From the 5350 Å observations made at Mt. Abu, India, since 1964 with a fixed photometer looking to polestar (where starlight is constant), it is noted that the background intensity has variations even after accounting properly for extinction and scattering. The situation may be worse in the infrared region. Hence assuming a certain value of K seems to be

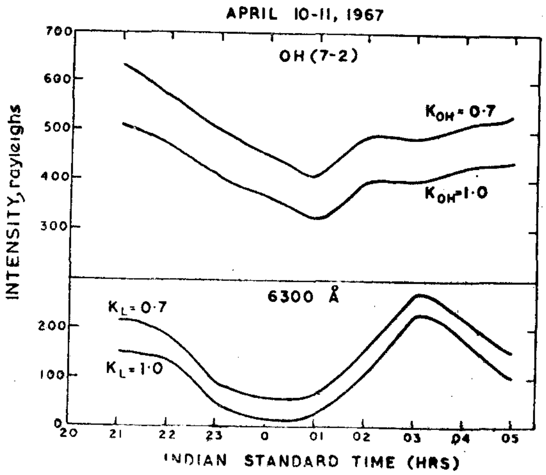

Fig. 1—Reduced night airglow data for one night with two assumed values of K each for 6300 Å and OH (7-2) band

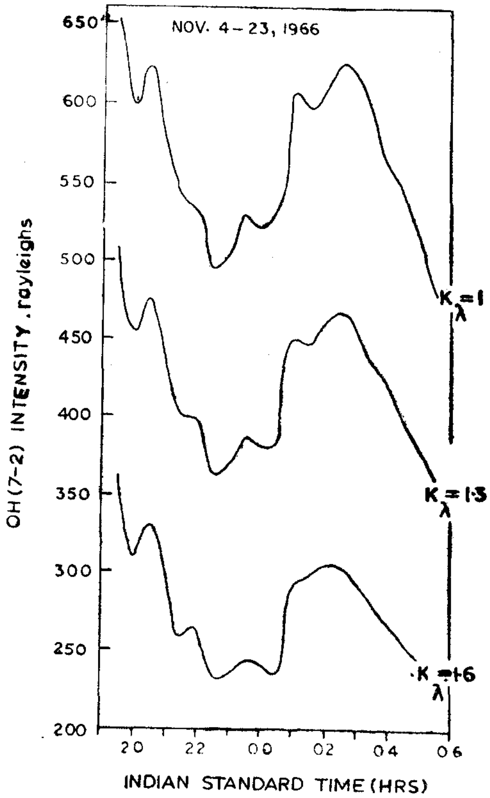

Fig 2—Reduced night airglow data with three assumed values of K for the OH (7-2) band. (The data are averages for one lunation)

TABLE 1—CHARACTERISTICS OF THE FILTERS USED WITH DIFFERENT PHOTOMETERS

Filter No.	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
λ_p	6306	5894	5750	5586	5300	6330	6080	6880	7270	6340	5880	5580	5350
λ_e	6300	5893	—	5577	—	6300	—	OH(7-2)	OH(8-3)	6300	5893	5577	—
T_{max}	69	79	74	78	14	71	77	42	32	40	43	24	27
H.W	47	41	34	38	130	105	48	120	100	135	97	88	45
S_λ	41.3	53.1	57.4	62.8	70.5	40.4	47.4	23.2	12.75	17.0	32.0	42.0	47.5
N_λ	5.133	15.60	17.97	19.37	15.81	4.710	10.36	1.412	0.9059	4.57	15.91	19.37	18.38

λ_p is the peak transmission wavelength of the filter in Å ; λ_e is emission wavelength in Å;

T_{max} is maximum % transmission of the filter ; H.W. is half-width of the filter;

S_λ is relative response of the photomultiplier ; N_λ is radiance of the ^{14}C source in units of 10^{-12} W cm^{-2}ster.$^{-1}$ Å^{-1}.

TABLE 2—RESPONSE OF THE PHOTOMETER (IN ARBITRARY UNITS) WITH SUCCESSIVE APPROXIMATION

Filter Wavelength, Å	I 6300	II 5893	III 5750	IV 5577	V 5300	VI 6300	VII 6080	VIII OH(7-2)	IX OH(8-3)	X 6300	XI 5893	XII 5577	XIII 5350
$\int S_\lambda N_\lambda T_\lambda d\lambda$	7.839	3.099	2.935	4.101	2.622	1.443	2.019	2.413	5.004	5.656	3.281	2.538	1.438
$N_\lambda \int S_\lambda T_\lambda d\lambda$	8.009	3.094	2.932	4.116	2.733	1.442	2.023	2.662	4.950	5.528	3.324	2.738	1.435
$S_\lambda \int N_\lambda T_\lambda d\lambda$	7.865	3.096	2.921	4.110	2.649	1.439	2.019	2.403	4.973	5.647	3.261	2.600	1.437
$S_\lambda N_\lambda \int T_\lambda d\lambda$	8.032	3.094	2.931	4.117	2.728	1.445	2.026	2.353	5.019	5.773	3.359	2.672	1.431
$S_\lambda N_\lambda T_\lambda$ (at 1/2 T_{max})	9.965	3.96	3.508	4.624	1.450	1.857	2.263	1.965	1.155	10.489	4.938	6.508	3.929

one of the weakest assumptions in the use of the formulae. The only solution is to determine the value of K experimentally or theoretically and use it in actual calculations. As an example, in Fig. 1 are shown reduced data for one night with two assumed values of K for 6300 Å and OH (7-2) night airglow emissions. Fig. 2 shows average intensities of OH (7-2) with 3 different K value for one lunation.

(3) Integrals appearing in Eqs. (9) and (19) have to be critically evaluated. However, it is quite convenient to approximate

$$\int S_\lambda N_\lambda T_\lambda d\lambda \qquad ...(20)$$

especially when the variations of N_λ and S_λ are smooth in the wavelength region (≈100 Å) where the filter has transmission. Integral (20) can be approximated to

$$S_\lambda \int N_\lambda T_\lambda d\lambda \qquad ...(21)$$

$$\text{or } N_\lambda \int S_\lambda T_\lambda d\lambda \qquad ...(22)$$

$$\text{or } S_\lambda N_\lambda \int T_\lambda d\lambda \qquad ...(23)$$

$$\text{or } S_\lambda N_\lambda T_\lambda \text{ (at 1/2 max. value of T)} \qquad ...(24)$$

Table 1 presents data of the filters and of the ^{14}C light source which we have used. Fig. 3 gives the responses of S_{10} and S_{20} type photomultiplier tubes and the ^{14}C light source used. Table 2 shows the results of calculations of the responses of the photometers with successive approximations using Eqs. (20)-(24) for filters listed in Table 1. The results lead to the conclusions:

(i) Values of the responses using Eqs. (20)-(24) do not differ by more than 2% when the filter width ⩽50 Å.

(ii) When the filters are rather broad the values calcultated from above equations differ by 5-10%.

(iii) Results obtained by Eqs. (20) and (24) (the worst approximation) differ by 10-20% for narrow filters and more than 20% for broader filters.

It may, therefore, be concluded that it is only necessary to evaluate $\int T_\lambda d\lambda$ while S_λ and N_λ can be considered constant within the narrow spectral range. $\int T_\lambda d\lambda$ can be evaluated by detailed calibration of the filter with a good spectrophotometer.

It must also be remembered that the filters should not have secondary transmissions to which the

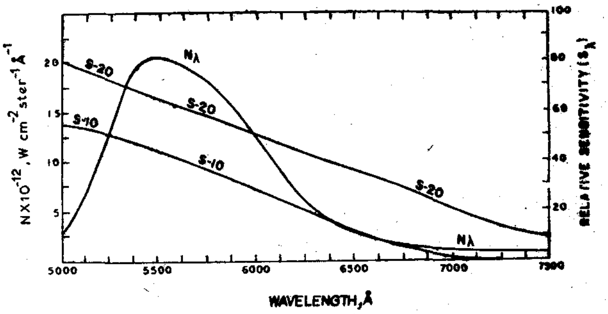

Fig. 3—Responses of S_{10} and S_{20} type photomultiplier tubes and the ^{14}C light source

photomultiplier is sensitive (In modern multilayer filters, care is taken to suppress all secondary transmissions). Also, we have not considered the calibration of the secondary standard of light (^{14}C light), which is one of the very important problems, and its variation with temperature[8] and time[9]. The source should be used preferably at one temperature and should be calibrated against a standard source at least once a year. One should also take note of the effects of (i) time, (ii) temperature, and (iii) inclination of the filter to the incident light, on the transmission characteristics of the filter.

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