Acceleration of Charged Particles in Magnetospheric Neutral Plasma Sheet

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The reconnection of magnetic field lines in the neutral plasma sheet has been studied in the light of momentum conservation and charge neutrality. The effect of field variations and plasma conductivity on the charged particle dynamics has been studied. It is found that the charged particles emerging out of the neutral plasma sheet are energized. It is argued that the reconnection process would make the plasma sheet unstable. The onset of likely instabilities in the neutral plasma sheet and their geophysical implication has been briefly outlined.

1. Introduction

The interaction between the supersonic solar wind with the dipole field of the earth gives rise to a collisionless shock front at a distance of about 10 $R_E$ in the solar direction. The shock front thus formed marks a limit of the perturbing influence of the geomagnetic field on the flow properties of the solar wind. The thermalized solar wind plasma flows downstream and sweeps the geomagnetic field lines to form an extended magnetospheric tail. Experimental measurements made on board the Imp-I satellite have revealed the existence of such an extended magnetic tail to great distances behind the earth in the antisolar direction. The solar wind interaction with the geomagnetic field and magnetospheric tail formation is shown by a schematic in Fig. 1. The plasma region between oppositely directed magnetic fields in the tail is known as the neutral plasma sheet. The tail field at about 20 $R_E$ has a strength of about (10-30) $\gamma$ and at the time of a magnetic storm its strength is found to increase by about 10 $\gamma$ ($1 \gamma = 10^{-5}$ gauss).

The enhancement in the tail field is often found correlated with the solar activity index and solar wind velocity. Further, these features have been found to be correlated with the auroral index, micropulsation activity, electron influx precipitation and emission of optical radiations. The morphological features of these geophysical phenomena and their correlation are suggestive of the possible role of neutral plasma sheet. With this in view, Speiser\textsuperscript{*} has studied the acceleration of charged particles in the geomagnetic tail using different models of the tail field. In a simple linear model, neglecting reconnection ($B_x = 0$), particles of either sign execute damped oscillations about the neutral plasma sheet, gain energy continuously from the electric field with electrons moving towards positive Z-direction and protons towards negative Z-direction. The major difficulty with this model is that the particles never come out of the current sheet and their energy increases without bound. This catastrophe of infinite energization was removed by accounting for the non-parallel tail fields. In dipole and tail model, particles of either sign oscillate about the neutral sheet, gain energy from the electric field and are ejected out of the sheet towards the earth. Those particles that are ejected below the current sheet impinge on the southern hemisphere while those moving upwards are directed to the northern hemisphere. In the extended tail model $B_x$ is assumed to vary linearly with distance along the tail decreasing from 1 $\gamma$ at 50 $R_E$ to zero at 500 $R_E$. Higher values of $B_x$ give rise to reconnection of field lines, which results into further energization of charged particles. The particle energization has been extensively studied by Cowley\textsuperscript{**}.

The intent of the present paper is to develop a better understanding of the neutral plasma sheet models, variation of electric and magnetic fields, energization of charged particles and direction of their ejection from the current sheet. Section 2 of this paper describes some of the important morphological features of the neutral plasma sheet which is essential for developing theoretical details. In Section 3 we have outlined theoretical details for two special conditions of sheet field; the normal component of the sheet field $B_x \neq 0$ and $B_x \to 0$. In the case $B_x \to 0$, the role of conservation of momentum accounts for the increasing width of neutral plasma sheet from dawn to dusk, whereas the violation of charge neutrality accounts for the detailed distribution of electric potential and field in the sheet. The dynamics of the charged particles, their energization and ejection have been considered in Section 4 of this paper. The number of oscillations executed by the charged particles before ejection, their ejection time, pitch angle and the distance traversed have been computed and details
Fig. 1.—Schematic showing the formation of magnetospheric neutral sheet
are shown in various figures. In Section 5 of this paper we have discussed the stability of neutral plasma sheet. It is shown that under process of reconnection the entire sheet would tend to be unstable. The role of tearing mode instability and damping of electrostatic waves have been discussed.

2. Morphology of Neutral Plasma Sheet and Formation of Neutral Points

The neutral plasma sheet is shown in the schematic of Fig. 1. The cross-sectional view of the magnetospheric tail is shown in Fig. 2(a). The hatched part in the centre of this figure shows the field reversal region. Fig. 2 (b) shows a side view of the neutral tail. In these diagrams the presence of non-parallel tail field is not shown. Speiser accounted for a finite perpendicular component of the tail field. In Fig. 3 (a) we have shown the dipole and tail model of Speiser. In the extended tail model the perpendicular component of the tail field $B_z$ is shown to vary from $1 \gamma$ at $50 R_E$ to almost zero at $500 R_E$.

The solar wind sweeps the frozen-in dipole field and brings it into the tail region. The plasma continuously drifts along the X-axis and in doing so gives rise to $\mathbf{V} \times \mathbf{B}$ field along the Z-axis. In the region close to the neutral point which is known as the diffusion region both the magnetic field and the velocity of the incoming plasma just approach zero so that for the electric field to be finite $E + \mathbf{V} \times \mathbf{B} = 0$. In this region Ohm's law can be written as

$$\mathbf{J} = \sigma [E + \mathbf{V} \times \mathbf{B}] \quad \ldots \ldots (1)$$

In order that the current density may be finite the conductivity near an X-type neutral point has to be finite. In the absence of particle collisions in the geomagnetic tail, $\sigma$ represents either the inertial conductivity or the gyro-conductivity. These conductivities are the usual scalar conductivities in which the mean time between collisions is replaced by either the life-time of the particle in the system or the gyro-period of the particle about $B_0$. Using these conductivities the electric field for a given region of the tail is found to agree with the observations of magnetospheric convection fields. Because of finite conductivity near an X-type neutral point there will be diffusion of field lines eventually giving rise to field line reconnection. The magnetic energy liberated during field annihilation is used in particle energization and reappears in the form of particle kinetic energy.

In the neutral sheet since the gyro-radius is large as compared to the distance over which magnetic field changes appreciably, adiabatic theory cannot be applied in describing the motion of charged particles. The individual particle trajectories have, therefore, to be computed. This would help in studying the role of neutral points in the acceleration mechanism, in the energy spectra of the accelerated particles and in the spatial distribution of these particles on ejection from the neutral sheet.

3. Theory of Particle Energization

The theoretical discussion of the particle dynamics and energization is given in two sub-sections. In Section 3.1 we give the brief theory relevant to that region of the neutral sheet where $B_z$ is significant and in Section 3.2 the details of particle dynamics near
the neutral line where \( B_x \approx 0 \) are given. In both these cases we have taken the co-ordinate system as shown in Fig. 2 (b).

### 3.1 Case when \( B_x \neq 0 \)

Let the tail field be directed along Y-axis with a small component along X-axis. The electric and magnetic fields in the tail configuration are written as

\[
\mathbf{b} = B_0 \left( \eta \, e_x - \frac{x}{d} \, e_y \right) \quad \ldots (2)
\]

\[
E = - E_0 \, \hat{a}_z \quad \ldots (3)
\]

where \( \eta = B_x / B_0 \), \( E_0 \) represents the magnitude of the electric field and \( 2d \) is the sheet thickness. The equations of motion of charged particles in such a field configuration are

\[
\ddot{x} = c_1 x \quad \ldots (4)
\]

\[
\ddot{y} = c_2 \eta \, y \quad \ldots (5)
\]

\[
\ddot{z} = - c_3 - c_1 x - c_2 \eta \, y \quad \ldots (6)
\]

where \( c_1 = q/m \, (B_0/d) \), \( c_2 = q/m \, (B_0) \), \( c_3 = \frac{q}{m} \, E_0 \)

and \( q, m \) are the charge and mass of plasma particles. These equations help us to visualize the dynamics of charged particles in the neutral sheet. The positively charged particles drifting into the neutral sheet are accelerated along negative Z-axis and gain a velocity \( \dot{z} \) proportional to \((-t)\). Eq. (5) depicts the charged particle acceleration along negative Y-axis which is proportional to \((-t)\). For negative values of \( z \), Eq. (4) implies oscillatory motion in \( x(t) \). These oscillations in the neutral sheet under suitable conditions can either damp or grow with time. When \( \dot{z} \) changes its sign from negative to positive the oscillation amplitude along X-axis increases monotonically with time and the particles are eventually ejected from the neutral sheet at small pitch angles. Using approximate analytical solutions the ejection time \( \tau \), the ejection velocity \( V_x \) and the distance travelled towards earth before ejection \( l_x \) are given by the relations

\[
\tau = \pi m q B_x \quad \ldots (7)
\]

\[
V_x = - \frac{2 E_0}{B_x} \quad \ldots (8)
\]

and

\[
l_x = - \frac{\sqrt{6} \, E_0}{(q/m) \, B_x^2} \quad \ldots (9)
\]

Further the number of oscillations \( n \) executed by the charged particles about the current sheet before ejection and the pitch angle \( \alpha \) of the emergent particles are given by

\[
n = \frac{1}{2 B_0} \left[ \frac{E_0}{2 \, (q/m) \, \eta d} \right]^{1/2} \quad \ldots (10)
\]

\[
\alpha = \frac{B_x \left( \frac{x_0}{U} \right)^2}{3 B_0} \quad \ldots (11)
\]

where \( u = E_0 / B_0 \) and \( x_0 \) is the initial X-component of the velocity of particles.

### 3.2 Case when \( B_x \to 0 \)

The problem of infinite energization of particles near an X-type neutral point where \( B_x \to 0 \) was solved by Alfvén by considering a finite width of the tail in Z-direction and by using a condenser plate analogy [Fig. 3 (b)]. Under the action of the magnetic field and dawn-to-dusk reconnection electric field, there will be \( \mathbf{E} \times \mathbf{B} \) drift of plasma particles into the neutral sheet where charge separation takes place giving rise to an electric current. It is shown by Alfvén that for self-consistency between the current provided by the incoming drifting plasma and the change in magnetic field across the current sheet the potential difference across the width of the tail is uniquely defined by the equation

\[
V_0 = \frac{B_x^2}{\mu_0 \eta q} \quad \ldots (12)
\]

Eq. (12) also follows directly by equating the Poynting's energy flow flux into the sheet from both sides to the energy carried by particles flowing out from the dusk and dawn boundaries of the sheet showing that Alfvén potential is in accordance with the conservation of energy.

In close proximity to the neutral line where \( B_x \) is exceedingly small, charged particles traverse the entire width of the neutral sheet and are ejected from the sides. It is shown by Cowley that in such field geometry (i.e., near an X-type neutral line) neither charge neutrality is maintained nor momentum is conserved. The conservation of momentum and charge neutrality result in a radical change in the electric field configuration and the tail structure.

#### 3.2.1 Conservation of momentum

The drift motion of charged particles outside the neutral sheet is independent of charge and mass of the incoming particles so that each blob of incoming plasma after charge separation in the neutral sheet provides a constant current throughout the entire sheet. Since the protons and electrons emerge out of the sides of the neutral sheet with the same energy spectrum ranging from zero to \( q V_0 \) (\( V_0 \) being the Alfvén potential) we find that the protons because of their larger mass, are responsible for a net flow of momentum towards the dusk boundary. Cowley has shown that in order to conserve the flow of momentum, the integral of \( B_0^2 / 2 \mu_0 \) along an equipotential decreases as we move from dawn to dusk. Since outside the neutral sheet...
the magnetic field is assumed constant, this would imply that the width of the neutral sheet increases as we move from dawn to dusk. Another way of arriving at this conclusion is in terms of the net reacting force acting on the tail on the dusk side due to the net momentum carried by protons. In order to account for the excess force, the magnetic force inside the tail on the dusk side must decrease which is possible only if the width of the neutral sheet on this side of the tail increases.

### 3.2.2 Charge neutrality in the current sheet

Accounting for the charge neutrality within the current sheet we find that the acceleration experienced by electrons in the neutral sheet is \((m_p/m_e)\) times the one experienced by protons, and, therefore, the electrons are quickly whisked away towards the dawn boundary leaving localization of a net positive charge near the dusk end. This non-neutrality of charge gives rise to X-component of the electric field outside the neutral sheet. The equipotentials for the resultant electric field are found to be concentrated near the dusk end [Fig. 3 (c)]. The charged particles outside the neutral sheet now drift towards the dusk side before entering the neutral sheet so that most of the Alfven potential drop is now localized in a small region of space near the dusk boundary with most of the space across the neutral sheet as a field-free space. Under the influence of \(E_x\), charged particles outside the neutral sheet will drift across the equipotentials towards the dusk side gaining energy. Due to high energy gain by protons outside the neutral sheet the particles violate the adiabatic condition of motion. The energy gain by electrons is comparatively small, and therefore, electrons continue to move adiabatically along the same equipotentials. Some of the drifting protons leave the tail before entering the current sheet and most of the remaining protons which enter into the current sheet are quickly ejected out of the sheet due to Z-component of the electric field. The electrons after entering into the current sheet near the dusk boundary move almost the entire width of the tail in nearly field free space as shown in Fig. 3 (c). The protons after entering the field reversal region start oscillating about the neutral sheet while electrons move adiabatically almost up to the neutral sheet. For quasi-neutrality of charge in the sheet, Cowley\(^7\) has shown that most of the Alfven potential drop takes place near the dusk end over a characteristic length of the order of few proton plasma wavelengths.

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**Fig. 4**—Variation of ejection velocity with \(B_x\) for different values of \(E_D\)

**Fig. 5**—Variation of distance traversed by particles towards earth before ejection with \(B_z\) for different values of \(E_D\)
4. Ejection of Charged Particles and Dynamical Parameters

Using the extended tail model we have calculated the dependence of the ejection velocity (same for electrons and protons) on \( B_x \) for different values of the electric field (Fig. 4). We find that the ejection velocity of a particle entering the geomagnetic tail at a particular distance increases linearly with the strength of the electric field. The distance traversed by the particles towards the earth before ejection has been calculated and its variation with \( B_x \) is shown in Fig. 5. The variation of the ejection time of both the species of particles with \( B_x \) is shown in Fig. 6. The ejection time of particles depends upon the mass of the particles. We find that the ejection time for both the particle species decreases as \( B_x \) increases. Thus away from an X-type neutral line, where \( B_x \) is large, the particle energization within the neutral sheet may not be significant. This is due to the fact that the time spent by the particles within the sheet during which
they could be accelerated to high energies by the electric field is rather small.

With the approximation that the X-component of the ejection velocity of the particles is of the order of $x_0$ (where $x_0$ represents the X-component of the velocity of the particle at a time when it just enters the current sheet), the pitch angle of the ejected particles is given by Eq. (11). In actual practice, however, the X-component of the velocity of the particle at the time of ejection is governed by phase of its oscillation. When $x$ just changes sign it is very likely that $x$ may be different from $x_0$. Using Eq. (11) we have, therefore, computed the variation of pitch angle of the ejected particles as a function of $U$ (for possible values of $x_0$ and $\eta$) which is shown in Figs. 7(a) and 7(b). For a particular $U$ and $x_0$ the pitch angle of the ejected particles is found to be small for smaller values of $B_x$. In such a region the energized charge particle flux distribution would be confined within the loss cone. Therefore, a flux of charged particles will be ejected very nearly along the field lines and these are likely to be precipitated in the auroral regions. It is these particles which give rise to various observed auroral features. Corresponding to large $x_0$ and small $U$, the pitch angle of the ejected particles are large. It is, therefore, unlikely that such particles would precipitate into the auroral atmosphere. Fig. 8 depicts the variation of the number of oscillations completed by a proton before ejection with tail field values corresponding to different values of $E_0$ and $B_x$. The number of oscillations completed by electrons are $\sqrt{m_p/m_e}$ times smaller.

5. Stability of Neutral Plasma Sheet

We have considered the reconnection model of the neutral plasma sheet which is in good agreement with the satellite observations and have studied the energization of charged particles emerging out of the sheet. Dessler's model, for example, argues that no significant merging of the field lines occurs. In the absence of concrete experimental evidence the Dessler model remains unsupported. The stability of the neutral plasma sheet was first proposed by Coppi, Laval and Pellat\(^1\) and they argue in favour of microscopic instabilities in the neutral plasma sheet and ignore the rate of conversion of magnetic energy into particle energy. The rate of merging of the field lines is chiefly governed by the boundary conditions and with the suitable boundary conditions the rate of merging may vary from zero to Alfvén speed. The rate of merging of field lines also depends on the rate at which the newly reconnected field lines can get away from the region of reconnection. The merging will be very rapid if the reconnected lines can shrink freely.

The neutral plasma sheet if assumed to be perfectly conducting favours the Dessler's model which predicts the metastable thin parallel neutral plasma sheet. However, the collision-free dissipation of neutral plasma sheet has been invoked which results into finite conductivity. In the presence of finite conductivity of neutral plasma sheet the magnetic fields (oppositely directed parallel fields) will slowly diffuse into the medium as the directed drift motion of the charge becomes randomized resulting into heat dissipation and faster decay of sheet current. The onset of this process would lower the conductivity of the neutral plasma sheet and increase the rate of diffusion of field lines. This is a particular type of resistive instability—the tearing mode—which has been studied by Schindler and his colleagues\(^1\). The field lines under the wake of tearing mode instability break into various loops as shown in Fig. 9. Observational support to this model has come from measurement of Behannon\(^1\) who reports the existence of positive (south-north) and negative (north-south) average normal components. From similar observations Mihalov et al.\(^1\) have also concluded the looping of field lines in the neutral sheet. Another
indirect support to tearing instability comes from
the occasional appearance of multiple arcs in auroral
display. The charged particle ejection from the neutral
plasma sheet is also structured in response to various loops formed in the neutral plasma sheet, (Fig. 9). The loop formation, in fact, is equivalent to the presence of many X-type neutral points which
in turn support various regions of charged particle
energization and their ejection from different regions
with different velocity and pitch angles.

The problem of wave particle interaction in the magnetospheric tail has been discussed by Axford and Dungey and Speiser. The interaction of these waves with streaming electrons has been studied by

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**Fig. 9**—Tearing instability due to ejection of charged particles from neutral plasma sheet

**Fig. 10**—Variation of $\frac{Y(Z)/Y_{Landau}}{}$ with distance $Z$ across the tail for different values of $K_D$
Gjoen\textsuperscript{19} after taking into consideration the case of one-dimensional propagation of electrostatic waves in \( Z \)-direction. Assuming the Alfven potential drop \( V_0 \) to take place over a distance \( D \) near the dusk boundary and considering the presence of small amplitude electrostatic waves, the potential variation \( \phi \) as a function of distance \( Z \) is given by

\[
\phi = V_0 \sin kZ / \sin kD \quad \text{(13)}
\]

where

\[
K^2 = \frac{nq^2}{m} \epsilon_0^2 \quad \text{(14)}
\]

and \( KD \neq p\pi, p \) being an integer.

The imaginary part of the wave frequency \( \gamma(Z) \) which controls the rate of energy exchange between the electrostatic wave and the streaming electrons is given by

\[
\gamma(Z) = \gamma_{\text{Landau}} \frac{1}{\sqrt{1 - \frac{2q \phi}{m V_{\text{phase}}^2}}} \quad \text{(15)}
\]

where \( \gamma_{\text{Landau}} \) is the normalized Landau damping factor. Taking the phase velocity, \( V_{\text{phase}} \), consistent with the general dispersion relation of electrostatic waves and ignoring space variation of \( K \), the wave number, we have studied the variation of \( \gamma(Z) \) as a function of distance \( Z \) in the neutral sheet near the dusk boundary which is shown in Fig. 10. The damping rate for \( KD \leq \pi/2 \) is found to increase from \( \gamma_{\text{Landau}} \) at the dusk boundary to a maximum for \( Z = D \) consistent with monotonic potential variation across the sheet.

6. Conclusions

The need for energetic particle flux to explain the magnetospheric and ionospheric phenomena such as convection processes, appearance of auroral forms, optical and X-ray emissions and radiowave absorption events, strongly supports an acceleration mechanism at work. The mechanism discussed in this paper therefore, lends support to interpret and correlate these phenomena. The fine structure of magnetospheric and ionospheric processes and their time evolution can be explained in terms of instabilities discussed in this paper with relative ease. Particularly the multiple arcs, periodic precipitation of particles and their spatial correlation lend support to the onset of tearing mode instability and looping of field lines.

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