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Propagation Characteristics of an Antenna in an Incompressible Plasma*

D. C. AGARWAL†

J.K. Institute of Applied Physics & Technology
University of Allahabad, Allahabad-2

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A study is made of the propagation characteristics of a line antenna of infinite length immersed in an incompressible plasma, and it is found that a complex surface wave accompanying the radiation of a plasma wave is generated. The phase and the attenuation constants are found to decrease as the ratio of the propagation constants for the electron plasma wave and the plane electromagnetic wave increases — a result quite similar to the case of the so called G-line.

1. Introduction

THE STUDY OF antennas in plasma[1-5] is important due to the fact that when a satellite traverses the ionospheric medium, a plasma sheath is generated around the antenna. The aim of this work is to study the propagation characteristics of an antenna of infinite length radiating in an incompressible isotropic plasma.

2. Theory

We assume that the antenna is infinitely long and cylindrical in shape (of radius a) and that it is an ideal conductor. It is surrounded coaxially by a cylindrical plasma sheath (radius b). Outside the sheath the medium is assumed to be isotropic incompressible plasma. In such an antenna system it can be shown that the current has only a longitudinal component having longitudinal symmetry. Taking the boundary conditions that the tangential component of the electric field is zero at the surface of the perfect conductor and the normal component of the microscopic velocity of electrons is zero at radial distance $r = b$, the dispersion relation for the waves propagating in longitudinal direction may be written [using positive eikonal, $\exp(j\omega t)$] as :

$$k_0' \left(1 - \frac{\omega_p^2}{\omega^2}\right) \frac{J_0(k_0'a)\, N_0(k_0'b) - J_0(k_0'b)\, N_0(k_0'a)}{J_0(k_0'a)\, N_1(k_0'b) - J_1(k_0'b)\, N_0(k_0'a)}$$

$$= k_e' \frac{H_0^{(2)}(k_e'b)}{H_1^{(2)}(k_e'b)} + \frac{\omega_p^2 k^2}{\omega^2 k_p'} \frac{H_0^{(2)}(k_p'b)}{H_1^{(2)}(k_p'b)} \quad \ldots(1)$$

where $(J_0\ J_1)$, $(N_0\ N_1)$ and $(H_0\ H_1)$ are ordinary, Neumann's and Hankel's Bessel functions respectively of orders 0 and 1 ; ω is the angular frequency and k is the propagation constant; (k_0,ω_0), (k_e,ω_e) and (k_p,ω_p) are the corresponding quantities for "free-space" plane electromagnetic wave and the electron plasma wave, respectively; and

$$k_0'^2 = k_0^2 - k^2 \quad \ldots(2)$$

Similar expressions for k_e' and k_p' can be obtained from Eq. (2) with k_0 replaced by k_e and k_p respectively, wherein

$$k_e^2 = k_0^2(1 - \omega_p^2/\omega^2) \quad \ldots(3)$$

$$k_p^2 = (\omega^2/u^2)\ (1 - \omega_p^2/\omega^2) \quad \ldots(4)$$

where u is the root mean square electron thermal velocity and is proportional to the square root of electron temperature. The second term in the right hand side of Eqs. (3) and (4), shows the influence of the compressibility of the plasma. Since it is difficult to solve Eq. (1) directly, we assume $\frac{k_p}{k_e} = \frac{c}{u}\ (\gg 1)$, c being the velocity of light, and in real plasma $k_0 b \ll 1$ and $k_p b \gg 1$. In particular, using Eq. (2) it is possible to assume approximately that

$$k_0' b,\ k_e' b \ll 1\ ;\ k_p b \gg 1 \quad \ldots(5)$$

This means that the thickness of ion sheath is much smaller than the wavelength of electromagnetic waves in free-space. However, it is much greater than that of the electron plasma wave. Moreover, we consider the electromagnetic mode along the cylinder and calculate the values when $k = k_e$.

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†Persent Address : Institute Für Physikalische Weltraumforschung, Heidenhofstrasse-8, D 78, Freiburg, West Germany.

From Eq.(5) and Eq. (1), and using the well known approximations to the Bessel functions occurring in Eq. (1), we obtain :

$$x^2 = \frac{(\omega_p/\omega)^2 \,[\ln\,(b/a) - j\,(1/k_p b)]}{\ln\,(x\,k_e a/2) + j\frac{\pi}{2} + (\omega_p/\omega)^2 \left\{\ln\,(b/a) - j\left(1/k_p b\right)\right\}} \quad ...(6)$$

where $x = (k_e'/k_e)^{1/2}$ ...(7)

Eq. (6) can be solved by an iterative approximation method with the initial root as $x_0 = -j$ and x^2 calculated. For the limiting case of incompressible plasma, that is, when $\left(\frac{k_p}{k_e}\right) \to \infty$ Eq. (6) (with $k_e a \ll 1$) reduces to :

$$x^2 = \frac{(\omega_p/\omega)^2 \ln\,(b/a)}{\ln\,|x| + \ln\,(k_e a/2) + (\omega_p/\omega)^2 \ln\,(b/a) + j\frac{\pi}{2}} \quad ...(8)$$

3. Discussion

If $k > k_e$, then from Eqs. (2) and (7) it is seen that x^2 is real and negative. This behaviour is similar to that of the so-called G-line propagation constant. The values of the phase part of the propagation constant obtained from Eq. (6) using the iteration method, is shown in Fig. 1. The corresponding attenuation constant is plotted in Fig. 2.

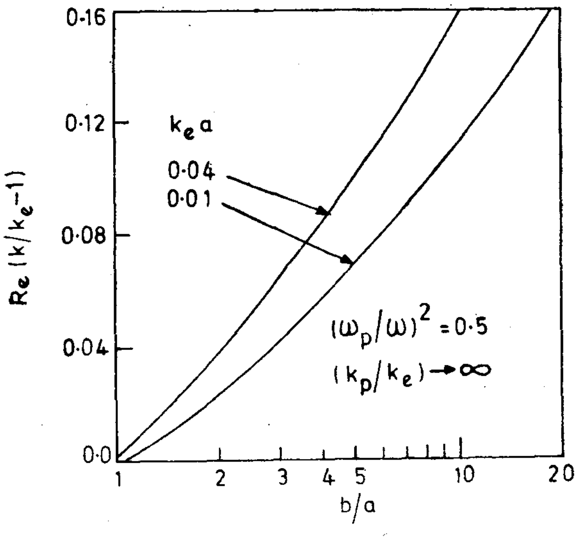

Fig. 1—Variation of phase constant, the real part $\mathrm{Re}\left(\frac{k}{k_e} - 1\right)$ with b/a

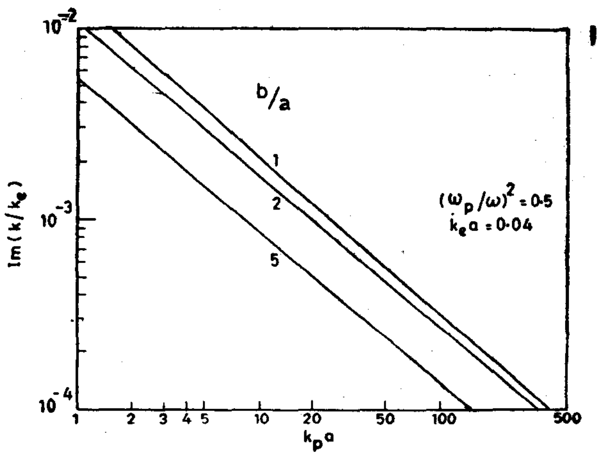

Fig. 2—Variation of attenuation constant, the imaginary part Im (k/k_e) with $(k_p a)$

From these plots it is evident that as k_p/k_e increases, the phase and attenuation constants decrease (quite similar to that of the G-line).

The electromagnetic wave, as $(k_p/k_e) \to \infty$ is the complex surface wave and its attenuation comes from the fact that the electromagnetic wave is changed into the plasma wave at the boundary of the plasma ion sheath and compressible plasma. Then it is radiated into free space as a leaky wave. Assuming that there is no ion sheath and the phase constant of the surface wave $k = k_e$ the wave equation for the plasma wave in cylindrical coordinates, with Z-axis along the axis of the antenna, may be written as

$$\nabla_p^2 + k_p^2 p = -\frac{N_e k_e}{\omega \epsilon_0} I \exp\,[-j\,k_e\,z\,\delta\,(r)] \quad ...(9)$$

where I is the total current, p is the pressure of the plasma N is electron density and $\delta\,(r)$ is a radial function of r.

Thus, p may be written as

$$p = \frac{j N_e k_e I}{4\omega\epsilon_0} H_0^{(2)}\left(\sqrt{k_p^2 - k_e^2}\cdot r\right) e^{-jkz} \quad ...(10)$$

In general $k_p \gg k_e$ so that plasma wave is radiated at angle θ (with respect to the direction of propagation of the complex surface wave) given by

$$\theta = \arccos\,(k_e/k_p) \approx \frac{\pi}{2} - (k_e/k_p) \quad ...(11)$$

The above analysis shows that the electromagnetic mode which decides the fundamental current distribution of the line antenna immersed in a compressible plasma becomes a complex surface wave which radiates a plasma wave. The complex wave is rapidily attenuated leaving behind the plasma wave beyond the boundary of the sheath.

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