A Method for Estimation of Eddy Diffusion from Atmospheric Electrical Parameters

H. R. SINGH, D. CHAND & M. PRASAD
Physics Department, University of Roorkee, Roorkee

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A close relationship between the atmospheric electrical parameters (electrical field, space charge and conductivity) and coefficient of eddy diffusion has been established by taking continuity equation for space charge flow under steady state conditions. The coefficient of eddy diffusion, $K$, is taken as height-dependent. The diurnal variation of $K$ is obtained by putting the diurnal variation values of electric field, space charge and conductivity in the final relation—a method of determining $K$ by the method of atmospheric electricity.

1. Introduction
The phenomenon of eddy diffusion is associated with the atmospheric particulates of differing origin. Information on this parameter is important for the understanding of the atmospheric transport of heat, momentum, ions, etc. The coefficient of eddy diffusion ‘$K$’ which is a measure of this exchange phenomenon, is known to play a crucial role in the explanation of the different types of variations in the atmospheric electrical parameters. As the distribution of space charge in the lower atmosphere in fair weather is affected by the turbulent exchange (eddy diffusion), the diurnal variation of electric field in fair weather on land seems to be related with the coefficient of eddy diffusion. The exchange (eddy diffusion) as a governing element in atmospheric electricity is also confirmed by the diurnal variation of the vertical atmospheric mass exchange.

The height dependence of the eddy diffusion coefficient, $K$, has been discussed earlier by many workers. Estimation of vertical turbulent eddy diffusivity has been made by comparing the observed profiles of radioactive elements (Rn, Tn) with those calculated ones near the ground

A linear relation for $K(z)$ with altitude has been found in all these estimations. An empirical relation for $K(z)$ can be found by meteorological variables for a certain region and it is of great interest to study the diurnal variation of eddy diffusivity with the help of atmospheric electricity variables—a direct correlation amongst $K(z)$ and other electrical variables. And the use of this height-dependent $K$ has been made by Chand and Varshneya in finding out the space charge distribution with height.

In the present paper, calculations have been made of the relationship among the elements of atmospheric electricity and the coefficient of eddy diffusion (taking it to be height dependent). And by substituting the diurnal variation of various elements of atmospheric electricity one can find out the diurnal variation of $K(z)$.

2. Theoretical Considerations
In steady state the distribution of space charge (in space and time) in the atmosphere is governed by the continuity equation

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$$

where $J$ is the current density and $\rho$ is the space charge density. As we know, the distribution of space charge between two atmospheric layers is determined by eddy diffusion near the ground, and therefore, the current density $J$ comprises both conduction and convection (by eddy diffusion) and is given by

$$J = - \left( \lambda F + K(z) \frac{\partial \rho}{\partial z} \right)$$

where $F$ is the electric field, $z$ is the altitude, and $K(z)$, the coefficient of eddy diffusion is a function of $z$. Negative sign appears because $J$ is measured in the same direction as $z$. Now, if the space charge transported into a unit volume of the atmosphere by the eddy diffusion is balanced by the space charge dissipated by the atmospheric electric conductivity in the same unit volume (i.e. in ionization equilibrium condition

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*exchange (Austausch) or eddy diffusion phenomenon

**Because we do not have simultaneous observations on the diurnal variation of all the three parameters, viz. electric field, space charge and conductivity, we are not giving the calculations at present.

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when $\frac{\partial \rho}{\partial t} = 0$ — a fairly reasonable assumption in fair weather as there is an equilibrium between production and recombination of ions, then the continuity equation, with the help of Eq. (2) becomes

$$\nabla \cdot \left( - \lambda \mathbf{F} - K(z) \frac{\partial \rho}{\partial z} \right) = 0 \quad \ldots (3)$$

or, assuming $\lambda$ to be constant

$$- \lambda \nabla \cdot \mathbf{F} = \nabla \cdot \left( K(z) \frac{\partial \rho}{\partial z} \hat{k} \right) \quad \ldots (4)$$

where $k$ is unit vector in Z-direction.

The electric field near the ground is given by Poisson's equation

$$\nabla \cdot \mathbf{F} = - 4\pi \rho \quad \ldots (5)$$

Now Eq. (4) reads as

$$4 \pi \lambda \rho = \nabla \cdot \left( K(z) \frac{\partial \rho}{\partial z} \hat{k} \right) \quad \ldots (6)$$

Taking only one-dimensional solution as $J$ is assumed to be independent of $X$ and $Y$ coordinates and assuming a linear relation for $K(z)$, such as

$$K(z) = k_0 z \quad \ldots (7)$$

where $k_0$ is a constant, a measure of the strength of the convection, and putting Eq. (7) in Eq. (6), the latter takes the form,

$$k_0 \frac{\partial \rho}{\partial z} + k_0 \frac{\partial \rho}{\partial z} - 4 \pi \lambda \rho = 0 \quad \ldots (8)$$

or

$$z \frac{\partial ^2 \rho}{\partial z^2} + \frac{\partial \rho}{\partial z} - \mu \rho = 0 \quad \ldots (9)$$

where

$$\mu = \frac{4 \pi \lambda}{k_0}$$

The general solution of the Eq. (9) is

$$\rho = A I_0 (2\sqrt{\mu} z) + B K_0(2\sqrt{\mu} z) \quad \ldots (10)$$

where $I_0$ and $K_0$ are the modified Bessel functions of the first and second kind respectively, of order zero, and $A$ and $B$ are arbitrary constants.

With the boundary conditions,

(i) $\rho = 0$, at $z = \infty$
(ii) $\rho = \rho_h$, at $z = h$

we get the arbitrary constant, $A = 0$ and

$$B = \frac{\rho_h}{K_0(2\sqrt{\mu} h)}$$

Hence, finally the distribution of space charge is given by

$$\rho = \frac{\rho_h K_0 (2\sqrt{\mu} z)}{K_0 (2\sqrt{\mu} h)} \quad \ldots (11)$$

Now, rewriting Eq. (5) and putting the value of $\rho$ from Eq. (11)

we have

$$\frac{dF}{dz} = \frac{- 4\pi \rho_h K_0 (2\sqrt{\mu} z)}{K_0 (2\sqrt{\mu} h)} \quad \ldots (12)$$

Integration of Eq. (12) is made with the help of following recurrence relation for modified Bessel functions of second kind:

$$\int x^n K_{n+1}(x) \, dx = - x^n K_n(x)$$

Therefore,

$$F = \frac{4 \pi \rho_h \sqrt{\mu} K_1 (2\sqrt{\mu} z)}{K_0 (2\sqrt{\mu} h)} + A \quad \ldots (13)$$

where $K_1$ is the modified Bessel function of second kind of order one.

Boundary condition to find the arbitrary constant $A$ is

$$F = F_0 \quad \text{at} \quad z = 0$$

which gives

$$A = F_0 - \frac{4 \pi \rho_h}{K_0 (2\sqrt{\mu} h) \cdot 2\mu}$$

Therefore, the solution of Eq. (12) is

$$F = \frac{4 \pi \rho_h}{2\mu \cdot K_0 (2\sqrt{\mu} h)} \left[ 2\sqrt{\mu} z \cdot K_1 (2\sqrt{\mu} z) - 1 \right] + F_0 \quad \ldots (14)$$

At $z = h$

$$F_h = \frac{4 \pi \rho_h}{K_0 (2\sqrt{\mu} h) \cdot 2\mu} \left[ 2\sqrt{\mu} \cdot h \cdot K_1 (2\sqrt{\mu} h) - 1 \right] + F_0 \quad \ldots (15)$$

where $\mu = \frac{4 \pi \lambda}{K_0}$

Eq. (15) gives the relationship among the electrical parameters (electric field, conductivity and space charge) and coefficient of eddy diffusion. By putting the diurnal variation values of various parameters we can find the diurnal variation of the coefficient of eddy diffusion at a height of $h$ (say at 1 m) near the ground.

3. Conclusion

The diurnal variation of atmospheric electrical parameters can be used to find out the diurnal variation of eddy diffusion coefficient by taking it to be height-dependent. The method incorporates the eddy diffusion (a meteorological activity) into the continuity equation for space charge flow under steady state conditions, and then shows close relationship.
among these parameters of two different origins—
atmospheric electricity and meteorology.

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References


