Wave Propagation through a Waveguide Filled with a Semiconductor in a Transverse Magnetostatic Field

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The characteristics of electromagnetic waves through a semiconducting material in a rectangular waveguide in the presence of a transverse magnetic field applied normal to the waveguide axis have been investigated. The general solution for the dispersion relation has been developed by adopting the principle of separation of variables. With certain limitation on applied magnetic field, phase and attenuation constants have been computed for the waveguide filled with semiconducting material. The propagation characteristics and effect of waveguide dimension on propagation are discussed for different types of modes. A comparative study of propagation characteristics is also done for liquid nitrogen and room temperatures.

1. Introduction

The study of the propagation of electromagnetic waves through bounded solid-state plasma has become increasingly important in recent years because of its wide applications in microwave components, such as, isolators, circulators, attenuators, couplers, phase shifters, etc. The non-reciprocal characteristics of a partially loaded waveguide with a semiconducting material in the presence of a transverse magnetic field was studied experimentally by Barlow and Koike\(^1\) and later by Toda\(^2,3\), Hirota and Suzuki\(^4\) and Arnold and Rosenbaum\(^5\). This property has been used to develop isolators in the microwave range. The propagation through a waveguide in the presence of transverse magnetic field was investigated theoretically by Engineer and Nag\(^6\). They concluded that no pure TE or TM type waves can propagate but anomalous modes are possible. Microwave propagation through a waveguide containing semiconductor with a very strong transverse magnetostatic field was studied by Nejib and Ruduski\(^7,8\). Some theoretical study was also made by Gupta et al\(^9,10\) to investigate the propagation characteristics of a waveguide containing various extrinsic semiconductors. Phase and attenuation characteristics have been computed by them for various frequencies in the presence of a strong magnetic field at liquid nitrogen and room temperatures.

The aim of the present work is to study the propagation of electromagnetic waves through a rectangular waveguide containing a semiconducting material of n-type germanium in the presence of finite transverse magnetostatic field. The propagation characteristics have been analyzed in terms of phase and attenuation constants with some limitations on the applied magnetic field.

2. Theoretical Considerations

Let us consider a rectangular waveguide of perfectly conducting walls completely filled with a semiconducting material. The external static magnetic field \(B_0\) is applied in the Y-direction of the waveguide. From Maxwell's equations, the following pair of differential equations for the \(E_z\) and \(H_x\) fields can be obtained as

\[
 \begin{align*}
 j_0 \omega \varepsilon_3 (\varepsilon_3 - \varepsilon_1) \\
 (\gamma^2 + \omega^2 \mu \varepsilon_1) (\gamma^2 + \omega^2 \mu \varepsilon_3) \frac{\partial^2}{\partial x \partial y} \\
 + \frac{1}{(\gamma^2 + \omega^2 \mu \varepsilon_3)} \frac{\partial^2}{\partial y^2} + 1 \right] H_x = 0 \\
 \frac{\partial^2}{\partial x^2} + \gamma^2 \varepsilon_1 + \omega^2 \mu \varepsilon_1 + \omega^2 \mu \varepsilon_3 \right] E_x \\
 \frac{\partial^2}{\partial y^2} + \gamma^2 \varepsilon_3 + \omega^2 \mu \varepsilon_3 \right] H_x = 0 \\
 \end{align*}
\]

where \(\varepsilon_1, \varepsilon_2,\) and \(\varepsilon_3\) are the components of dielectric tensor

\[
 [\varepsilon] = \begin{bmatrix} \varepsilon_{11} & 0 & \varepsilon_{12} \\ 0 & \varepsilon_{22} & 0 \\ -\varepsilon_{31} & 0 & \varepsilon_{33} \end{bmatrix}
\]
with

\[ \epsilon_{11} = \epsilon_{33} = \epsilon_1 = \epsilon \left[ 1 - \frac{j \sigma}{\omega \epsilon (1 + (R_H B_0 \sigma)^2)} \right] \]

\[ \epsilon_{22} = \epsilon_2 = \epsilon \left[ 1 - \frac{j \sigma}{\omega \epsilon} \right] \]

\[ \epsilon_{33} = \epsilon_{31} = \epsilon_3 = \frac{j R_H B_0 \sigma^2}{\omega (1 + (R_H B_0 \sigma)^2)} \]

and

\[ \epsilon = \epsilon_0 \epsilon_L \]

The electric and magnetic fields are assumed to vary as \( \exp(j \omega t - \gamma z) \). The constants \( \epsilon, \epsilon_0, \mu, \mu_L \) represent the scalar dielectric permittivity of the semiconductor, the scalar permittivity of free space, the semiconductor permeability assumed to be that of free space and the relative dielectric constant of the semiconductor material, respectively. Since the signal frequency is assumed to be very small in comparison to the collision frequency, \( \sigma \) is taken to be equal to the dc conductivity of the semiconductor. \( R_H \) is the Hall coefficient.

To solve Eqs. (1) and (2) for \( E_z \) and \( H_z \), we use the principle of separation of variables. Let

\[ E_z = p(x) e^y \]  \hspace{1cm} \ldots (4)

\[ H_z = q(x) h(y) \]  \hspace{1cm} \ldots (5)

Applying the same procedure given by Gabriel and Brodwin\(^{14}\) and the imposition of the appropriate boundary conditions, we can obtain the equations for \( p \) and \( q \)

\[ \left( \Psi_1 \frac{\partial^2}{\partial x^2} + \Psi_4 \right) p + \left( \Psi_3 \frac{\partial^2}{\partial x^2} + \Psi_4 \right) q = 0 \]  \hspace{1cm} \ldots (6)

\[ \left( \Psi_5 \frac{\partial^2}{\partial x^2} + \Psi_7 \right) p + \left( \Psi_8 \frac{\partial^2}{\partial x^2} + \Psi_8 \right) q = 0 \]  \hspace{1cm} \ldots (7)

where

\[ \Psi_1 = \frac{j \omega (\epsilon_1 - \epsilon_3) \gamma (\frac{mn}{a})}{(\gamma^2 + \omega^2 \mu \epsilon_1)(\gamma^2 + \omega^2 \mu \epsilon_3)} \]

\[ \Psi_2 = \frac{j \omega \epsilon_3 \left( \frac{mn}{a} \right)}{(\gamma^2 + \omega^2 \mu \epsilon_3)} \]

\[ \Psi_3 = \frac{1}{(\gamma^2 + \omega^2 \mu \epsilon_3)} \]

\[ \Psi_4 = 1 - \left( \frac{mn}{a} \right)^2 \]

\[ \Psi_5 = \frac{\epsilon_1}{(\gamma^2 + \omega^2 \mu \epsilon_1)} \]

\[ \Psi_6 = \frac{-\epsilon_1 \left( \frac{mn}{a} \right)^2}{(\gamma^2 + \omega^2 \mu \epsilon_1)(\gamma^2 + \omega^2 \mu \epsilon_3)} \]

\[ \Psi_7 = \frac{j \omega \gamma (\epsilon_2 - \epsilon_3) \left( \frac{mn}{a} \right)}{(\gamma^2 + \omega^2 \mu \epsilon_1)(\gamma^2 + \omega^2 \mu \epsilon_3)} \]

\[ \Psi_8 = \frac{j \omega \mu \epsilon_3 \left( \frac{mn}{a} \right)}{(\gamma^2 + \omega^2 \mu \epsilon_3)} \]

\( a \) is the cross-sectional dimension of the waveguide parallel to the external magnetic field and \( m \) is an integer.

Eqs. (6) and (7) can be combined into a determinant equation in the following form after some manipulation

\[ \left[ A \frac{d^4}{dx^4} + B \frac{d^2}{dx^2} + C \right] \{ p(x) q(x) \} = 0 \]  \hspace{1cm} \ldots (8)

where

\( A = \epsilon_1 \)

\( B = 2 \gamma^2 \epsilon_1 + \omega^2 \mu \epsilon_1 \epsilon_2 + \omega^2 \mu (\epsilon_3^2 + \epsilon_2^2) - (\epsilon_1 + \epsilon_3) \left( \frac{mn}{a} \right)^2 \)

\( C = \gamma^4 \epsilon_1 + \gamma^2 \left( \omega^2 \mu (\epsilon_3^2 + \epsilon_2^2) + \omega^2 \mu \epsilon_1 \epsilon_3 - (\epsilon_1 + \epsilon_3) \right) \times \left( \frac{mn}{a} \right)^2 + \epsilon_2 \left( \frac{mn}{a} \right) \}

\( \times \left( \frac{mn}{a} \right)^2 + \epsilon_2 \left( \frac{mn}{a} \right) \}

\( \times \left( \frac{mn}{a} \right)^2 + \epsilon_2 \left( \frac{mn}{a} \right) \}

2.1 Case of Magnetic Field \( B_z \) Such that \( (R_H B_0 \sigma)^2 \ll 1 \) and \( \epsilon_1 \neq 0 \)

Because of coupled wave equations it is difficult to solve the boundary value problem for this geometry in the presence of finite magnetic field. For the sake of simplicity we assume that the value of \( B_0 \) such that \( (R_H B_0 \sigma)^2 \ll 1 \); then the diagonal components of the permittivity tensor, in Eq. (3) are all found to be equal, that is \( \epsilon_1 = \epsilon_2 \) and off-diagonal elements of the permittivity tensor \( \epsilon_{13} \) and \( \epsilon_{23} \) are non-zero. Since Eq. (8) is of the fourth order, hence a linear combination of four constants \( A_1, A_2, A_3 \) and \( A_4 \) of the following form will in general be required:

\[ E_z = (A_1 e^{p_1 x} + A_2 e^{p_2 x} + A_3 e^{p_3 x} + A_4 e^{p_4 x}) \times \sin \left( \frac{mn}{a} \right) y \exp(j \omega t - \gamma z) \]  \hspace{1cm} \ldots (9)

where \( p_1 \) and \( p_4 \) are transverse wave numbers in the
With the help of Eq. (9), the field components and appropriate boundary conditions, we obtain the following transcendental equation of the form
\[ 2p_1 p_2 (a'^2 - b'^2) (1 - \cosh p_1 b \cosh p_2 b) = - \left( p_1^2 (a' + b')^2 + p_2^2 (a' - b')^2 \right) \sinh p_1 b \times \sinh p_2 b \]
where
\[ a' = \frac{1}{2} \varepsilon_1 \omega^2 \mu \varepsilon_1 \]
\[ b' = \frac{1}{2} \left[ \left( \frac{\omega^2 \mu \varepsilon_2}{\varepsilon_1} \right)^2 - 4 \omega^2 \mu \varepsilon_2 \left( \frac{\pi}{a} \right)^2 \right] \]
and \( b \) is the cross-sectional dimension of the waveguide along \( X \)-axis.

If we make the assumption that \( p_1 b \) and \( p_2 b \ll 1 \), then Eq. (10) can be written as
\[ 4b'^2 (p_1^2 + p_2^2) + 4a'^2 b' (p_1^2 - p_2^2) + b^2 p_1^2 p_2^2 \times (b'^2 - a'^2) = 0 \]  
(11)
which can be simplified as
\[ \gamma^2 = 2 \left[ \left( \frac{\pi}{a} \right)^2 - \omega^2 \mu \varepsilon_1 + a' \right] + \frac{4 b'^2}{b^2 (b'^2 - a'^2)} \]
\[ + \left[ \left( \frac{\pi}{b} \right)^2 - \omega^2 \mu \varepsilon_1 - a' \right] \times b'^2 + \frac{8 b'^2}{b^2 (b'^2 - a'^2)} \]
\[ \times \left[ \left( \frac{\pi}{a} \right)^2 - \omega^2 \mu \varepsilon_1 \right] \]  
(12)
Eq. (12) is solved to give explicit relations for phase constant \( \beta \) and attenuation constant \( \alpha \) as,
\[ \beta^4 - A' \beta^2 - C' = 0 \]  
(13)
\[ \alpha^4 + A' \alpha^2 - C' = 0 \]  
(14)
where
\[ A' = \left[ \omega^2 \mu \varepsilon + \frac{2}{b^2} - \frac{6a^2}{m^2 \pi^2 b^2} - \frac{2a^2 \mu \{ R_H B_0 \sigma a \}^2}{b^2 m^2 \pi^2 \varepsilon \left[ 1 + \frac{a^2}{\omega \varepsilon} \right]} \right. 
\[ - \frac{\mu \left( m^2 \pi^2 b^2 \right)^2 \{ R_H B_0 \sigma a \}^2}{b^2 \varepsilon \left[ 1 + \frac{a^2}{\omega \varepsilon} \right]} - \frac{8 \varepsilon}{b^2 \mu \{ R_H B_0 \sigma a \}^2} \]
\[ C' = \frac{1}{4} \left[ \omega \mu \sigma + \frac{2a^2 \mu \sigma \{ R_H B_0 \sigma a \}^2}{b^2 m^2 \pi^2 \omega \varepsilon \left[ 1 + \frac{a^2}{\omega \varepsilon} \right]} \right. 
\[ + \frac{\mu a \left( m^2 \pi^2 b^2 \right)^2 \{ R_H B_0 \sigma a \}^2}{b^2 \omega \varepsilon \left[ 1 + \frac{a^2}{\omega \varepsilon} \right]} - \frac{8 \sigma}{b^2 \mu \{ R_H B_0 \sigma a \}^2} \]  
(15)
For small values of \( B_0, \varepsilon_2 = 0 \) and \( a' = b' = 0 \), hence from Eq. (12) we get the expression
\[ \gamma^2 = \left( \frac{m\pi}{a} \right)^2 - \omega^2 \mu \varepsilon_1 \]
which is the same dispersion relation as that obtained for isotropic case. Thus, in the presence of small magnetic field there are some propagating modes which are similar to the unperturbed TE modes as discussed by Engineer and Nag.  

### 2.2 Case of Strong Magnetic Field

Under the influence of strong external applied magnetic field the non-diagonal terms of dielectric tensor in Eq. (3) become zero and
\[ X(b''/a) \]
which can be simplified as
\[ \gamma^2 = \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 - \omega^2 \mu \]
and
\[ \gamma^2 = \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 - \omega^2 \mu \varepsilon_2 \]  
(17)

We conclude from the above equations that TE modes are similar to the equation for free space waveguide, whereas TM modes are modified. The expression for phase and attenuation constants in the TM mode can be obtained from Eq. (18) in the following form:
\[ \beta^2 = \frac{1}{2} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 - \omega^2 \mu \varepsilon_2 \right] \pm \left[ \left( \frac{m\pi}{b} \right)^2 \right] \]
\[ \omega \varepsilon_2 \varepsilon_1 \]  
(19)
\[ \alpha^2 = \frac{1}{2} \left[ \left( \frac{m\pi}{a} \right)^2 - \left( \frac{n\pi}{b} \right)^2 - \omega^2 \mu \varepsilon_2 \right] \nu \left[ \left( \frac{m\pi}{a} \right)^2 \right] \]
\[ \omega \varepsilon_2 \varepsilon_1 \]  
(20)

### 3. Results and Discussion

Using the dispersion Eqs. (13), (14), (19) and (20) for waveguide filled with a semiconducting material
of n-type germanium for the conditions of applied external magnetic fields we have computed the phase and attenuation constants for different values of signal frequency and other parameters of waveguide and the plasma medium.

The results have been shown in Figs. 1-3 for magnetic field $B_0$ such that $(R_H B_0 \alpha)^2 \ll 1$ and $\varepsilon_0 \neq 0$ and Figs. 4-6 for infinitely strong magnetic field. The parameters of semiconductor that are used in the computation are given below.\textsuperscript{11,15-18}

(a) 300°K and the value of $B_0$ such that $(R_H B_0 \alpha)^2 \ll 1$ and $\varepsilon_0 \neq 0$

(i) Flux density of steady magnetic field $B_0 = 0.45$ weber/m$^2$
(ii) Plasma density $N = 0.91 \times 10^{20}$ electrons/m$^3$
(iii) Plasma frequency $\omega_p = 2.46 \times 10^{13}$ rad/sec
(iv) Size of rectangular waveguide $b = 0.0106m$, $a = 0.0212m$
(v) Conductivity $\sigma = 6.25$ ohm m
(vi) Mobility $\mu_r = 0.425$ m$^2$/V sec
(vii) Effective electron mass $m^*_e = 0.12 m_0$
(viii) Collision frequency $\nu = 1.38 \times 10^{13}$/sec
(ix) Relative static dielectric constant of $G$, $\varepsilon_L \cdot \varepsilon_0 = 16.0$
(x) Free space permeability $\mu = 1.256 \times 10^{-4}$ henry/m
(xi) Free space permittivity $\varepsilon_0 = 8.85 \times 10^{-12}$ farad/m

(b) 77°K and the value of $B_0$ such that $(R_H B_0 \alpha)^2 \ll 1$ and $\varepsilon_0 \neq 0$

(i) $B_0 = 4.0$ kilogauss
(ii) $N = 3.9 \times 10^{19}$ electrons/m$^3$

(c) 300°K and strong magnetic field

(i) $N = 10^{20}$ electrons/m$^3$
(ii) $\omega_p = 2.57 \times 10^{11}$ rad/sec
(iii) $\sigma = 5.7$ ohm m
(iv) $\mu_r = 0.36$ m$^2$/V sec
(v) $\nu = 1.63 \times 10^{13}$/sec

(d) 77°K and strong magnetic field

(i) $N = 4.0 \times 10^{19}$ electrons/m$^3$
(ii) $\omega_p = 1.6 \times 10^{11}$ rad/sec
(iii) $\sigma = 22.0$ ohm m
(iv) $\mu_r = 3.5$ m$^2$/V sec
(v) $\nu = 1.5 \times 10^{13}$/sec

Fig. 1—Variation of $\alpha$ and $\beta$ with $\omega$ for different modes of excitations in the presence of magnetic field $B_0$ such that $(R_H B_0 \alpha)^2 \ll 1$

Fig. 2—Variation of $\alpha$ and $\beta$ with $\omega$ for different waveguide heights in the presence of magnetic field $B_0$ such that $(R_H B_0 \alpha)^2 \ll 1$

Fig. 3—Variation of $\alpha$ and $\beta$ with $\omega$ at 77°K and 300°K in the presence of magnetic field $B_0$ such that $(R_H B_0 \alpha)^2 \ll 1$.
Figure 1 shows the variation of phase and attenuation constants with signal frequency for different modes of excitations at room temperature. We see that for all modes the phase constant varies linearly with the frequency. The attenuation constant is found to be approximately constant for \( m = 1 \). However, for higher modes, the attenuation constant is found to be constant at higher frequencies, whereas it decreases rapidly at lower frequencies. Fig. 2 depicts the variation of the phase and attenuation constants with frequency for different waveguide heights at room temperature. One can see that the phase constant increases linearly with frequency. However, the attenuation constant is practically independent of the frequency and the width of the waveguide. Fig. 3 shows the variation with frequency of phase and attenuation constants at two different temperatures, namely, at liquid nitrogen temperature (77° K) and at room temperature (300° K). Due to higher values of conductivity at 77° K for n-type germanium, the attenuation constant has higher values at liquid nitrogen temperature as compared to that at room temperature.

From the graphs it is concluded that there does not exist any characteristic cut-off frequency and that the propagation as well as attenuation is possible over all frequency ranges. However, it is found that the effect of the finite value of \( \varepsilon_R \) subject to the condition that \((R_B B_0 \sigma)^2 \ll 1\) on the values of phase and attenuation constants is not significant (though it is not shown separately in Figs. 1-3).

Figure 4 shows the variation of phase and attenuation constants with frequency for different modes of excitation at room temperature in the presence of strong transverse magnetic field. It is found that the value of cut-off frequency increases as the mode number increases. The cut-off frequency is mainly dependent upon the waveguide dimension and the relative dielectric constant of the semiconducting material. The effect of varying waveguide dimension shown in Figs. 5 and 6 depicts the propagation characteristics at liquid nitrogen and room temperatures. It is found that the propagation characteristics in the presence of strong transverse magnetic field are similar to those depicted in Fig. 3.

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References