

The array dimension is taken as  $k_0 d = 5$  such that the array may be operative in the range from 6 to 300 MHz. Using Eq. (7), the horizontal radiation pattern is obtained for  $\theta = \pi/2$  and is given by

$$F\left(\frac{\pi}{2}, \phi\right) = A \left[ 1 + \sum_{m=2}^5 \left\{ \exp[j(m-1)(\Psi + k_0 d \cos \phi_j)] + \exp[j(m-1)\{\Psi + k_0 d \cos(\phi - \alpha)\}] \right\} \right] \dots(8)$$

The horizontal radiation patterns showing the variation of  $F(\pi/2, \phi)$  with  $\phi$  for two different values of  $\Psi$  angles 0 and  $\pi$  are shown in Fig. 2.

For the study of the vertical radiation patterns, we have chosen the angle  $\phi = \alpha/2 = \pi/4$  as a particular case. Substituting the above values, the expression for the vertical radiation pattern reduces to

$$F\left(\theta, \frac{\pi}{4}\right) = A \left[ 1 + 2 \sum_{m=2}^5 \exp j(m-1)\{\Psi + k_0 d \times \cos \frac{\alpha}{2} \sin \theta\} \right] \dots(9)$$

The variation of  $F(\theta, \pi/4)$  with  $\theta$  is shown in Fig. 3 for two values of  $\Psi$  as 0 and  $\pi$ .

In Fig. 2 which shows the variation of  $F(\pi/2, \phi)$  with  $\phi$  in the horizontal plane, a mirror symmetry about  $180^\circ$  is observed. For  $\Psi = 0$ , highly directive beams are obtained along  $0^\circ$  and  $90^\circ$  and due to mirror symmetry, also along  $180^\circ$  and  $270^\circ$  with half-power beam-width angle of  $12^\circ$ . It is interesting to note that no side-lobes appear and the minima occur at  $45^\circ$  and  $135^\circ$ . However, for  $\Psi = \pi$ , the maximum occurs along  $45^\circ$  with the half-power beam-width angle of  $15^\circ$ . Further, it may be observed that a secondary lobe with field-intensity of 67% of the main lobe and a few other minor lobes of low field-intensities also appear.

The vertical radiation patterns shown in Fig. 3 also exhibit a mirror symmetry about  $180^\circ$ . The field pattern for  $\Psi = 0$  is found to be sufficiently directive. The half-power beam-width angle of the main lobe is about  $20^\circ$ . The maxima occur along  $0^\circ$  and  $180^\circ$  with small secondary lobes appearing along  $30^\circ$ ,  $70^\circ$ ,  $110^\circ$  and  $150^\circ$ . However, for  $\Psi = \pi$ , the central direction of the main lobe is shifted by  $90^\circ$  where it is bifurcated. The maximum intensities are at  $60^\circ$  and  $120^\circ$ . The half-power beam-width angle of the bifurcated main beam is as large as  $84^\circ$ . There occur side-lobes with intensity reaching upto 33% of that of the main lobe.

Although the same scale has been chosen for plotting the curves for  $\Psi = 0$  and  $\Psi = \pi$ , it must be

mentioned that only the relative intensities are depicted and in particular, the absolute maximum field intensity in the case  $\Psi = 0$  is much greater than in the case  $\Psi = \pi$ .

For the chosen array dimension of the V-shaped array, the radiation patterns are highly directive in the horizontal plane both for  $\Psi = 0$  and  $\Psi = \pi$  and also in the vertical plane for  $\Psi = 0$ .

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#### Mutual Impedance of a Dipole Antenna in Magnetoplasma\*

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The mutual impedance of a dipole antenna in an anisotropic plasma has been calculated following the quasi-static approximation method.

IN AN anisotropic media the self impedance and the mutual impedance of an antenna are different. Self impedance of a dipole antenna in an anisotropic plasma has been studied by several investigators<sup>1</sup>. The importance of the study of the mutual impedance of a dipole antenna in an anisotropic plasma has been discussed by Miyazaki<sup>2</sup>. The purpose of the present communication is to calculate the mutual impedance of the dipole antenna in an anisotropic plasma following the quasi-static approximation method.

\*This work was done by the author when he was at the Department of Electrical Engineering, Tohoku University, Sendai, Japan.

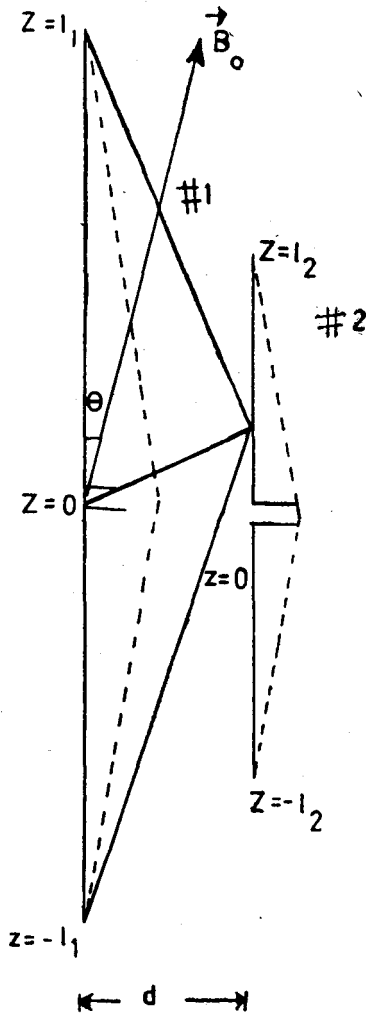


Fig. 1—Parallel dipole antenna

The dipole antenna under consideration is shown in Fig. 1. The current distribution is assumed to be triangular in nature. For quasi-static approach the dimension of the antenna is taken to be short compared with the free space wavelength. The distance between the two dipoles is also assumed to be smaller than the wavelength. Also, the plasma surrounding the antenna is taken to be sharply bounded and that outside the boundary it is uniform. Then following Balmain<sup>1</sup> and Schelkunoff and Friis<sup>3</sup> the mutual impedance with the above assumed current distribution in a static magnetic field can be written as

$$\begin{aligned}
 Z_{21} = & \frac{a}{j\omega 2\pi \epsilon_0 K' l_1 l_2 F^{1/2}} \left[ \frac{1}{F^{1/2}} \left\{ N(l_2 - l_1) \right. \right. \\
 & - N(-l_1) + N(l_2 + l_1) - N(l_1) - 2N(l_2) \\
 & \left. \left. + 2N(0) \right\} - \left( \frac{G}{2F} + l_2 \right) \right] \\
 & \times \ln \frac{M(l_2 - l_1) M(l_2 + l_1) M^2(0)}{M(-l_1) M(l_1) M^2(l_2)} \\
 & - l_1 \ln \frac{M(l_2 + l_1) M(-l_1)}{M(l_1) M(l_2 - l_1)} \quad \dots(1)
 \end{aligned}$$

where

$$\begin{aligned}
 N(\alpha) &= [F\alpha^2 + G\alpha + H]^{1/2} \\
 M(\alpha) &= 2 [F(F\alpha^2 + G\alpha + H)]^{1/2} + 2F\alpha + G \\
 F &= 1 + (a^2 - 1) \cos^2 \theta \\
 G &= 2d(a^2 - 1) \sin \theta \cos \theta \\
 H &= \alpha^2 \{1 + (a^2 - 1) \sin^2 \theta\} \\
 a^2 &= \frac{K'}{K_0}, K_0 = 1 - \frac{X}{U}, K' = 1 - \frac{XU}{U^2 - Y^2} \\
 X &= \omega_P / \omega^2, Y = \omega_H / \omega, U = 1 - j\nu / \omega
 \end{aligned}$$

$l_1$  and  $l_2$  are the lengths of the antennas.

Case 1

When  $l_1$  and  $l_2$  are equal to  $l$ ,  $\theta=0$  and the static magnetic field is parallel to the dipoles, Eq. (1) reduces to

$$\begin{aligned}
 Z_{21} = & \frac{1}{j\omega\pi \epsilon_0 K' l} \left[ \frac{\ln(al + \sqrt{a^2 l^2 + d^2})^2}{a(2al + \sqrt{4a^2 l^2 + d^2})} \right. \\
 & \left. + \frac{1}{2al} \left\{ 3d + \sqrt{4a^2 l^2 + d^2} - 4\sqrt{a^2 l^2 + d^2} \right\} \right] \quad \dots(2)
 \end{aligned}$$

When  $d \rightarrow 0$ , Eq. (2) agrees with that of Balmain<sup>1</sup>.

Case 2

When  $l_1=l_2=l$ , but the magnetic field is perpendicular to the dipole, Eq. (1) reduces to

$$\begin{aligned}
 Z_{21} = & \frac{a}{j\omega\pi \epsilon_0 K' l} \left[ \left\{ \frac{\ln(l + \sqrt{l^2 + a^2 d^2})^2}{ad(2l + \sqrt{4l^2 + a^2 d^2})} \right. \right. \\
 & \left. \left. + \frac{1}{2l} \left\{ 3ad + \sqrt{4l^2 + a^2 d^2} - 4\sqrt{l^2 + a^2 d^2} \right\} \right\} \right] \quad \dots(3)
 \end{aligned}$$

Now, if  $d \rightarrow 0$ , Eq. (3) is reduced to the expression for the self impedance of antennas. However, in this case Eq. (3) does not reduce to the expression for the self impedance of the dipole antenna as derived by Balmain<sup>1</sup>.

Case 3

When the antennas are in horizontal direction (collinear to each other, the position of antennas is as

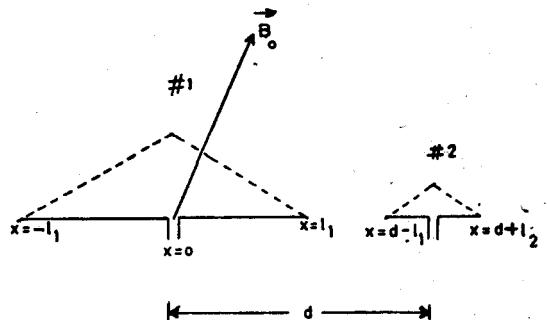


Fig. 2—Dipole antennas arranged in a line

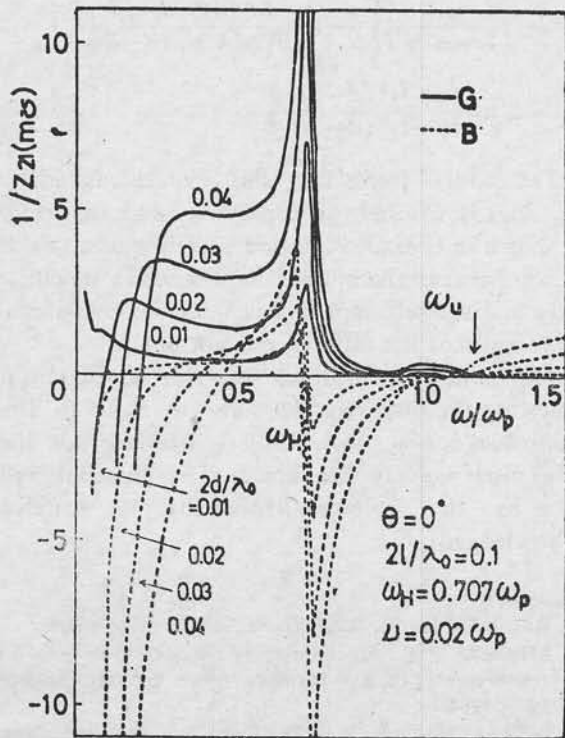


Fig. 3—Plot of mutual impedance versus admittance (parallel antenna,  $\theta=0^\circ$ )

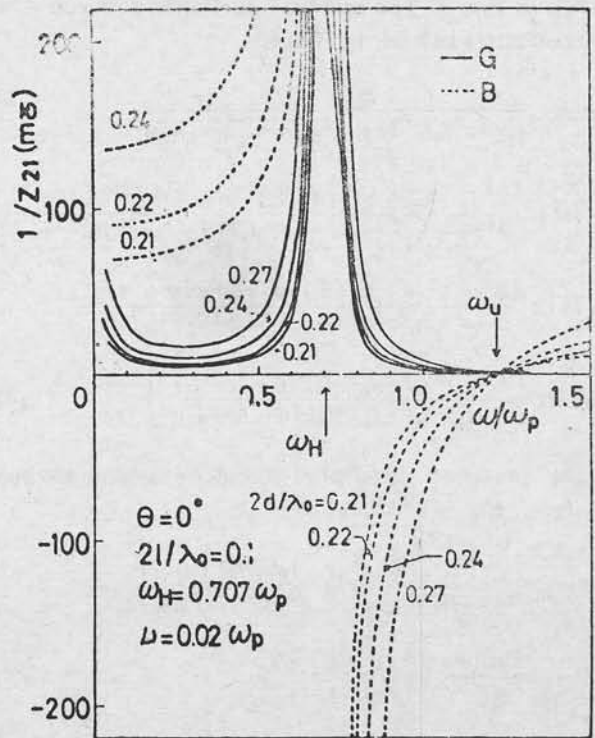


Fig. 5—Plot of mutual impedance versus admittance (colinear antennas,  $\theta=0^\circ$ )

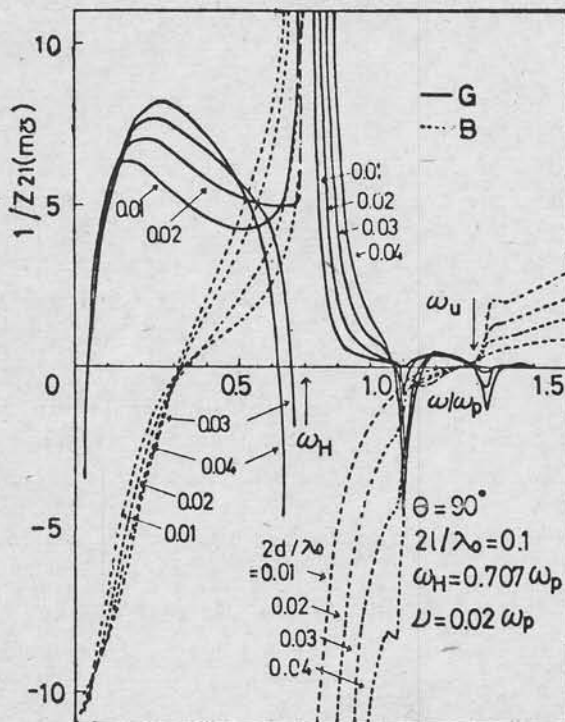


Fig. 4—Plot of impedance versus admittance (parallel antennas,  $\theta=90^\circ$ )

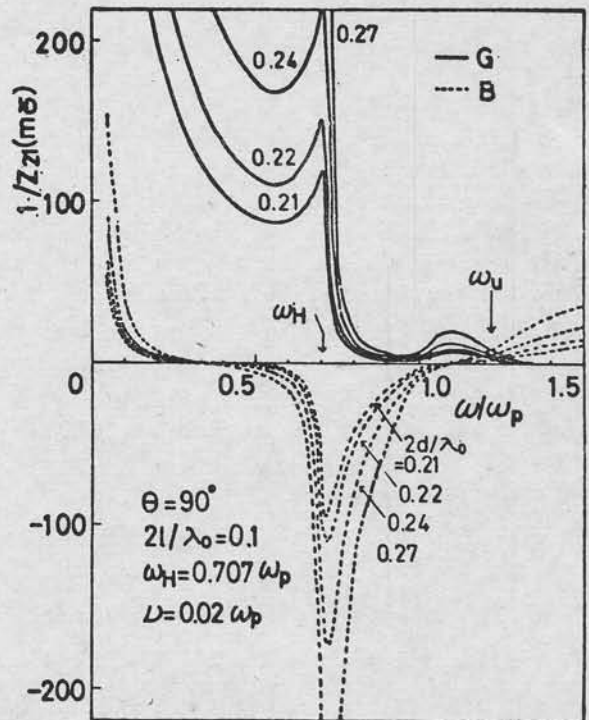


Fig. 6—Plot of mutual impedance versus admittance (colinear antennas,  $\theta=90^\circ$ )

shown in Fig. 2. The mutual impedance between the two antennas can be derived as

$$Z_{21} = \frac{a}{j\omega 4\pi \epsilon_0 K' l_1 l_2 \sqrt{\sin^2 \theta + a^2 \cos^2 \theta}} - \left[ d \ln \frac{(d-l_1)^2 (d+l_1)^2 (d+l_2)^2 (d-l_2)^2}{d^4 (d+l_1+l_2) (d-l_1+l_2) (d+l_1-l_2) (d-l_1-l_2)} + l_1 \ln \frac{(d+l_1)^2 (d-l_1+l_2) (d-l_1) (d-l_1-l_2)}{(d-l_1)^2 (d+l_1+l_2) (d+l_1-l_2)} + l_2 \ln \frac{(d+l_2)^2 (d+l_1-l_2) (d-l_1-l_2)}{(d-l_2)^2 (d+l_1+l_2) (d-l_1+l_2)} \right] \dots(4)$$

In the presence of parallel magnetic field and when  $l_1=l_2=l$ , Eq. (4) reduces to

$$Z_{21} = \frac{1}{j4\pi\omega\epsilon_0 K' l} \left[ \frac{d}{l} \ln \frac{(d+l)^4 (d-l)^4}{d^6 (d+2l) (d-2l)} + \frac{2 \ln (d+l)^2 (d-2l)}{(d-l)^2 (d+2l)} \right] \dots(5)$$

and in the presence of perpendicular magnetic field and with  $l_1=l_2=l$  Eq. (4) reduces to

$$Z_{21} = \frac{a}{j4\pi\omega\epsilon_0 K' l} \left[ \frac{d}{l} \ln \frac{(d+l)^4 (d-l)^4}{d^6 (d+2l) (d-2l)} + 2 \ln \frac{(d+l)^2 (d-2l)}{(d-l)^2 (d+2l)} \right] \dots(6)$$

The mutual impedance has been calculated with Eqs. (2), (3), (5) and (6) respectively and the results are shown in Figs. 3, 4, 5, and 6. On making a comparison between the mutual impedance as calculated above and the self impedance calculated by Balmain<sup>1</sup>, it is found that the difference is not much.

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