Some Observations on Very Low Frequency Spectra of Leader Pulses

S. P. GUPTA
Department of Physics, Banaras Hindu University, Varanasi-5

Received 20 May 1974; revised received 1 October 1974

A detailed experimental study has been carried out to see the influence of pause time on vlf spectra radiated by stepped leaders of lightning discharge. It is found that the vlf amplitude spectrum of a series of leader pulses will be broad when the mean pause time between successive leader pulses is of the same order as its own standard deviation. However, if the standard deviation is very small compared to the mean pause time of stepping, the spectra would have a number of peaks at repetition frequency of the stepings and its harmonics. Moreover, two types of stepped leader (α- and β-type) have been distinguished in terms of their spectral shapes.

1. Introduction

The stepped nature of the first leader in a cloud-to-ground discharge is one of the most intriguing aspects of a lightning flash. Indeed, one of the most significant findings of the Schonland School of workers is the existence of the stepped leaders (hereafter referred to as SL). Ever since its discovery, many workers have studied the various aspects of the SLs. The gross characteristics of SL have been listed by Uman who also reviews briefly the theories put forward by different workers. The electrostatic field changes associated with the SL are observed to be smooth unless the equipment used is specially designed to emphasize the rapid changes. On the other hand, the radiated field changes due to SL are readily observable. This is because the discontinuous progress of the discharge results in discontinuities in the changes of the electric moment with time. Since the radiated fields are proportional to the second time derivatives of the electric moment, they become appreciable in the case of SL.

The frequency range of electromagnetic radiation from SL extends from vlf to uhf (microwave) and the studies of Malan show that as we proceed to higher frequencies the importance of radiation from SL vis-a-vis that from return strokes increases progressively. At microwave frequencies the SL contribution far outweighs that of return stroke. In this paper we do not, however, concern ourselves with hf radiation and we deal only with the vlf region of the spectrum.

Arnold and Pierce have investigated the vlf characteristics of leader pulses by assuming a double exponential expression for the surge current. Their theoretical deductions lead to some interesting conclusions: (i) the vlf amplitude spectrum of a train of leader pulses will be broad when the mean pause time (hereafter referred to as MPT) of stepping, is of the same order as its own standard deviation and (ii) if the standard deviation is very small compared to the MPT, the spectrum would have pronounced maxima at its repetition frequency and its harmonics.

Recently, Gupta et al. presented first experimental evidence in support of the theoretical predictions by Arnold and Pierce. The purpose of this paper is to furnish experimental evidence in more detail which supports the theoretical conclusions of Arnold and Pierce and suggests a correlation between the type of spectrum obtained and the type of leader (whether α-type or β-type).

2. VLF Spectra Radiated by Stepped Leaders

2.1 Theoretical Considerations

To start with theoretical discussion, we first assume that all the individual leader pulses are identical in all aspects except that successive pulses occur at different times. If \( E(\omega) \) represents the amplitude spectrum of a single leader pulse, then the mean amplitude spectrum of a series of leader pulses is given by

\[
| L(\omega) | = E(\omega) \cdot G(\omega),
\]

where \( G(\omega) = \frac{1}{\sqrt{1 - \cos \omega \tau}}, \) and \( \tau \) is the pause time of a series of leader pulses. As usual \( \omega \) signifies the angular frequency and \( G(\omega) \) shows maxima and minima in the \( \omega \) plane. To find the frequencies where these points of inflexion occur we write

\[
\frac{\partial G(\omega)}{\partial (\omega \tau)} = 0
\]

which gives \( \sin (\omega \tau) = 0 \)
or
\[ f = \frac{n}{2\pi}, \quad n = 1, 2, 3 \ldots \text{etc.} \]

By differentiating once more, we see that \( G(\omega) \) shows maxima (i.e. infinites) when \( n \) is odd and minima when \( n \) is even.

The above simple treatment refers to the situation when the pause time \( \tau \) is the same between any two successive steps. But in reality the pause time shows considerable variation and the treatment becomes more complex. We define a mean for the pause time (\( \bar{\tau} \)) and a variation (\( \sigma \)) in the mean as:
\[ T_{N+1} = T_N = \bar{\tau} + \sigma_n \]

In this case, following the treatment of Arnold and Pierce\(^8\) and of Large\(^9\), the mean amplitude spectrum of \( N \) leader pulses can be written as:
\[ | L(\omega) | = | E(\omega) | N^{1/2} G(\omega) \]
where
\[ G(\omega) = \frac{\sinh (\sigma^2 \omega^2 / 2)}{\cosh (\sigma^2 \omega^2 / 2) - \cos \omega \tau} \]

Here \( \sigma \) is the standard deviation in \( \tau \)

2.2 Theoretical Results

Thus the amplitude spectra of SL can be computed with the help of Eq. (2) for any given combination of the MPT of steppings (\( \bar{\tau} \)) and its standard deviation (\( \sigma \)). As an illustration we chose \( \tau = 50 \mu \text{sec} \) and make three combinations each with one of the 3 values of \( \sigma \) as 30, 3 and 0 \( \mu \text{sec} \).

The normalized frequency spectrum \( \frac{E(\omega)}{E(\omega)_{\text{max}}} \) of a single leader pulse and that of the entire series of leader pulses \( \frac{L(\omega)}{L(\omega)_{\text{max}}} \) for the above 3 combinations of \( \tau \) and \( \sigma \) are shown in Fig. 1. By this normalization we remove the influence of \( N \) on \( L(\omega) \) while retaining the effect of \( G(\omega) \) on the spectral shape of \( L(\omega) \).

The spectrum of a single leader pulse has been taken from Arnold and Pierce\(^8\). From Fig. 1, it is seen that when the MPT of steppings is of the order of its standard deviation, the leader spectrum does not differ much from that of the single pulse. But when the standard deviation is very small compared to the MPT, the bandwidth of the spectrum is reduced and harmonics occur. When standard deviation (\( \sigma \)) is zero, sharply defined maxima and minima occur. In what follows below we present the experimental evidence in support of these theoretical conclusions.

3. Experimental Arrangements and Recording of the Data

The experimental arrangement, consists of an automatic atmospheric recorder\(^4\) and a set of 6 narrow band tuned receivers with a bandwidth of 400 Hz between half power points. The tuned frequencies of the 6 narrow band receivers were kept in the range 5-30 kHz at an interval of 5 kHz. The inputs for all the amplifiers (atmospheric recorder and 6 amplifiers) were derived from a master cathode follower connected to a vertical antenna. Responses from the two receivers tuned to 5 and 10 kHz were displayed on a two beam Orion oscillograph whereas the responses of the remaining 4 tuned amplifiers and of the wide band amplifier were displayed on a five beam C. R. T. (7 YP). The outputs from all the amplifiers, displayed on both the tubes, have been photographed simultaneously by two camera units driven by a synchronizing pulse derived from the initiating atmospheric. The outputs are visible only for a few msec, determined by the duration of the Z modulation pulse, applied to the control grids of the cathode ray tubes. The records obtained from the broad band amplifier (i.e. waveform) are of sufficient resolution to enable measurements of the MPT (\( \tau \)) and standard deviation (\( \sigma \)). The outputs of the tuned amplifiers, of course, correspond to the amplitude spectrum at the tuned frequencies. In other words, the 6 tuned amplifiers give us the Fourier amplitude spectrum at 6 spot frequencies.

4. Results

Four typical examples of the experimental records obtained by above technique are shown in Figs. 2(a)-2(d). Each of these figures is a simultaneous photograph of 7 cathode-ray beams. The output of the broad band amplifier is denoted, in each figure, by BB. All the remaining 6 signatures in each figure correspond to the outputs of the six tuned amplifiers. The frequencies chosen are indicated in these figures against the signatures. The broad band amplifier outputs show clearly a number of pulses which we
Fig. 2 - Oscillograms showing stepped leader radiated fields and their spectra. The tuned amplifier frequencies are given in kHz. BB indicates broad band amplifier responses. The time base for BB is 2 msec.
attribute to the radiation pulses from SL. The average pause times are in the neighbourhood of 100 \( \mu \)sec and are typical of leaders. It is well known that during the later parts of an intracloud discharge similar pulses are radiated but then pause times are in the range of a few msec\(^2\). Thus, the pulses seen in oscillograms could not have been due to \( K \)-changes of an intracloud discharge. Since the duration of the records shown in these figures is only 2 msec, it is evident that only a part of the SL process is recorded by our equipments.

The peak amplitudes of the tuned amplifier responses represent the amplitude spectra. Since the calibrations for tuned amplifiers were not available in absolute units, the amplitude spectra obtained by the tuned amplifiers are normalized so that the maximum value is unity and are shown in Figs. 3 (a)-3 (d) by continuous lines. These are shown in the figures as “experimental results”.

From the broad band responses, both the MPT (\( \tau \)) and standard deviation (\( \sigma \)) could be easily determined. The MPTs (\( \tau \)) are measured by noting the time intervals between successive positive or negative peaks of the radiation pulses. Our results, however, showed some variation depending on whether the measurements referred to the positive or negative peaks. We, therefore, considered both the types of peaks in evaluating the mean pause times and their standard deviation. The values of these two parameters (\( \tau \) and \( \sigma \)) are also shown in these figures.

Once \( \tau \) and \( \sigma \) are measured, the amplitude spectra can once again be estimated from the theory outlined in Sec. 2.2. Assuming that the spectrum due to
GUPTA : VLF SPECTRA OF LEADER PULSES

a single pulse is same as that quoted by Arnold and Pierce\textsuperscript{16}, the spectra for the SL process have been estimated and are shown in Figs. 3(a)-3(d) by means of broken lines which are labelled as ‘estimated’ in these figures.

It, thus, follows that the two curves in each of the Figs. 3(a)-3(d) show how far our experimental results corroborate the theory of Arnold and Pierce\textsuperscript{16}.

5. Discussion

Specifically, we note that the spectra obtained from tuned amplifier outputs shown in Figs. 3(a) and 3(b) indicate a definite tendency in the parent leader to generate harmonics. This is quite consistent with the values of MPT ($\tau$) and standard deviation ($\sigma$) measured from the broad band amplifier responses [In Fig. 3(a), $\tau = 110 \mu$sec, $\sigma = 28 \mu$sec, and in Fig. 3(b), $\tau = 105 \mu$sec, $\sigma = 23 \mu$sec].

Similarly, we note that the measured spectra in Figs 3(c) and 3(d) show no tendency to generate any harmonics. In other words the spectra in Figs. 3(c) and 3(d) are rather broad. This result is also consistent with the measured values of $\tau$ and $\sigma$ [In Fig. 3(c), $\tau = 96 \mu$sec, $\sigma = 46 \mu$sec and in Fig. 3(d), $\tau = 94 \mu$sec, $\sigma = 56 \mu$sec].

Even though only 4 examples of our measurements are shown in Figs. 2 and 3, a total of about 100 records have been obtained. Roughly, half of the total number of SLs recorded showed a clear tendency to generate harmonics. However, we have not observed very deep nulls in the spectra. This probably shows that perfectly regular stepping (i.e. $\sigma = 0$) is a very rare occurrence.

Hence the agreement between the ‘experimental’ and ‘estimated’ spectra shown in Figs. 3(a)-3(d) is reasonable. Considering the fact that our results pertain only to a portion of the SL process, the agreement between the ‘experimental’ and ‘estimated’ spectra may indeed be viewed as satisfactory.

5.1 $\alpha$- and $\beta$-Type of Stepped Leaders and Their Spectra

It is known that the SLs are of at least two types, called $\alpha$- and $\beta$-type. This classification of SLs was first made by the Schonland school of workers\textsuperscript{18}. Type $\alpha$-leaders, show a low and fairly uniform velocity throughout their passage between cloud-base and ground. The steps are short and are so weak in luminosity that they are not easily photographed unless the discharge is very close or the camera has a large aperture lens. In type $\beta$-leaders, the top of the channel is extensively branched. The $\beta$-steps move with greater but variable velocity. The steps are long and fairly bright.

It seems quite logical to suggest that the SLs associated with vlf spectra which show narrow bandwidth and/or harmonics should be identified with the $\alpha$-type. This is because the $\alpha$-type leaders show more or less regular steppings and we expect the standard deviation in pause times to be small.

By the same logic, we should identify leaders whose vlf spectra show broad maxima with the $\beta$-type leaders.

6. Conclusions

From the analysis of about 100 atmospheric records due to leader pulses we arrive at the following conclusions.

(i) The vlf spectra of SLs are of two types. In one type, the spectra show a narrow bandwidth or show a tendency to generate harmonics. In the other type, the vlf spectra show broad maxima.

(ii) Our experimental results fully support the theory of Arnold and Pierce\textsuperscript{16}.

(iii) Deep nulls in the spectra are not observed.

(iv) It is possible, in principle, to distinguish between $\alpha$- and $\beta$-type SLs from their respective vlf spectra.

Acknowledgement

Thanks are due to Dr Manoranjan Rao and late Prof. B. A. P. Tantry for the valuable suggestions and discussions about the problem. The author is grateful to the University Grants Commission, New Delhi for financial support.

References

7. RAETHER, H., Arch. Elektroteck, 34 (1940), 59.