Characteristics of a Waveguide Filled with Moving Lossy Warm Plasma

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The propagation of electromagnetic waves through a waveguide containing a lossy warm plasma which moves down the waveguide with a constant velocity \( V \) in the presence of strong transverse magnetic field is investigated. Equations describing the fields and normal modes (TE and TM modes) which can exist in a waveguide are first formulated in the primed system by making use of Maxwell's equations in the primed system and by the use of appropriate boundary conditions. Then by using Lorentz transformations and the principle of phase invariance, fields and normal modes are obtained in the unprimed system. The dispersion relations and cut-off frequencies for both modes are found. From wave equation corresponding to TE modes in the unprimed system, it is concluded that the dispersion relation as well as cut-off frequency corresponding to TE modes is independent of the transverse magnetic field, drift velocity and compressible property, and is the same as that for free space waveguide. However, the propagation and cut-off characteristics in the TM modes are found to be changed significantly as compared to the free space waveguide. The field as well as power flow inside the waveguide are derived for TM modes.

1. Introduction

The propagation of electromagnetic waves through waveguides containing moving media has been the subject of extensive investigation by many workers\(^4\). Collier and Tai\(^2\) have studied the propagation characteristics of electromagnetic waves in a cylindrical waveguide filled with uniformly moving isotropic medium under the assumption that the medium velocity was much smaller than the velocity of light. Shiozawa\(^6\) and Daly\(^7\) later pointed out that the same problem could be solved without imposing any restriction on medium velocity by applying Lorentz transformations and the principle of phase invariance. Jain et al.\(^8\) have studied theoretically the propagation of electromagnetic waves through a parallel plane waveguide containing lossless warm plasma assuming it to be anisotropic in the presence of transverse magnetic field and that the plasma is moving with respect to guide walls with uniform velocity. In this paper, we discuss the characteristics of electromagnetic waves passing through a rectangular waveguide containing lossy warm plasma moving with the velocity \( V \) with respect to the guide wall in the presence of strong transverse magnetic field.

2. Development of Theory

Consider a rectangular waveguide of dimensions \( a \) and \( b \) of perfectly conducting walls containing lossy warm plasma, which is supposed to be moving with respect to the guide walls with a constant velocity \( V \) in the \( z \)-direction. A static magnetic field is applied in the \( x \)-direction. We consider two frames of reference, the primed system in the plasma and the unprimed system which is attached to the guide walls. With the help of linearized hydrodynamic equation of motion, equation of continuity and the equation of state, Maxwell's equations can be put into the following form

\[
\nabla \times \vec{H}' = j \omega' e_0 \bar{e} \cdot \vec{E}' + \frac{1}{\omega' m' (1-j \gamma' / \omega')} \frac{\partial p'}{\partial x'}
\]

\[
\nabla \times \vec{E}' = -j \omega' \mu_0 \bar{e} \vec{H}'
\]

where

\[
\bar{e}' = e'_1 \hat{x}' + e'_2 \hat{y}' + e'_3 \hat{z}'
\]

\[
e'_1 = 1 - \frac{\gamma' \omega'}{\omega' (\omega' - j \gamma')} \text{ and } \omega' = \frac{n'}{m' e_0}
\]

Here \( \mu_0, e_0 \) are permeability and permittivity of free space. Eqs. (1) and (2) are basic equations which characterize the uniaxial and compressible property of plasma inside the guiding structure in the primed system. Using Eqs. (1) and (2), and assuming \( z' \) dependence of the type \( e^{-\gamma' z'} \) one can express transverse fields with respect to the \( x' \)-direction as

\[
\begin{align*}
L' \begin{pmatrix} E'_{1x} \\ E'_{2x} \end{pmatrix} &= \begin{pmatrix} 0 & \frac{\partial}{\partial y'} \\ \frac{\partial}{\partial x'} & 0 \end{pmatrix} \begin{pmatrix} \omega \mu_0 \gamma' & \partial \\ -\gamma' & 0 \end{pmatrix} \begin{pmatrix} \omega \mu_0 \gamma' & \partial \\ -\gamma' & 0 \end{pmatrix} \begin{pmatrix} E'_2 \\ H'_2 \end{pmatrix} \\
L' \begin{pmatrix} H'_{1y} \\ H'_{2y} \end{pmatrix} &= \begin{pmatrix} j \omega e_0 \gamma' & \frac{\partial}{\partial y'} \\ -j \omega e_0 \gamma' & 0 \end{pmatrix} \begin{pmatrix} \omega \mu_0 \gamma' & \partial \\ -\gamma' & 0 \end{pmatrix} \begin{pmatrix} E'_2 \\ H'_2 \end{pmatrix}
\end{align*}
\]

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and $y' = \alpha' + j \beta'$; $\alpha'$, $\beta'$ being attenuation and phase constants in the primed system. $E'_x$, $H'_x$ and $p'$ are found to satisfy the following equations:

$$
\left[ \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \gamma'^2 + k^2 e' \right] E'_x = \frac{e' e'}{\omega^2 m' e_0}
$$

$$
\times \left[ \frac{k''}{(1-jv'/\omega')} - \frac{\omega^2}{a'^2} \right] \frac{\partial p'}{\partial x'}
$$

... (4)

It is seen that $E'_x$ and $H'_x$ satisfy separate wave equations as expected.

For TE modes, set $E'_x = 0$. Then $p'$ is found identically zero from Eqs. (4) and (6). The only potential function, $H'_x$, which must satisfy Eq. (5) gives the dispersion relation for TE modes and the characteristics are found to be the same as those of free space waveguide.

3. Characteristics of TM Modes

The dispersion relation for the TM modes is obtained by substituting Eq. (6) into Eq. (4), by applying the appropriate boundary conditions as:

$$
\beta_{TM}^2 + \left[ K'^2 - \left( 1 - \frac{\omega_p^2}{\omega^2 + v'^2} \right) k^2 \right] \frac{\omega_p^2 K_{12}(K_{12}^2 a'^2 - \omega^2)}{(K_{12}^2 a'^2 - \omega^2)^2 + (\omega v')^2} - \frac{k_0^2 \omega_p^2 K_{12}^2 a'^2 (K_{12}^2 a'^2 - \omega^2 + v'^2)}{\omega^2 (K_{12}^2 a'^2 - \omega^2 + v'^2)^2 + v'^2(2\omega^2 - K_{12}^2 a'^2)^2} = 0
$$

... (7)

where

$$
K_1 = \frac{m \pi}{a} \text{ and } K'^2 = \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2
$$

In order to derive the dispersion relations corresponding to various modes in the moving system, we use the transformation relating the parameters in the primed system to those in the unprimed system. Using these transformations, one obtains the following dispersion relations corresponding to various modes in the unprimed system:

$$
B^4 - 2u \Omega B^3 + [6 u^2 \Omega^2 + M] B^2 - 2u \Omega [2u^2 \Omega^2 + M] B + [u^4 \Omega^4 + u^2 M \Omega^2 - N] = 0
$$

... (8)

where

$$
\begin{align*}
\Omega' &= \Omega - uB \\
r' &= (1 - u^2)^{-1}
\end{align*}
$$

and

$$
B = \frac{\beta_{TM} C}{\omega_p}, \quad \Omega = \frac{\omega}{\omega_p}, \quad z = \frac{v}{\omega_p}, \quad \delta = \frac{a}{C}, \quad \Omega_1 = K_1 C, \quad \Omega_c = KC, \quad u = \frac{V}{C}
$$

Eq. (8) shows that the dispersion relation for TM modes gets modified by terms dependent on collision frequency, drift velocity and compressible property of plasma.

By setting $u = 0$ in Eq. (8), we get

$$
B^4 + \left[ \frac{\Omega_2^2}{\omega^2 + z^2} \Omega^2 - \Omega^2 \Omega_1^2 \delta^2 (\Omega_1^2 \delta^2 - \Omega^2 + z^2) \right] B^2 - \frac{z^2 \Omega^2}{\omega^2 + z^2 + \Omega^2 (\Omega_1^2 \delta^2 - \Omega^2 + z^2)^2 + z^2 (2\Omega^2 - \Omega_1^2 \delta^2)^2 + \Omega_1^2 (\Omega_1^2 \delta^2 - \Omega^2)^2} \left[ \frac{\Omega_2^2}{\omega^2 + z^2} \Omega^2 - \Omega^2 \Omega_1^2 \delta^2 (\Omega_1^2 \delta^2 - \Omega^2 + z^2) \right] \Omega^2 = 0
$$

... (9)
which is same as that when the plasma inside the waveguide is assumed to be at rest. This indicates the correctness of dispersion relation (8).

The cut-off frequencies for TM modes are found by setting $B$ in Eq. (8) equal to zero.

$$u^4 \Omega_0^4 + u^4 M \Omega_0^2 - N = 0 \quad \ldots \ldots (11)$$

where $M$ and $N$ can be obtained by putting $B=0$ in Eq. (9), and $\Omega=\bar{\Omega}_0$. Thus, when the plasma is in relative motion with respect to guide walls, the cut-off frequency for TM mode is changed by terms depending on plasma parameters.

It is also desirable to see how the power flow changes. Since the velocity vector $V$ only has an $x$-component, $\rho V_z$ is always zero. The pressure and velocity fields do not contribute to the power flow in the waveguide. With the help of Eq. (3), potential function and transformation relation, it is easy to establish the orthogonal relation for TM modes with a different mode index. Thus, for a TM mode the power flow is given by

$$W_{TM} = \frac{1}{4} \int \int E_x H_y^* dx dy = \frac{1}{4} \frac{\omega_0^2 \beta_{TM}}{k_x^2 - K_x^2} |E_0^y|^{ab} \ldots \ldots (12)$$

It is clear that $W_{TM}$ does not contain the acoustic speed $'a'$ explicitly. However, the temperature affects the power flow through the propagation number $\beta_{TM}$ in Eq. (12). Also, we observe that the power flow which depends solely on the field components have the same form as those obtained for stationary media because the expression for the electromagnetic field components do not change in the form when the plasma is moving, the only essential difference being in the value of $\beta_{TM}$.

4. Results and Discussion

Eq. (8) has been used to compute $B$ for parameters appropriate to the laboratory plasma, i.e. $\omega_0 = 0.5$, $\Omega_1 = 0.2$, $\delta = 0.02$, $u = 0.5$ and $z = 0.5$. The $(B-\Omega)$ diagram for TM modes is shown in Fig. 1 for a rectangular waveguide filled with moving lossy warm plasma. For comparison, Eq. (10) has also been used to compute $B$ for stationary media ($u = 0$) and is shown in Fig. 2. It is found that for lossy and compressible plasma, propagation is possible over all frequency ranges with cutoff frequency dependent on plasma parameters. The cut-off frequency for the lossy warm plasma shifted towards the high frequency side as compared to the cut-off frequency obtained by Tuan$^8$ and Allis et al.$^9$. It is also observed that in the low frequency range, the propagation constant increases, showing the forward
wave to have positive group and phase velocities, and then decreases in the non-propagating region, showing the existence of a backward wave with positive phase velocity and negative group velocity. Finally, at the higher microwave frequency range, the value of $B$ approaches that of free-space waveguide showing that there is no interaction between electrons and electromagnetic waves. It is concluded that lossy characteristics of plasma destroy the non-propagating region. From Fig. 1 it is observed that though relativistic motion of the medium also makes propagation possible at all frequencies, yet there is no dispersion. However, the cut-off frequency corresponding to the perturbed waveguide modes is shifted to the high frequency side due to the relativistic motion. The inclusion of relativistic motion in the uniaxial plasma model changes the propagation and cut-off properties of TM modes in case of lossless plasma and lossless warm plasma under the influence of strong transverse magnetic field. The resonance frequency also disappears in this case and characteristics are found to be the same in both the cases. It is also observed that for warm plasma moving non-relativistically under the influence of strong transverse magnetic field, electron temperature and drift velocity do not affect the cut-off frequency and are found to be the same as those of free-space waveguides.

5. Conclusion

The propagation characteristics of electromagnetic waves in a rectangular waveguide filled with a uniaxial, warm moving plasma have been studied. The propagation as well as cut-off characteristics of TE modes are independent of drift velocity and compressibility of the medium because there is no interaction between the moving electrons and the fields in the presence of strong transverse magnetic field whereas for TM modes they get modified by terms depending on plasma parameters and the relative movement between plasma and the guide walls. The cut-off frequency is unaffected by electron temperature and strong transverse magnetic fields for a moving non-relativistic medium. It is also seen that lossy characteristics of a plasma make propagation possible over the non-propagating region of lossless temperate plasma. The warmer the plasma the higher is the cut-off frequency.

References