Effect of Dissipation & Diffusion on the Propagation of Atmospheric Gravity Waves in Thermosphere

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An attempt has been made to investigate the extent to which diffusion, viscosity, thermal conduction and ion drag, influence the propagation of atmospheric gravity waves at the thermospheric heights. The perturbation in the electron density is calculated when the diffusion processes are taken into account and it is found that the diffusion can be ignored only when the wave period is less than about 5 min. The viscosity is responsible for damping the slow waves for which the Boussinesq approximation is valid and the wavelength is small. Thermal conduction is more important in damping waves having periods near the Brunt period. The effect of ion drag depends largely on the frequency and is highly complex.

1. Introduction

The theory of internal atmospheric gravity waves in inviscid non-conducting atmospheres without ion drag is straightforward and reasonably successful in describing the dynamics of the upper atmosphere. These waves are responsible for the production of irregular motions and ionization disturbances in the D, E and F regions of the ionosphere. A fairly extensive review on the subject can be found in a recent article by Yeh and Liu.

Unfortunately, the effects of viscosity and thermal conductivity assume great importance above 95 km and cause damping of these waves. Moreover, in the upper atmosphere, hydromagnetic drag (ion drag) is of considerable importance and is found to prevent the exponential increase of the neutral gas velocity around the F-layer maximum. Further, its role is complicated by the fact that it is anisotropic; i.e., the drag on westerly winds (induced by gravity waves) can be different from the drag on the southerly winds. Thus, it is necessary to take these effects into account to formulate a more realistic theory. Recently, attempts have been made to study the propagation of atmospheric gravity waves in a viscous, thermally conducting atmosphere, wherein both these processes have increased in importance inversely with density. Simultaneous inclusion of all the three dissipative processes in the basic equations has been attempted by Klostermeyer. However, in most of these investigations the damping was computed for a few specific waves under consideration. The purpose of this paper is to derive a reasonably simple expression for the attenuation constant so that the computations can be carried out for several model atmospheres. An attempt is also made to investigate to what extent each of these dissipative mechanisms, namely, viscosity, thermal conduction and ion drag, becomes effective.

Another problem arises when one considers the interaction of these waves with the ionization. Usually it is assumed that the electrons are driven by the neutral wave along the direction of the geomagnetic field lines and the influence of pressure gradients on longitudinal motion of electrons (motion along the magnetic field) is often neglected. This effect, commonly known as diffusion, is very rapid near the F2 region and must be incorporated while studying the ionospheric response to the atmospheric gravity waves.

2. Theoretical Background

We assume that the atmosphere can be described by the equations of conservation of mass, momentum and energy. For an isothermal atmosphere, these are:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = q_1 \]  
\[ \rho \frac{DV}{Dt} + \nabla \rho - \varrho g + 2\alpha \Omega \times V = f \]  
\[ \rho T \frac{D\varphi}{Dt} = Q \]

where \( \varrho = \) neutral mass density
\( q_1 = \) rate of production of mass per unit volume (assumed to be localized)
\( V = \) fluid velocity
The asterisk over $V_z$ is used to denote the complex conjugate operation. Using the polarization relation of Hines [his Eqs. (23)-(26)], Eq. (8) can be simplified to give

$$\langle W \rangle = \frac{1}{2} \text{real} (p' V_z^*)$$

where

$$\langle G_z \rangle = \frac{1}{2} \text{real} (p' V_z^*)$$

The propagation of waves in a lossy atmosphere is accompanied by dissipation. The total rate of energy dissipation is

$$W_t = T_0 \frac{d}{dt} \int p_s \, dV$$

where the integration is carried out over all sourceless regions. The temperature $T_0$ is the temperature of the equivalent thermodynamic system with the same entropy. The total dissipation can be computed by making use of Eqs. (1) and (3). On a unit volume basis, the dissipation due to thermal conductivity, viscosity and ion drag can be shown to be given by

$$W_t = \lambda (\nabla T')^2 / T_0 + \sigma : \nabla V' + \nu_n \rho_i (V' - V_i)$$.}

where

$$V' = \text{perturbed fluid velocity}$$

$$T' = \text{perturbed temperature}$$

$$T_0 = \text{background temperature}$$

A correct theoretical description of the atmospheric waves requires, therefore, a full-wave treatment to the set of Eqs. (1)-(3), as has been described, for instance, by Klostermeyer. In order to avoid extensive numerical computations, however, we assume that at the F-region heights, the acoustic gravity wave mode which transports energy upwards predominates. This assumption has been justified to some extent by Volland. The phase propagation and attenuation of the atmospheric waves can then be obtained from a ray approximation or, in other words, from the height dependent complex wave number of the upgoing acoustic gravity waves. For this purpose we assume that the waves propagate upwards in a horizontally stratified atmosphere such that the horizontal component of the wave number, $k_h$, is real and constant at all heights. However, the wave dissipation will make the vertical component of the wave number, $k_z$, complex. If the dissipation is small, then the imaginary part of $k_z$ or the attenuation constant can be computed approximately by using

$$k_z^2 = \langle W \rangle / 2 \langle G_z \rangle$$

where $\langle W \rangle$ is the time averaged dissipation per unit volume given by Eq.(6) and $\langle G_z \rangle$ is the vertical component of energy flux in an inviscid atmosphere.

The attenuation constant $k_z^2$ signifies the gradual extinction of the wave as it propagates upward. Since $W$ (due to three dissipative processes) given by Eq. (6) is additive, its effect on $k_z^2$ should be computed separately for each of the three dissipative processes.

The energy flux is just $\Gamma = p' V'$, and its time average in the vertical direction may be computed approximately by using

$$\langle \Gamma_z \rangle = \frac{1}{2} \text{real} (p' V_z^*)$$

The asterisk over $V_z$ is used to denote the complex conjugate operation. Using the polarization relation of Hines [his Eqs. (23)-(26)], Eq. (8) can be simplified to give

$$\langle \Gamma_z \rangle = \left( | \rho' | / 2 \sigma_0 \right) \omega_k \omega^2$$

where $\omega$ is the angular wave frequency and $\omega_k$ is the angular Brunt frequency. The time-averaged energy loss per unit volume due to thermal conduction is again the perturbed quantities in complex notation,

$$\langle W_{tc} \rangle = \left( \lambda / 2 T_0 \right) (\nabla T') \cdot (\nu T') \cdot (\nu T')^*$$

Making use of the ideal gas law and the polarization relation of Hines, we obtain
This ratio in the Boussinesq limit, applicable to gravity waves with \( \omega^2 < \omega^2_B \) and \( k^2 > 1/4 H^2 \), is obtained from Eqs. (14) and (17) as

\[
\left( \frac{k^2_T}{k^2_T} \right)_{\text{vis}} = \left( \frac{\lambda M y}{\eta R} \right) \frac{V^2_{ph}}{c} \]

Where \( V_{ph} = \omega/k \) is the horizontal phase velocity. This indicates that the relative importance of thermal conduction and viscosity depends on the ratio \( (V_{ph}/c) \) in a Boussinesq fluid, since all other parameters on right hand side of Eq. (18) are constants.

For the purpose of computing energy loss due to ion drag, we assume that the ion responds to the acoustic gravity waves in a quasi-equilibrium way along the magnetic field lines, i.e.,

\[
V_i' = (V' \cdot \mathbf{I}_b) \mathbf{I}_b
\]

Where \( \mathbf{I}_b \) is a unit vector along the earth’s magnetic field. Further, let us choose a coordinate system such that \( X \)-axis is eastward, \( Y \)-axis is northward, and the \( Z \)-axis is upward. Let \( I \) be the magnetic dip. The contribution to \( k^2_z \) due to ion drag is then

\[
\left( \frac{k^2_T}{k^2_T} \right)_{\text{id}} = \frac{2 \gamma c}{\omega k_c} \left[ k^2_z + (k^2_y \sin I + k_z \cos I)^2 \right]
\]

Where \( \gamma = \frac{v_{in} p_i}{\rho_0} \) represents the collision frequency of a neutral particle with all ions. As seen from Eq. (19), the attenuation due to ion drag depends not only on the elevation angle of \( k \) but also on the azimuthal angle of \( k \). The ion drag attenuation in the acoustic limit and in the Boussinesq limit can be similarly reduced from Eq. (19) to give respectively

\[
\left( \frac{k^2_T}{k^2_T} \right)_{\text{id}} = \frac{\nu}{2 \omega k_c} \left[ k^2_z + (k_y \sin I + k_z \cos I)^2 \right]
\]

And

\[
\left( \frac{k^2_T}{k^2_T} \right)_{\text{id}} = - \frac{\nu}{2 \omega k_c} \left[ k^2_z + k_y^2 \sin^2 I \right]
\]

Which agrees with the formula given by Landau and Lifshitz. In the Boussinesq approximation Eq. (15) reduces to

\[
\left( \frac{k^2_T}{k^2_T} \right)_{\text{vis}} = - \frac{\nu}{2 \omega k_c} \frac{\omega^2_B}{2 \rho_0 k_s} k_x
\]

The relative importance of thermal conduction and viscosity can be assessed by taking the ratio

\[
\left( \frac{k^2_T}{k^2_T} \right)_{\text{vis}} / \left( \frac{k^2_T}{k^2_T} \right)_{\text{id}}
\]
tic waves are longitudinal. On the other hand for a Boussinesq fluid, we find that \((kz)_{\|}^2\) vanishes when \(k\) is perpendicular to \(I_1\) in the magnetic meridian. This is also to be expected since waves in the Boussinesq limit are known to be transverse.

3. Atmospheric and Wave Parameters

In order to investigate numerically the behaviour of the attenuation constant \(k^2\) due to viscosity, thermal conduction and ion drag, we need to calculate the coefficients \(\eta\), \(\lambda\) and \(v\). For this purpose, it is assumed that \(O\) and \(O^+\) ions are the main neutral and ionic constituents in the thermosphere. From expression given by Dalgarno and Smith\(^\text{21}\) and by Stubbe,\(^\text{22}\) we then obtain in mks units\(^\text{23}\)

\[
\begin{align*}
\eta &= 3.34 \times 10^{-7} T^{0.71} \\
\lambda &= 6.71 \times 10^{-4} T^{0.71} \\
v &= 7.22 \times 10^{-17} T^{0.77} N_i
\end{align*}
\]

where \(N_i\) is the ion number density. The atmospheric parameters are taken from the model atmosphere CIRA.\(^\text{24}\) Model 5 has been used, which is representative of the mean solar activity. Calculations are performed for Delhi (geogr. lat., 28°38'N; geomag. inclination, 42°). The undisturbed ion density is taken to be \(10^6\) cm\(^{-3}\), as the peak value at 300 km. The waves have angular frequencies in the range \(\omega = 10^{-3}-10^{-2}\) sec\(^{-1}\) and horizontal propagation vector in the range \(k_h=0.5 \times 10^{-5}-10^{-4}\) m\(^{-1}\) covering most of the range of the gravity wave spectrum.

4. Results and Discussion

4.1 Viscosity and Thermal Conduction

Figs. 1 and 2 show the imaginary parts of the vertical propagation vector (\(\text{Im} k_z\)) as a function of the horizontal wave number (or horizontal wavelength) for different periods \(T\) at 200 and 300 km heights. The quantity \(\text{Im} k_z\) determines the attenuation of the upgoing acoustic gravity wave and does not include the term \(1/(2H)\), which is responsible for the exponential growth of the wave amplitude in a non-dissipative isothermal atmosphere. Consequently, the amplitude is proportional to

\[
\exp \left[ \int \left( \frac{1}{2H} - \text{Im} k_z \right) \, dz \right]
\]

where \(z\) is the height. A comparison between the terms \(\text{Im} k_z\) and \(1/(2H)\) thus indicates whether a wave is considerably attenuated or not. To make it obvious the numerical values of \(1/(2H)\) are presented in Figs. 1-3 as dotted lines in addition to \(\text{Im} k_z\) curves.

Each curve in Fig. 1 indicates severe damping at high horizontal wave numbers (or short wavelengths). For practical purposes a wave with \(\text{Im} k_z = 10^{-3}\) km\(^{-1}\)
or less experiences little damping at that height. If on the other hand \( \text{Im } k_x = 10^{-3} \text{ km}^{-1} \) or larger at a certain height, the wave will be severely damped and will not be able to propagate upward above that height. As seen from Fig. 1, for a given period, \( \text{Im } k_x (\text{viscous}) \) is larger than \( \text{Im } k_x (\text{thermal}) \) for small \( k_x \) (long wavelength), while the converse is true for larger \( k_x \) (short wavelength). This indicates that the thermal conduction is more effective in damping the long wavelength waves while viscosity is more effective in damping the short wavelength waves. The dependence of damping on height can be obtained by comparing Figs. 1 and 2. For the same period and the horizontal wave number the corresponding attenuation constant at 300 km is roughly 2-5 to 5 times higher than that at 200 km. The dependence of attenuation constant on wave frequency (or period) is shown in Fig. 3 for two heights. The curves are drawn for a fixed horizontal wavelength with both viscosity and thermal conduction taken into account. All curves show a broad minimum near which damping is minimum. The distinct minima in the curves of the gravity waves \( (T = 20-45 \text{ min}) \) results mainly from two processes which can roughly be described as follows:

(i) Viscosity and thermal conduction cause an increase of \( \text{Im } k_z \) with decreasing horizontal phase velocity \( V_{\text{ph}} (\approx \omega/k_x) \).

(ii) The waves become evanescent with increasing \( V_{\text{ph}} \) leading to an asymptotic increase of \( \text{Im } k_z \).

Such results have first been published by Volland\(^{18}\) and show that the upper atmosphere reacts like a filter for acoustic gravity waves. These results have also been utilized by Klostermeyer\(^{15}\) to explain the dispersion in the nighttime horizontal phase velocity as observed by Herron.\(^{16}\)

4.2 Ion Drag

The above results show that viscosity and thermal conduction have a strong control on those waves which propagate above 300 km height. We now study the effect of ion-drag on those waves for which the thermosphere appears to be more transparent than for other waves. For this purpose we have chosen the period to be 30 min. Fig. 4 shows the graph of attenuation constant versus the azimuthal angle of propagation measured in the eastward direction from the magnetic north for different dip angles \( \phi \) at 300 km height which is assumed to be the daytime reflection level for the majority of the observed data.\(^{14,27,28}\)

As mentioned earlier the daytime peak electron density has been taken as \( 10^6 \text{ cm}^{-3} \) as a representative value at 300 km altitude. It can be seen from Fig. 4 that for a given period and magnetic dip, the loss due to ion drag is minimum when the wave propagates with an azimuthal angle of 0° (southward) or 180°.
4.3 Diffusion

While calculating the damping due to ion drag, we assumed that the electrons are driven by the neutrals along the direction of the field lines and neglected the influence of the pressure gradients (diffusion) on the longitudinal motion of the electrons. This assumption has also been made by various authors while discussing the interaction of the atmospheric gravity waves with the ionization. Under this assumption, the perturbation in the electron density due to an atmospheric gravity wave propagating at the F2 region height is given by

\[
N'_o = \left( \frac{1}{\omega} \right) (V \cdot I_b) I_b \left( k - i I_z \frac{\partial}{\partial z} \right) N_{eo}
\]

where \( N'_o \) is the perturbation in the electron density induced by the gravity wave and \( N_{eo} \) is the background ionization density. Eq. (20) in its simple form does explain the observed characteristics of the gravity wave spectra qualitatively.\(^{11,32,33}\) However, for an accurate comparison between the theory and the experiment, the effect due to diffusion must be included while calculating the perturbation in electron density. Eq. (20), when this effect is included, takes the form\(^8\)

\[
N'_o = (-i\omega) \left\{ \nabla \cdot (N_{eo} V_i) + D_a \frac{\partial^2 N'_o}{\partial z^2} \right\}
\]

where \( \zeta \) is an axis parallel to the magnetic field and \( D_a \) is the diffusion constant and is given by

\[
D_a = \frac{2kT_i}{m n_i},
\]

\( k \) being the Boltzmann constant. For plane waves, the above equation reduces to

\[
N'_o = (\omega + i D_a k_b^2)^{-1} (V \cdot I_b) I_b \left( k_b - i I_z \frac{d}{dz} \right) N_{eo}
\]

where \( k_b = k - I_b \)

The above equation suggests that the diffusion introduces a phase shift in the perturbation from that due to the dynamic effect alone.

A rough estimate of the diffusion term can be made by taking some numerical values for the above terms, i.e.

Taking \( D_a = 2 \times 10^{-6} \text{ m}^2/\text{sec} \) and \( k_b = 3 \times 10^{-2} \text{ m}^{-1} \)

we get \( D_a k_b^2 = 2 \times 10^{-3} \text{ sec}^{-1} \)

which is of the same order as \( \omega \).

Thus diffusion can be ignored only when
\( \omega > 10 D_\eta k_z^3 \)

This corresponds to a wave period of 5 min.

5. Conclusions

In this paper, we have presented a method whereby the attenuation constant of acoustic gravity waves can be computed approximately. The results indicate that the viscosity is responsible for damping slow waves for which the Boussinesq approximation is valid and the wavelength is small. On the other hand, thermal conduction is more important in damping waves with periods near the Brunt period. The dissipation due to ion drag depends both on the frequency and the azimuth of the wave, being smaller for N-S propagating waves and larger for E-W propagating waves. The loss due to ion drag is comparable to that due to viscosity or thermal conduction during daytime conditions. The three dissipative processes are expected to have strong influence on the gravity wave spectrum in the thermosphere. Results show that the periods below the Brunt period and above about 60 min are filtered out in the thermosphere through damping. These results are in general agreement with the spectra obtained experimentally, although the presence of the neutral winds and temperature gradients give rise to additional complications regarding the low cut off period. The diffusion can be neglected only when the wave period is less than 5 min. The main effect of the diffusion is to introduce a phase shift from that due to dynamic effect alone.

It may be pointed out that although the computations have been carried out by assuming an isothermal atmosphere, the method itself does not have this restriction.

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