Effective Collision Frequency of Electrons in Helium & Neon

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The efforts in the past to derive the electron collision frequencies were confined to evaluations starting with the momentum transfer cross-section. In this way no weightage is given to the other experimentally derived parameters. Naccache and McDowell [Naccache P F & McDowell M R C, J. Phys., 7B (1974), 2203] have derived parametrized phase shifts which give weightage to all the experimentally measured cross-sections. The present study deals with the computation of effective collision frequencies in He and Ne starting with the phase shifts. These results are compared with the earlier results, and are found to be in complete agreement.

1. Introduction

Rare gases form an important part of the terrestrial and planetary atmospheres. The effective collision frequencies of electrons in rare gases are of interest as they lead to the derivation of transport properties like thermal and electrical conductivities, viscosity, diffusivity and mobility.

Workers have determined earlier the effective collision frequencies of electrons for collisions with rare gases based on the experimental results of momentum transfer cross-section. There has also been an increasing tendency to calculate theoretically the momentum transfer and the total cross-sections. They show good agreement with the existing experimental values. In this paper the authors have used the theoretically calculated values of phase shifts derived from experimentally determined collision cross-sections to obtain the effective electron collision frequencies in He and Ne. These calculations hold good for the thermal energy region (below 1 eV), which is much below the threshold energies needed for dissociation, excitation and ionization.

In a swarm which is an ensemble of electrons with a wide distribution of electron energy, the average electron energy is sufficiently low so that the few electrons in the high energy tail which can excite and ionize the atoms, can be neglected. With this restriction, a gas is assumed to be weakly ionized.

The collision frequency of mono-energetic electrons with gas atoms or molecules is given by

\[ \nu_m(v) = n \times Q_m(v) \]  

where \( n \) is the number density of electrons, \( v \) the velocity of electrons and \( Q_m(v) \) the momentum transfer cross-section.

Following Majumdar, the effective collision frequency \( \nu_{ef} \) which will represent exactly the collision effect equivalent to constant \( \nu \) in Appleton-Hartree equation in the limiting cases, is obtained by averaging \( \nu_m(v) \) over a Maxwellian velocity distribution of electrons and is given as follows.

(i) When \( \omega > v \) and \( (\omega \pm \omega_H)^2 > v^2 \)

\[ \frac{\nu_{ef}}{n} = \frac{4}{3v_0^2} \int_0^\infty \nabla_m(\epsilon) e^{i\omega \epsilon} e^{i\epsilon/2} d\epsilon \]  

(ii) When \( \omega < v \) and \( (\omega \pm \omega_H)^2 < v^2 \)

\[ \frac{\nu_{ef}}{n} = \frac{4}{3v_0^2} \int_0^\infty \nabla_m(\epsilon) e^{i\omega \epsilon} e^{i\epsilon/2} d\epsilon \]

where

\( \epsilon = \frac{m_e v^2}{2k_BT_e} \)

\( \omega \) Angular radio frequency

\( \omega_H \) Angular electron gyrofrequency

\( k_B \) Boltzmann constant

and other symbols have their usual meanings.

2. Helium and Neon

The laboratory plasma experiments yield: (i) the total cross-section, (ii) the diffusion cross-section and (iii) the differential cross-section of electrons. McDowell and recently Naccache and McDowell have made phase shift analysis of electron-helium and electron-neon scattering in the elastic region. They obtained a set of \( s, p, d \), phase shifts consistent with the experimental data of cross-sections (i), (ii) and (iii) referred to above. The phase shifts so obtained have also been found to be in good agreement with the theoretical values. For He, the phase shifts for...
the non-resonant part of the energies of the $s$, $p$ and $d$ waves respectively are:

$$\gamma_0 (k^2) = \pi$$

$$\gamma_0 (k^2) = \arctan \left\{ \frac{k a_{13} (1 + a_3 k^2)}{\left[ 1 - 1 \cdot 45661 a_{13}^2 k - 0.925 k^2 \ln k^2 \right] + a_3 k^2 + a_4 k^2} \right\}$$  \hspace{1cm} \ldots (4)$$

$$\gamma_1 (k^2) = k^2 \left[ \frac{\beta_01 + a_5 k + a_6 k^2 + a_7 k^3}{1 + a_3 k^2} \right]$$  \hspace{1cm} \ldots (5)$$

$$\gamma_2 (k^2) = \beta_02 k^2 \left[ \frac{1 + a_9 k + a_{10} k^2}{1 + a_3 k^2} \right]^{-2}$$  \hspace{1cm} \ldots (6)$$

where

$$\beta_l = \frac{\pi \alpha}{(2l - 1)(2l + 1)(2l + 3)}; \quad l = 1, 2 \quad \ldots (7)$$

$k$ Wave number,

$\alpha$ Polarizability factor

$a_{13}, \ldots, a_{11}$ Fixed parameters

The contribution of the waves with $l > 2$ is negligibly small and is not taken into account.

The momentum transfer cross-section, $Q_m(k)$, is given by,\(^8\)

$$Q_m(k) = \frac{4\pi}{k^2} \sum (l + 1) \sin^2 (\gamma_l - \gamma_{l+1})$$  \hspace{1cm} \ldots (8)$$

where $\gamma_l$ is the phase shift of the $l$th wave and for $l \leq 2$. Using Eqs. (4)-(8) and Eq. (1), $\nu_{\text{eff}}/n$ has been solved numerically for He and Ne and is plotted in Figs. 1 and 2 as a function of $T_e$. Fig. 1 corresponds to the condition (i) given in Eq. (2) and Fig. 2 corresponds to the condition (ii) given in Eq. (3).

In the same figures we have plotted the earlier published results of Banks\(^1\) and Itikawa\(^2\) for comparison.

The phase shifts for Ne are given by,\(^6\)

$$\tan \gamma_0 (k^2) = - a_{01} k$$

$$- \left( \frac{\pi \alpha}{3} \right) k^2 - \frac{4}{3} a_{01} a k^3 \ln k + \sum_{i=1}^{5} a_{0i} k^{l+1}$$  \hspace{1cm} \ldots (9)$$

and

$$\gamma_l (k^2) = \frac{\pi \alpha k^2}{(2l - 1)(2l + 1)(2l + 3)} + \sum_{i=1}^{m} a_{li} k^{l+2}$$  \hspace{1cm} \ldots (10)$$

where the symbols have the same meanings as in Eqs. (4)-(7).

### 3. Discussion

The effective collision frequencies have been calculated in two ways. First we obtain a straightforward solution by solving Eqs. (2) and (3) using the

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**Fig. 1**—Effective collision frequency (e-He and e-Ne) versus electron temperature for the condition : $\omega^2 \gg \nu^2$ and $(\omega + \omega_{\text{eff}})^2 \gg \nu^2$ (The upper curves are for helium and the lower one is for neon).

**Fig. 2**—Effective collision frequency (e-He and e-Ne) versus electron temperature for the condition : $\omega^2 \gg \nu^2$ and $(\omega + \omega_{\text{eff}})^2 \ll \nu^2$ (The upper curves are for helium and the lower one is for neon).
Simpson’s rule. Secondly, when the condition (i) [Eq. (2)] is satisfied, Eq. (2) can be simplified to obtain an analytical expression as follows.

In the low energy region, following O’Malley\textsuperscript{16} we have

\[
Q_m(k) = 4\pi \left[ A^2 + \left( \frac{4\pi}{5} \right) aA\kappa \left( \frac{8}{3} \right) a^2 A^2 k^2 \ln(k) + Bk^2 + \cdots \right]
\]  

\text{(11)}

where \( A \) is the scattering length and \( B \) is a constant.

Using Eqs. (11) and (2), we get

\[
\frac{\nu_{\text{eff}}}{n} = \frac{16}{3} \sqrt{\frac{2\pi k_B T}{m_e}} \left[ 2A^2 + \frac{3}{2} \pi a^3 b^2 A^2 b^2 + \frac{4}{3} aA^2 b^2 \right]

\times (11 - 6\gamma) + 16 \pi a^3 b^2 \ln b + 6 B b^2 \]  

\text{(12)}

where

\[ \beta = \sqrt{E/E_R}; \ E_R = 13.605 \text{ eV}, \text{ and } \gamma \text{ is the Euler’s constant.} \]

The computations made using Eq. (12) agree very well with those obtained by solving Eq. (2) directly.*

The results of Itikawa for the effective collision frequencies are based on the experimental momentum transfer cross-sections given by Crompton \textit{et al.}\textsuperscript{11} for He and Ne. Banks has compiled the cross-section data obtained experimentally by several workers. He arrived at a compromise model which gives the \( \nu_{\text{eff}}/n \) profile shown in Fig. 1. It is found from the Figs. 1 and 2 that the cross-section data for (both He and Ne) used by Itikawa and those used by us are in excellent agreement. The cross-sections based on earlier experiments and used by Banks, on the other hand, are quite different. Our calculated values of mobility and diffusion coefficient (for both He and Ne) based on the results of \( \nu_{\text{eff}} \) are in close agreement with the experimental values given by Nelson and Davis\textsuperscript{12} and Pack and Phelps\textsuperscript{13} respectively.

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**References**


* The calculated values are available with the authors.