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Instabilities of the Whistler Mode in the Magnetosphere*

G RENUKA & K S VISWANATHAN
Department of Physics, University of Kerala, Kariavattom, Trivandrum 695 581

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The nature of the convective instability has been investigated for whistler wave propagation at frequencies less than and close to the cyclotron frequency for an anisotropic and loss cone velocity distribution for the electrons. The dispersion equation has been solved and an expression is given for the growth or decay rate of the waves for convective instability. Detailed numerical calculations have been made for the growth or decay rate of the wave for different values of the anisotropy ratio $T_\perp/T_\parallel$ of the perpendicular and parallel temperatures, and the McIlwain parameter L. It is found that for parallel propagation, the waves are unstable for frequencies less than the cyclotron frequency and the instability depends only on the anisotropy parameter $T_\perp/T_\parallel$ and not on the loss cone index j.

1. Introduction

A systematic study of the instabilities of the various types of waves that can propagate in the magnetosphere will help to unravel some of the unsolved problems like the auroral precipitation, acceleration of particles, etc. in Space Physics. Electrostatic and electromagnetic instabilities, both for parallel wave propagation as well as for oblique propagation, have been considered earlier by several workers[1-4] and these have been reviewed in a monograph by Hasegawa.[5] It is now well established from the vast amount of satellite data on Van Allen particles that there are several mechanisms operating in the magnetosphere tb produce a complex variety of non-Maxwellian and unstable plasma distributions in the earth's ionized environment. However, the nature of plasma instabilities and the ranges in which they occur, both for frequency as well as for wave vector, coming from these distributions have not been, so far, exhaustively studied.

Electrostatic or electromagnetic modes generally become unstable either because the velocity distribution is anisotropic or beams travelling with velocities greater than the phase velocities of the waves are present in the plasma. The whistler is a circularly polarized electromagnetic wave, that has been observed to propagate at very low frequency (vlf) in the range 3-30 kHz and propagation is generally possible for the whistlers when their frequency is less than the cyclotron frequency of the electrons. The whistler instability has been studied earlier by Leimohn[6] for an anisotropic Cauchy distribution function and by several others[7-10] for absolute instability of the whistler mode.

In this work, we study the nature of the Landau damping or the convective instability for the whistler mode, using a loss cone and anisotropic velocity distribution. Denoting by $T_\parallel$ and $T_\perp$ the prrallel and perpendicular temperatures, respectively, we have studied the whistler instability for values of $(T_\perp/T_\parallel)$ from 1 to 40, and in this range it is found that the whistler becomes unstable when the frequency of the wave is less than and equal to $(0{\cdot}9\,|\,\omega_c\,|)$. The numerical results suggest that the instability starts even for smaller values of $T_\perp/T_\parallel$, as the ratio ω/ω_c is diminished further and generally whistlers with small frequencies $(\omega/\omega_c \sim 0{\cdot}5)$ become unstable even for values of $T_\perp/T_\parallel$ of the order of two. It has also been found that for parallel whistlers, the loss cone index does not significantly affect the damping or growth rates of the waves and it is the temperature anisotropy that plays a crucial role in stimulating the instability.

2. Convective Instability

The dispersion relation for a vlf wave of the form exp $(ikz - i\omega t)$, interacting with electrons has been earlier given by Leimohn (Ref. 6, page 863) and is given by

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$$c^2k^2/\omega^2 = 1 + \sum_{i,e} \frac{\pi \omega_p^2}{k\omega} \int_0^\infty d\,v_\perp \int_{-\infty}^{+\infty} - \frac{v^2\,\partial F/\partial v_\perp}{(v_\parallel - v_c)}\, dv_\parallel$$

$$+ \sum_{i,c} \frac{\pi \omega_p^2}{k\omega} \int_0^\infty d\,v_\perp \int_{-\infty}^{+\infty} \frac{(v_\parallel\,\partial F/\partial v_\perp - v_\perp\,\partial F/\partial v_\parallel)v_\perp^2\, k dv_\parallel}{\omega\,(v_\parallel - v_c)} \quad ...(1)$$

where

$$v_c = (\omega \pm \omega_c)/k \quad ...(2)$$

In Eqs. (1) and (2), the summation is over all species (electrons e and ions i); the + and − signs in v_c refer to the right-handed and left-handed circularly polarized modes, respectively; ω and $\mathbf{k}$ represent the frequency and wave vector, respectively of the electromagnetic wave; ω_p and ω_c are the plasma frequency and cyclotron frequency and F denotes the frequency distribution of the particles. In the region of the radiation belts, the electrons have anisotropic velocity distribution, and the components of their parallel and perpendicular velocities along a line of force are different. Besides, under conditions of geomagnetic storm, one or two of the adiabatic invariants could be violated and the electrons can suffer pitch angle scattering into the loss cone.

We shall therefore, in the following, investigate the interaction of the electrons having a loss cone and anisotropic velocity distribution with a vlf electromagnetic wave propagating along a line of force. We shall assume that the electrons obey a loss cone distribution of the following form[11]

$$F_j\,(v_\parallel, v_\perp) = (j\,!\,\pi^{3/2}\,u\,w^{(2j+2)})^{-1}$$

$$\times\, v_\perp^{2j} \exp\{-v_\parallel^2/u^2 - v_\perp^2/w^2\} \quad ...(3a)$$

$$= A_j\, v_\perp^{2j}\, B_t \quad ...(3b)$$

where

$$A_j = \{j\,!\,\pi^{3/2}\,u\,w^{(2j+2)}\}^{-1} \quad ...(4a)$$

$$B_t = \exp\{-v_\parallel^2/u^2 - v_\perp^2/w^2\} \quad ...(4b)$$

$$u = \{2\,T_\parallel/m\}^{1/2} \quad ...4(c)$$

$$w = \{2\,T_\perp/(1+j)\,m\}^{1/2} \quad ...(4d)$$

$$v_\perp = v \sin\alpha \quad ...(4e)$$

and

$$v_\parallel = v\cos\alpha \quad ...(4f)$$

In Eqs. (3a) to (4f), u and w are the thermal velocities; j is the index of the loss cone ($j > 0$); $v_\parallel$ and $v_\perp$ are the velocity components along and perpendicular to the magnetic field and α is the particle's pitch angle.

For studying parallel right-handed electromagnetic waves (whistlers), we regard the ions as essentially stationary and evaluate the contribution arising from the electron term only. The derivatives $\partial F/\partial v_\parallel$ and $\partial F/\partial v_\perp$ can easily be evaluated and on substituting the expressions for these in the dispersion Eq. (1), we get

$$c^2\,k^2/\omega^2 = 1 + \frac{\pi\,\omega_p^2}{k\omega}\int_0^\infty d\,v_\perp$$

$$\times \int_{-\infty}^{+\infty} - \frac{2A_j\,B_t\,(j - v_\perp^2/w^2)\,v_\perp^{(2j+1)}}{v_\parallel - v_c}\, d\,v_\parallel + \frac{\pi\omega_p^2}{k\omega}$$

$$\times \int_0^\infty d\,v_\perp \int_{-\infty}^{+\infty} \frac{2A_j\,B_t\,k\,v_\parallel\,v_\perp^{(2j+3)}}{\omega\,(v_\parallel - v_c)}\, d\,v_\parallel$$

$$\times \left(\frac{1}{u^2} - \frac{1}{w^2} + \frac{j}{v_\perp^2}\right) \quad ...(5)$$

The integrals occurring in Eq. (5) can be easily evaluated and one can easily verify that

$$G_1 = (Z_1\,k/\omega)\,(1/u^2 - 1/w^2) \int_0^\infty d\,v_\perp \int_{-\infty}^{+\infty} Z_2\, v_\perp^2\, v_\parallel\, d\,v_\parallel$$

$$= \frac{\omega_p^2}{\omega^2}\left(\frac{w^2}{u^2} - 1\right)(1+j)\left\{1 + \frac{2v_c^2\,M}{u\,\pi^{1/2}}\right\} \quad ...(6a)$$

$$G_2 = (Z_1/w^2) \int_0^\infty dv_\perp \int_{-\infty}^{+\infty} v_\perp^2\, Z_2\, d\,v_\parallel$$

$$= \frac{2\omega_p^2}{k\omega}\cdot\frac{M\,v_c}{u\,\pi^{1/2}}\,(1+j)$$

$$G_3 = -\,j\,Z_1 \int_0^\infty d\,v_\perp \int_{-\infty}^{+\infty} Z_2\, d\,v_\parallel \quad ...(6b)$$

$$= -\,\frac{2\omega_p^2}{k\omega}\cdot\frac{M\,v_c\,j}{u\pi^{1/2}} \quad ...(6c)$$

and

$$G_4 = (j Z_1 k/\omega) \int_0^{\infty} d v_\perp \int_{-\infty}^{+\infty} Z_2 v_{\parallel} d v_{\parallel}$$

$$= \frac{\omega_p^2 j}{\omega^2} \left\{ 1 + \frac{2 v_c^2 M}{u \pi^{1/2}} \right\} \quad ...(6d)$$

where

$$M = \int_0^{\infty} \frac{\exp\{-v_{\parallel}^2/u^2\}}{(v_{\parallel}^2 - v_c^2)} d v_{\parallel}$$

$$= (i \pi/2v_c) W(v_c/u) \quad ...(7a)$$

$$Z_1 = 2\pi \omega_p^2 A_j/k\omega \quad ...(7b)$$

and

$$Z_2 = B_t v_\perp^{(2j+1)}/(v_{\parallel} - v_c) \quad ...(7c)$$

The error integral $W(s)$ is defined by

$$W(s) = \frac{2si}{\pi} \int_0^{\infty} \frac{\exp(-t^2)}{(s^2 - t^2)} dt \quad (I_{ms} > 0) \quad ...(8)$$

Substituting these in Eq. (5), we find that the dispersion equation takes the form

$$c^2 k^2/\omega^2 = 1 + \frac{\omega_p^2}{\omega^2}\left(\overline{1+j}\ \frac{w^2}{u^2} - 1\right)$$

$$+ i \left\{ \frac{\pi^{1/2} \omega_p^2 W}{u\omega} \left[\frac{1}{k} + \frac{v_c}{\omega}\left(\overline{1+j}\ \frac{w^2}{u^2} - 1\right)\right]\right\} \quad ...(9)$$

We shall try to investigate the behaviour of a wave (either its spatial damping or growth) of a particular frequency ω and propagating along a line of force. We are more specifically interested in convective instability, and shall assume that ω is real whereas k is complex. Further, we shall consider waves whose frequencies are less than the cyclotron frequency, so that $|(v_c/u)|$ is small. In this case, $W(v_c/u)$ can be expanded by means of the following expansion (Ref. 5, page 39), which holds good for small values of v_c/u.

$$W(v_c/u) = \exp\{-v_c^2/u^2\} + \{2i(v_c/u)/\pi^{1/2}\}$$

$$\times \left\{ 1 - \frac{2}{3}(v_c/u)^2 + \frac{4}{15}(v_c/u)^4 - \frac{8}{105}(v_c/u)^6 + ... \right\} \quad ...(10)$$

We shall now study the damping or the growth rate of a wave of frequency ω, such that ω/ω_c is less than unity and close to it. Let us write

$$k = k_r + i k_i \quad ...(11)$$

Substituting this in Eq. (9), we find that the dispersion equation can be written in the form

$$\epsilon(k, \omega) = A_c + i B_c \quad ...(12)$$

where

$$A_c = k_r^2 - \frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2}\left(\overline{1+j}\ \frac{w^2}{u^2} - 1\right)$$

$$- \frac{N}{k_r}\left(\frac{k_i}{k_r} W_r - W_i\right) \quad ...(13)$$

and

$$B_c = 2 k_i k_r - \frac{N}{k_r}\left(\frac{k_i}{k_r} W_i + W_r\right) \quad ...(14)$$

where W_r and W_i are the real and imaginary parts of $W(v_c/u)$ and are approximately given by

$$W_r = \exp\left\{-\left(\frac{\omega - \omega_c}{u k_r}\right)^2\right\} + \frac{2}{\pi^{1/2}} \frac{k_i}{k_r}\left(\frac{\omega - \omega_c}{u k_r}\right)$$

$$- \frac{4}{\pi^{1/2}} \frac{k_i}{k_r}\left(\frac{\omega - \omega_c}{u k_r}\right)^3 \quad ...(15)$$

and

$$W_i = \frac{2}{\pi^{1/2}}\left(\frac{\omega - \omega_c}{u k_r}\right)\left\{1 - \frac{2}{3}\left(\frac{\omega - \omega_c}{u k_r}\right)^2\right\} \quad ...(16)$$

Further

$$N = \frac{\pi^{1/2} \omega_p^2 \omega}{u c^2}\left[1 + \frac{\omega - \omega_c}{\omega}\left(\overline{1+j}\ \frac{w^2}{u^2} - 1\right)\right] \quad ...(17)$$

For studying convective instabilities, we keep ω as real and k complex, and in this case the solution of the dispersion equation is given by the simultaneous solutions of the equations

$$A_c = 0 \quad ...(18)$$

and

$$B_c = 0 \quad ...(19)$$

The latter condition immediately determines the imaginary part k_i of k and assuming $|k_i| \ll k_r$, one obtains that

$$k_i = \frac{N \exp\left\{-\left(\frac{\omega-\omega_c}{uk_r}\right)^2\right\}}{\left\{2k_r^2 - \frac{4N}{k_r\pi^{1/2}}\left(\frac{\omega-\omega_c}{u\,k_r}\right)\left[1-\frac{4}{3}\left(\frac{\omega-\omega_c}{u\,k_r}\right)^2\right]\right\}} \quad ...(20)$$

If this is substituted in Eq. (13) one could obtain the corresponding value of k_r from the following equation.

$$k_r^2 = \omega^2/c^2 + \frac{\omega_p^2}{c^2}\left(\overline{1+j}\,\frac{w^2}{u^2} - 1\right) + \frac{Nk_i}{k_r^2}\exp\left\{-\left(\frac{\omega-\omega_c}{uk_r}\right)^2\right\} - \frac{2N}{\pi^{1/2}k_r}\left(\frac{\omega-\omega_c}{uk}\right) + \frac{4N}{3\pi^{1/2}k_r}\left(\frac{\omega-\omega_c}{uk_r}\right)^3 \quad ...(21)$$

The relation in Eq. (20) can be obtained alternatively also from the dispersion relation

$$\epsilon\,(k,\omega) = 0 \quad ...(22)$$

If we write $k = k_r + i\,k_i$ in the equation

$$\epsilon\,(k,\,\omega) = \epsilon_1\,(k,\omega) + i\,\epsilon_2\,(k,\omega) \quad ...(23)$$

we obtain

$$\epsilon\,(k,\omega) = \epsilon_1\,(k_r,\omega) + i\,k_i\,\frac{\partial\epsilon_1}{\partial k} + i\,\epsilon_2\,(k,\omega) \quad ...(24)$$

and hence one gets

$$k_i = -\,\epsilon_2\,(k,\omega)/(\partial\epsilon_1/\partial k) \quad ...(25)$$

Eq. (25) is in fact identical with Eq. (20). Eqs. (20) and (21) were solved numerically by a process of iteration, for any given value of ω, and after the value of k_r has been determined, k_i can be obtained from Eq. (20).

3. Numerical Results and Discussion

The dispersion Eqs. (20) and (21) were solved numerically using the TDC 316 computer for different values of $T_\perp$ and $T_\parallel$ so that the effect of the anisotropy ratio $T_\perp / T_\parallel$ on the damping rate or growth rate could be ascertained at different points in the equatorial plane. The magnetic field B at any point was represented by the dipole field given by the formula

$$B = (B_0/L^3\cos^6\lambda)\,(1+3\sin^2\lambda)^{1/2} \quad ...(26)$$

where L represents the McIlwain parameter, B_0 is the equatorial magnitude of the magnetic field and λ is the geomagnetic latitude. The cyclotron frequency and plasma frequency are, respectively, given by

$$\omega_c = |\,e\,|\,B/mc \quad ...(27)$$

and

$$\omega_p^2 = 4\,\pi\,ne^2/m \quad ...(28)$$

The different parameters used in our calculations were as follows.

$B_0 = 0{\cdot}31$ gauss

$j = 0,\ 1$ and 3

$T_\perp = 20$ keV

$\omega/\omega_c = 0{\cdot}99,\ 0{\cdot}9,\ 0{\cdot}8,\ 0{\cdot}75,\ 0{\cdot}7,\ 0{\cdot}65$ and $0{\cdot}5$

$L = 2{,}3$ and 7

$\lambda = 0$ degree

and $T_\perp / T_\parallel$ was given a wide range of values starting from 1 upto 40. The temperature associated with 1 eV is $1{\cdot}60 \times 10^4$ °K. The particle density n chosen for $L = 2$, 3 and 7 are 500, 500 and 5 particles per cm^3, respectively, representing the fact that the density is higher in the plasmapause.[12]

In Tables 1-4, we reproduce the numerical values of the growth or decay rate (k_i) of the waves for different values of the parameter $T_\perp / T_\parallel$, L, j and ω/ω_c. It can be seen from the Tables 1 and 2 that for $L = 2$ and $j = 3$, k_i is positive for all values of the anisotropy parameter $T_\perp / T_\parallel$ at $\omega/\omega_c = 0{\cdot}99$. For $\omega/\omega_c = 0{\cdot}9$, $0{\cdot}8$, $0{\cdot}75$ and $0{\cdot}7$ the wave shows damping (k_i is positive) as well as growth (k_i is negative). For $\omega/\omega_c = 0{\cdot}8$, the waves are growing and unstable for $T_\perp / T_\parallel \geq 5$ but are Landau damped for values of $T_\perp / T_\parallel$ below this threshold value and the damping rate k_i is of the order of 10^{-5}. For $T_\perp / T_\parallel = 5$, the growth rate is extremely small ($0{\cdot}14102 \times 10^{-10}$) and is 10^{-5} smaller than the growth rate corresponding to higher values of this parameter. This suggests that for a small region of values in the neighbourhood of 5 for $T_\perp / T_\parallel$, the waves are practically undamped and could propagate with very little growth rate or damping. For $\omega = 0{\cdot}9\,\omega_c$, the waves are Landau damped for values of $T_\perp / T_\parallel$ upto 10, but for higher values of this parameter, they are unstable. For $\omega/\omega_c = 0{\cdot}75$ and $0{\cdot}7$, the threshold value of $T_\perp / T_\parallel$ are 4 and $3{\cdot}3333$, respectively. At $L = 3$, $j = 1$ and $\omega = 0{\cdot}5\,\omega_c$, the wave starts growing even at

$T_\perp / T_\parallel = 2$ and the corresponding growth rate is given by

$$k_i = 0{\cdot}69869 \times 10^{-11}$$

Table 3 gives the k_i and the corresponding k_r values for $\omega/\omega_c = 0{\cdot}75$, $L = 7$ and $j = 1$ and 3. The k_i and k_r values show no dependence on the loss cone index j. This shows that a loss cone distribution does not produce any instability for parallel propagation, but for perpendicular propagation of the different types of waves, it may be important. For $T_\perp / T_\parallel = 2$, k_i is $0{\cdot}20757E-06$ and k_r is $0{\cdot}10016E-04$. This corresponds to a wavelength of 6·272 km which is in good agreement with the observed whistler frequency (3-30 kHz). The computer calculations also suggested that the values of k_i and k_r for all $T_\perp / T_\parallel$, j, L and ω are in agreement with the assumption that $|k_i| \ll k_r$. As an example, for $T_\perp / T_\parallel = 10$, the numerical values of k_r and k_i are $0{\cdot}16717E-03$ and $-0{\cdot}50364E-08$, respectively, and in this case the ratio $|v_c/u| = |(\omega-\omega_c)/u\,k_r|$ is 0·00896, which is much less than unity.

The dependence of k_i on the parameter L can be visualized easily from Table 4. The growth rate values decreases with increasing L for $\omega/\omega_c = 0{\cdot}7$ and $j = 0$.

In Fig. 1, we give the plot of k_i versus $T_\perp / T_\parallel$ for $L=7$, $\omega/\omega_c = 0{\cdot}99$, 0·9 and 0·8. The part of the graph shown in thick lines corresponds to damping, whereas the part shown in dotted lines corresponds to growth of the wave (k_i is negative). The wave is completely damped for $\omega/\omega_c = 0{\cdot}99$ and the damping rate at first increases upto $T_\perp / T_\parallel = 4$ and thereafter it

Table 1—Numerical Values of the Growth or Decay Rate k_i for $j = 3$, $L = 2$, $\omega/\omega_c = 0{\cdot}99$, 0·9 and 0·8

$T_\perp/T_\parallel$	k_i cm^{-1}		
	$\omega/\omega_c = 0{\cdot}99$	$\omega/\omega_c = 0{\cdot}9$	$\omega/\omega_c = 0{\cdot}8$
1·2500	0·17480E−05	0·48796E−05	0·54815E−05
1·6667	0·16035E−05	0·36988E−05	0·37989E−05
2·0000	0·15317E−05	0·31213E−05	0·27969E−05
4·0000	0·13190E−05	0·14927E−05	0·72729E−06
5·0000	0·12541E−05	0·25514E−05	−0·14102E−10
10·0000	0·10235E−05	−0·93618E−11	−0·47178E−06
20·0000	0·74146E−06	−0·90729E−06	−0·19615E−06
40·0000	0·43005E−06	−0·75724E−06	−0·69136E−07

Table 2—Numerical Values of the Growth or Decay Rate k_i for $j = 3$, $L = 2$, $\omega/\omega_c = 0{\cdot}75$, 0·7 and 0·65

$T_\perp/T_\parallel$	k_i cm^{-1}		
	$\omega/\omega_c = 0{\cdot}75$	$\omega/\omega_c = 0{\cdot}7$	$\omega/\omega_c = 0{\cdot}65$
3·0769	0·89617E−06	0·97243E−08	−0·14789E−06
3·3333	0·96956E−06	−0·22096E−10	−0·17990E−06
4·0000	−0·10884E−10	−0·25660E−06	−0·16531E−06
4·4444	−0·23885E−06	−0·26427E−06	−0·14622E−06
5·0000	−0·35274E−06	−0·24448E−06	−0·12501E−06

Table 3 — Numerical Values of the Growth or Decay Rate k_i and k_r for $L = 7$, $\omega/\omega_c = 0{\cdot}75$, $j = 1$ and 3

$j = 1$

$T_\perp/T_\parallel$	k_r cm^{-1}	k_i cm^{-1}
1·4286	0·40403E−04	0·13963E−07
2·0000	0·10016E−04	0.20757E−06
2·5000	0·12410E−04	0·11368E−06
3·0769	0·16711E−04	0·42894E−07
3·3333	0·19585E−04	0·23491E−07
4·0000	0·28209E−04	−0·26642E−12
4·4444	0·35101E−04	−0·56370E−08
5·0000	0·44719E−04	−0·82902E−08
10·0000	0·16717E−03	−0·50364E−08
20·0000	0·54197E−03	−0·18070E−08

$j = 3$

$T_\perp/T_\parallel$	k_r cm^{-1}	k_i cm^{-1}
1·4286	0·40403E−04	0·13963E−07
2·0000	0·10016E−04	0·20757E−06
2·5000	0·12410E−04	0·11368E−06
3·0769	0·16711E−04	0·42894E−07
3·3333	0·19585E−04	0·23491E−07
4·0000	0·28209E−04	−0·26642E−12
4·4444	0·35101E−04	−0·56370E−08
5·0000	0·44719E−04	−0·82902E−08
10·0000	0·16717E−03	−0·50364E−08
20·0000	0·54197E−03	−0·18070E−08

Table 4—Numerical Values of the Growth or Decay Rate k_i for $j = 0$, $\omega/\omega_c = 0{\cdot}7$, $L = 2$,3 and 7

$T_\perp/T_\parallel$	k_i cm^{-1}		
	$L=2$	$L=3$	$L=7$
3·0769	0·97243E−08	0·16679E−06	0·13197E−07
3·3333	−0·22096E−10	−0·69386E−11	−0·53226E−12
4·0000	−0·25660E−06	−0·76775E−07	−0·60457E−08
4·4444	−0·26427E−06	−0·78806E−07	−0·62049E−08
5·0000	−0·24448E−06	−0·72751E−07	−0·57276E−08
10·0000	−0·96175E−07	−0·28515E−07	−0·22446E−08

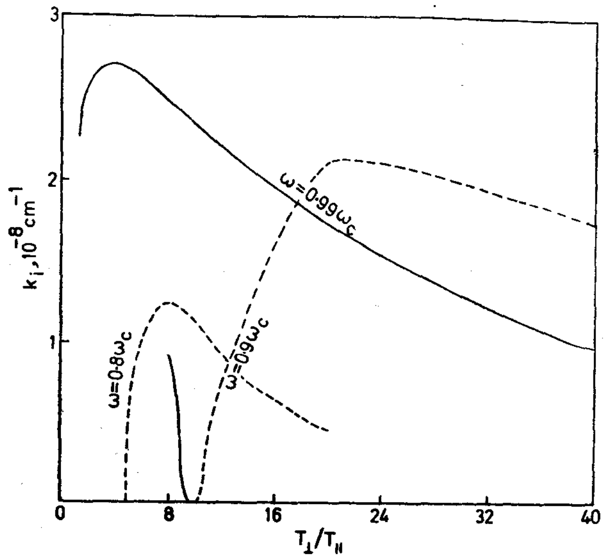

Fig. 1 — Plots of growth or damping rate versus $T_\perp / T_\parallel$ for $L = 7$, $\omega/\omega_c = 0{\cdot}99$, 0·9 and 0·8

decreases with increasing value of the anisotropy ratio. For $\omega = 0{\cdot}9\ \omega_c$, the wave shows damping upto $T_\perp / T_\parallel = 10$ and afterwards it starts growing. At $T_\perp / T_\parallel = 10$, the growth rate is $0{\cdot}22350 \times 10^{-12}$ (marked as zero in Fig. 1) and it increases for $T_\perp / T_\parallel$ upto 20 and thereafter it decreases for increasing $T_\perp / T_\parallel$. For $\omega = 0{\cdot}8\ \omega_c$, the growing part of the wave alone is shown. At $T_\perp / T_\parallel = 5$, the growth rate is extremely small ($0{\cdot}32781 \times 10^{-12}$) and it is maximum for $T_\perp / T_\parallel = 8$ and afterwards it decreases with increasing values of the parameter $T_\perp / T_\parallel$.

Thus it can be concluded that for a loss cone and anisotropic velocity distribution, an instability occurs when ($T_\perp / T_\parallel$) exceeds a certain threshold value. The computed values of the growth rate are in agreement with the one earlier given by Landau and Cuperman[8] and Cuperman and Salu[9] if the effect of loss cone index is neglected.

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