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Propagation of TM Modes in a Parallel Plane Waveguide Filled with Moving Collisional Plasma

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The reflection and transmission of TM modes in a parallel plane waveguide are investigated when half-space ($Z < 0$) between the planes is filled with homogeneous, collisional and moving plasma. It is observed that the reflection and transmission coefficients and transmission frequency are modified and depend on the velocity (V) of the plasma medium and collision frequency (ν) of the electrons. However, the frequency of the reflected wave is not affected by ν, but is a function of V only.

1. Introduction

The problems of reflection and transmission of electromagnetic waves by moving dielectric half-space and by waveguides filled with moving anisotropic plasma have received considerable attention of various authors.[1-8] Recently, the reflection and transmission of electromagnetic waves by moving half space plasma medium have been studied by Yeh,[9] without taking collisions of electrons with neutral particles into account.

In this paper, the effects of collisions of electrons with neutral particles on the reflection and transmission coefficients for TM modes have been reported. The field equations for TM modes are also derived without imposing any restriction on the velocity of the plasma medium.

2. Formulation of the Problem

A parallel plane waveguide of perfectly conducting walls infinite in the Y and Z directions and separated by a distance d in the X-direction is filled with homogeneous, warm, collisional and moving plasma in the space $Z < 0$. Two reference frames, the primed frame being fixed in the plasma medium and the unprimed frame attached to the waveguide, are considered.

The permittivity of the warm and collisional plasma medium in the primed frame is given by[10]

$$\epsilon'_p = \epsilon_0 \left[1 - \frac{\omega'^2_p}{\omega' (\omega' - i\nu')} \right] \qquad \ldots(1)$$

where ϵ_0, ω'_p, ω' and ν' are, respectively, the permittivity of free space, plasma frequency, angular frequency and collision frequency of the electrons with neutral particles in the primed system.

Following Jordan,[11] the field expressions (in CGS system) for TM waves between parallel, perfectly conducting planes in the unprimed system are:

$$B_y = B_0 \cos\left(\frac{m\pi}{d} x\right) \exp\left[i(KZ - \omega t)\right]$$

$$E_x = \frac{CK}{\omega} B_y$$

$$E_z = -\frac{iC}{\omega}\left(\frac{m\pi}{d}\right) \tan\left(\frac{m\pi}{d} x\right) B_y \qquad \ldots(2)$$

where C, ω, B_0, m and K are, respectively, the velocity of light, angular frequency of the wave, an arbitrary constant, an integer which corresponds to the order of mode and the wave vector in the negative direction of Z-axis.

The field equations for incident, reflected and transmitted TM modes in the primed frame, which is moving with a constant velocity V in the positive direction of Z-axis have the form:

Incident fields

$$B'_y = B'_0 \cos\left(\frac{m\pi}{d} x'\right) \exp\left[i(K'Z' - \omega' t')\right]$$

$$E'_x = \frac{CK'}{\omega'} B'_y$$

$$E'_z = -\frac{iC}{\omega'}\left(\frac{m\pi}{d}\right) \tan\left(\frac{m\pi}{d} x'\right) B'_y \qquad \ldots(3)$$

Reflected fields

$$B_{y'}^{(r)} = R' \cos\left(\frac{m\pi}{d}\, x'\right) \exp\left[-i(K'Z' + \omega' t')\right]$$

$$E_{x'}^{(r)} = -\frac{CK'}{\omega'}\, B_{y'}^{(r)}$$

$$E_{z'}^{(r)} = -\frac{iC}{\omega'}\left(\frac{m\pi}{d}\right) \tan\left(\frac{m\pi}{d}\, x'\right) B_{y'}^{(r)} \qquad \ldots(4)$$

Transmitted fields

$$B_{y'}^{(t)} = T' \cos\left(\frac{m\pi}{d}\, x'\right) \times \exp\left[-i(\omega' \sqrt{\epsilon'_p \mu_0}\, Z' + \omega' t')\right]$$

$$E_{x'}^{(t)} = -\frac{CK'}{\omega'} \sqrt{\epsilon'_p \mu_0}\, B_{y'}^{(t)}$$

$$E_{z'}^{(t)} = -\frac{iC}{\omega'}\left(\frac{m\pi}{d}\right) \tan\left(\frac{m\pi}{d}\, x'\right) B_{y'}^{(t)} \qquad \ldots(5)$$

Here R', B'_0 and T' are arbitrary constants, K' is the propagation constant and μ_0 is the permeability of the free space.

From Eqs. (3)-(5), matching the tangential electric and magnetic fields at the boundary $Z' = 0$ we have

$$B'_0 + R' = T'$$

$$B'_0 - R' = -\frac{\omega'}{K'} \sqrt{\epsilon'_p \mu_0}\; T'$$

From these equations we obtain

$$R' = \frac{B'_0 (K' + \omega' \sqrt{\epsilon'_p \mu_0})}{(K' - \omega' \sqrt{\epsilon'_p \mu_0})};\quad T' = \frac{2K' B'_0}{(K' - \omega' \sqrt{\epsilon'_p \mu_0})} \qquad \ldots(6)$$

If in the unprimed frame, the reflected and transmitted fields are :

Reflected

$$B_y^{(r)} = R \cos\left(\frac{m\pi}{d}\, x\right) \exp\left[-i(K^r Z + \omega^r t)\right]$$

$$E_x^{(r)} = -\frac{CK^r}{\omega^r}\, B_y^{(r)}$$

$$E_z^{(r)} = -\frac{iC}{\omega^r}\left(\frac{m\pi}{d}\right) \tan\left(\frac{m\pi}{d}\, x\right) B_y^{(r)} \qquad \ldots(7)$$

Transmitted

$$B_y^{(t)} = T \cos\left(\frac{m\pi}{d}\, x\right) \exp\left[\, i(K^t Z - \omega^t t)\right]$$

$$E_x^{(t)} = \frac{CK^t}{\omega^t}\, B_y^{(t)}$$

$$E_z^{(t)} = -\frac{iC}{\omega^t}\left(\frac{m\pi}{d}\right) \tan\left(\frac{m\pi}{d}\, x\right) B_y^{(t)} \qquad \ldots(8)$$

where R and T are the arbitrary constants, K^r, ω^r and K^t, ω^t are the propagation constants and angular frequencies of the reflected and transmitted waves respectively. Now to get the transformation of fields and other parameters from unprimed frame to primed and vice versa, we have made use of phase invariance,[12] the covariance of Maxwell's equations with respect to the Lorentz transformations[13] and satisfying the boundary conditions; the transformation equations for the case in which the plasma medium in waveguide is moving in the Z-direction with constant velocity V are given by : (for unprimed to primed frame)

$$B'_{\parallel} = \left(B - \frac{V \times E}{C}\right)_{\parallel};\; B'_{\perp} = \alpha\left[B - \frac{V \times E}{C}\right]_{\perp}$$

$$E'_{\parallel} = \left(E + \frac{V \times B}{C}\right)_{\parallel};\; E'_{\perp} = \alpha\left[E + \frac{V \times B}{C}\right]_{\perp}$$

$$K' = \alpha\left(K - \frac{\omega V}{C^2}\right);\; \omega' = \alpha(\omega - VK)$$

$$B'_0 = \alpha B_0\left(1 - \frac{VK}{\omega}\right);\; \nu' = \alpha\nu$$

$$\text{with} \quad \alpha = (1 - \beta^2)^{-1/2};\; \beta = \frac{V}{C} \qquad \ldots(9)$$

where the subscripts $\parallel$ and $\perp$ indicate, respectively, the directions parallel and perpendicular to the relative motion of the two systems. For primed to unprimed frame

$$B_{\parallel} = \left(B' + \frac{V \times E'}{C}\right)_{\parallel};\; B_{\perp} = \alpha\left[B' + \frac{V \times E'}{C}\right]_{\perp}$$

$$E_{\parallel} = \left(E' - \frac{V \times B'}{C}\right)_{\parallel};\; E_{\perp} = \alpha\left(E' - \frac{V \times B'}{C}\right)_{\perp}$$

$$K = -K_0;\qquad \omega^r = \alpha(\omega' - VK')$$

$$K^r = \alpha\left(K' - \frac{\omega' V}{C^2}\right);\; \omega^t = \alpha(\omega' - V\omega' \sqrt{\epsilon'_p \mu_0})$$

$$K^t = \alpha\left(\frac{V\omega'}{C^2} - \omega'\sqrt{\epsilon'_p\,\mu_0}\right);$$

$$R = \alpha R'\left(1 - \frac{VK'}{\omega'}\right)$$

$$T = \alpha T'\,(1 - V\sqrt{\epsilon'_p\,\mu_0}) \qquad ...(10)$$

From Eqs. (9) and (10) we obtain

$$\omega^r = \alpha^2\omega\,(1+\beta)^2\,;\; K^r = -\,\alpha^2 K_0\,(1+\beta)^2$$

$$\omega^t = \alpha^2\omega\,(1+\beta)\,(1-\beta Q)$$

$$K^t = \alpha^2 K_0\,(1+\beta)\,(\beta - Q)$$

$$R = \alpha^2 B_0\,(1+\beta)^2\,\frac{(1-Q)}{(1+Q)}$$

$$T = 2\alpha^2 B_0\,(1+\beta)\,\frac{(1-\beta Q)}{(1+Q)} \qquad ...(11)$$

where

$$Q = \left[\frac{\left\{\alpha^2\left[(1+\beta)^2 + (\nu/\omega)^2\right] - (\omega_p/\omega)^2\right\}(1+\beta) - (i\nu/\omega)(\omega_p/\omega)^2}{\alpha^2\,(1+\beta)\,[\,(1+\beta)^2 + (\nu/\omega)^2\,]}\right]^{1/2}$$

Putting the values of ω^r, K^r, ω^t, K^t, R and T from Eq. (11) into Eqs. (7) and (8), one can get the field equations for TM modes.

3. Reflection and Transmission Coefficients

The reflection coefficient (C_R) and transmission coefficient (C_T) are defined as

$$C_R = \frac{\hat{Z}.P_R}{\hat{Z}.P_I} \qquad ...(12)$$

and

$$C_T = \frac{\hat{Z}.P_T}{\hat{Z}.P_I} \qquad ...(13)$$

where $\hat{Z}$ is the unit vector in the Z-direction and

$$P_I = \frac{1}{2}\left[\,E \times B^*\,\right];\; P_R = \frac{1}{2}\left[\,E^r \times B^{*r}\,\right];$$

$$P_T = \frac{1}{2}\left[\,E^t \times B^{*t}\,\right] \qquad ...(14)$$

Making use of Eqs. (7) and (8), Eqs. (12) and (13) reduce to

$$C_R = -\left(\frac{R\,R^*}{B_0^2}\right)\left(\frac{\omega}{\omega^r}\right)\left(\frac{K^r}{K}\right) \qquad ...(15)$$

$$C_T = \left(\frac{T\,T^*}{B_0^2}\right)\left(\frac{\omega}{\omega^t}\right)\left(\frac{K^t}{K}\right) \qquad ...(16)$$

For collisionless case (i.e. $\nu = 0$) Eq. (15) is identical to the one obtained by Yeh.[9] In Eq. (16) ω^t is complex, and so Eq. (13) is not applicable for C_T, as it is only meaningful for harmonic, time-dependent fields (i.e. ω^t=real). Taking $Q = Q_r + iQ_i$, for collisional case ($\nu \neq 0$) Eq. (15) with the help of Eq. (11) reduces to

$$C_R = \frac{(1+\beta)^2}{(1-\beta)^2}\left[\frac{1 - 2Q_r + (A^2+B^2)^{1/2}}{1 + 2Q_r + (A^2+B^2)^{1/2}}\right] \qquad ...(17)$$

where

$$Q_r = \left[\frac{1}{2}\left\{(A^2 + B^2)^{1/2} + A\right\}\right]^{1/2}$$

$$A = \frac{(1+\beta)^2 + \left(\frac{\nu}{\omega_p}\right)^2\left(\frac{\omega_p}{\omega}\right)^2 - (1-\beta^2)\left(\frac{\omega_p}{\omega}\right)^2}{(1+\beta)^2 + \left(\frac{\nu}{\omega_p}\right)^2\left(\frac{\omega_p}{\omega}\right)^2}$$

$$B = -\left(\frac{\nu}{\omega_p}\right)\frac{(\omega_p/\omega)^3\,(1-\beta)}{(1+\beta)^2 + \left(\frac{\nu}{\omega_p}\right)^2\left(\frac{\omega_p}{\omega}\right)^2}$$

4. Results and Discussion

Eq. (17) is plotted in Fig. 1 for different velocities of plasma medium, under three different conditions, namely, (a) $\nu = 0$ (the collisionless case), (b) $(\nu/\omega_p) = 0{\cdot}1$ and (c) $\nu/\omega_p = 1$; for a value of $(\omega_p/\omega)^2 = 0{\cdot}5$. A similar plot for a value of $(\omega_p/\omega)^2 = 1{\cdot}5$ is shown in Fig. 2.

From Eq. (11) we see that the frequency of the reflected modes is only a function of velocity of the plasma medium, while for the transmitted modes

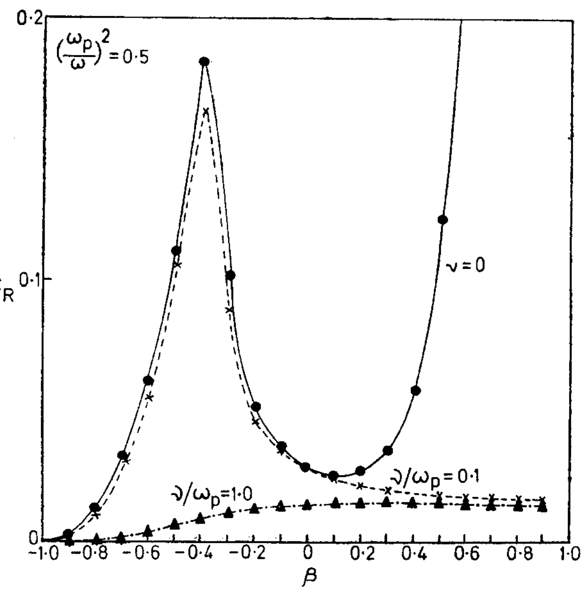

Fig. 1 — Variation of reflection coefficient (C_R) with β for ν/ω_p = 0, 0·1 and 1·0, when $(\omega_p/\omega)^2 = 0{\cdot}5$

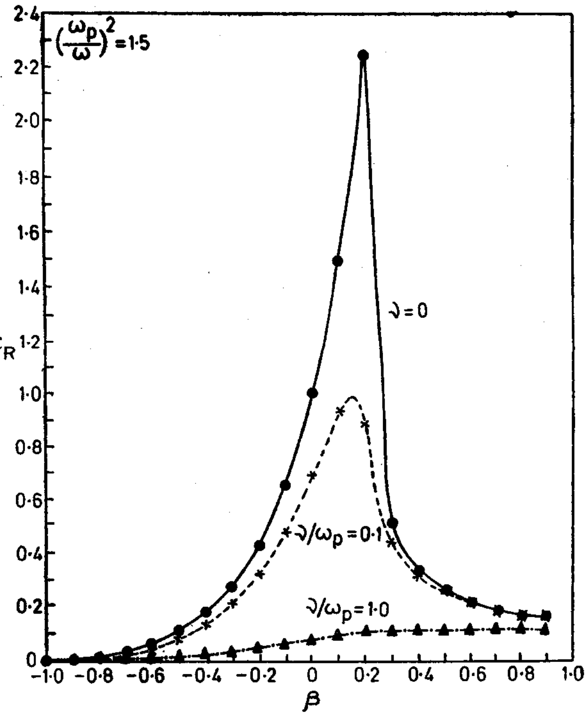

Fig. 2 — Variation of reflection coefficient (C_R) with β for $\nu/\omega_p = 0$, 0·1 and 1·0, when $(\omega_p/\omega)^2 = 1·5$

the frequency is a complicated function of ν, ω_p and V. The variations of C_R (reflection coefficient) with V and ν are shown in Figs. 1 and 2.

From Fig. 1 we observe that $\nu = 0$ and $\nu/\omega_p=0·1$. The value of C_R varies in a similar fashion in the range $-1·0 < \beta < 0·1$, but for $\beta > 0·1$ and $\nu = 0$ the value of C_R increases and becomes greater than 1·0 when $\beta > 0·65$, whereas for $\nu/\omega_p = 0·1$, there is a small decrease in the value of C_R. For the case when $\nu/\omega_p = 1·0$, the value of C_R first increases in the limit $-1·0 < \beta < 0·1$ and then it remains constant for $\beta > 0·1$.

Fig. 2 shows that the value of C_R is nearly the same for $\nu/\omega_p=0·1$ and $\nu=0$ in the limit $-1·0< \beta$ and $\beta < -0·7$, but in the range $-0·7 < \beta < 0·3$, C_R is less for $\nu/\omega_p = 0·1$. For $\nu/\omega_p = 1·0$, the variation of C_R is similar to the case when $(\omega_p/\omega)^2 = 0·5$. If $\beta = -1·0$, C_R reduces to zero for all the cases which is obvious.

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