Triangle Size Effects in Spaced Receiver Drift Experiment-Errors in Estimating Parameters of Ground Diffraction Pattern

H CHANDRA
Physical Research Laboratory, Ahmedabad 380 009

Received 14 March 1978

Role of errors in correlation of spaced receiver fadings in the computation of parameters of the ground diffraction pattern has been studied. The increase in the size of the diffraction pattern with the receiver separation could be explained. Apparent alignment of the patterns along the hypotenuse, when a small right-angled isosceles triangle is used, could also be accounted for. Appropriate correlation value must be the criterion for choosing antenna separation rather than geometry, and correlations higher than 0.8 at zero time lag must be avoided in this type of experiment.

1. Introduction

Drift and the parameters of the ground diffraction pattern obtained by full correlation analysis of spaced fading records have been studied using more than the conventional three-aerial configuration in recent years. The summary of these reports is: (i) drift direction is independent of triangle size or geometry; (ii) true drift speed \( V \) is low for small triangle size (the apparent drift speed, however, is independent of separation); (iii) semi-major axis and the axial ratio of the ground diffraction pattern increase with separation; and (iv) diffraction pattern is apparently aligned along the hypotenuse if a small right-angled triangle is used.

Large equilateral triangle size has been adopted by different workers. However, it is not understood so far as to why results of full correlation analysis should depend on separation or on antenna geometry. From theoretical investigations, Chandra demonstrated that small decrease in correlation values due to instrumental limitations could account for the observed triangle size effect in true drift velocity \( V \). Role of such errors in correlation values in the computations of the parameters of the ground diffraction pattern is studied in this paper. It is shown that this could account for the observed dependence of the size of the diffraction pattern on separation as well as the apparent alignment along hypotenuse, if observed using a small right-angled triangle.

2. Theory

For simplicity, only one-dimensional case is considered and the correlation functions are assumed to be Gaussian. Following Briggs, spaced time correlation function of a general form has been adopted which in case of a Gaussian model becomes:

\[
\rho (\xi_0, \tau) = \exp \left\{ \frac{-(\xi_0 - V\tau)^2 + V^2 \tau^2}{\xi_1^2} \right\} \quad (1)
\]

where

- \( \xi_1 \) The scale of ground pattern
- \( \xi_0 \) The receiver separation
- \( V \) The velocity of the pattern
- \( V_0 \) A measure of the random changes in the pattern
- \( \tau \) Time lag

The auto- and cross-correlation functions are, therefore,

\[
\rho (0, \tau) = \exp \left\{ \frac{-(V^2 + V_0^2) \tau^2}{\xi_1^2} \right\} \quad (2)
\]

and

\[
\rho (\xi_0, 0) = \exp \left\{ \frac{-(\xi_0 - V\tau)^2 + V_0^2 \tau^2}{\xi_1^2} \right\} \quad (3)
\]

The cross-correlation function at zero time lag would be given by

\[
\rho (\xi_0, 0) = \exp \left\{ \frac{-(\xi_0^2 + \xi_1^2)}{\xi_1^2} \right\} \quad (4)
\]

which is independent of the velocities and depends on the separation and scale size only. Following Phillips and Spencer, the time lag \( \tau \) for equivalent autocorrelation used in the calculations of parameters of the pattern can be found by solving Eqs. (2) and (4). Thus

\[
\tau = \frac{\xi_0}{\sqrt{V^2 + V_0^2}} \quad (5)
\]

which shows that for a drifting pattern this time lag is a function of separation only. The scale size of the pattern is generally determined from the following relations.
scale size $= \left(\frac{\xi_0}{\tau}\right) \tau_{0.5} = (V^2 + V_s^2)^{1/2} \tau_{0.5}$ ...(6)

which is independent of separation. Here $\tau_{0.5}$ represents the time lag for half autocorrelation. Thus on the basis of correlation analysis the scale size of the pattern must be same irrespective of the separation used in the experiment. Generalizing the results to two-dimensional case, one can say that, for a given drifting pattern, scale size in different directions would be obtained from the values of $\tau$ in these directions and that will be independent of the separation or geometry. Thus a parameter like axial ratio or the orientation is also independent of geometry or separation of the receivers.

3. Effect of Instrumental Limitations

As discussed in the earlier paper for too low a separation correlation values would be very high and due to instrumental limitations like in the digitization (both electronical and manual) or differences in different receiver channels, one will underestimate the correlation function. This will result in the overestimation of time lag ($\tau$). Hence underestimation of the scale size would result which is exactly the result reported in such investigations. Since under such conditions smaller separation would be affected more and hence if the geometry is right-angled triangle, it will be the two sides where underestimation will be felt more than the hypotenuse and an apparent alignment of the pattern along hypotenuse will occur. It must be noted that Beynon and Wright had used rather small separation ($\lambda/2$) in their measurements than what is usually the practice. Kumar and Rai had used 90 m separation at Udaipur and reported that only a few cases had shown such apparent alignment along the hypotenuse. With adequate separation of about 140 m at Adelaide, Golley and Rossiter did not find such triangle size effects for D-region records while same separation gave triangle size effects for E-region records.

To illustrate these results, a sample case has been worked out here. Auto- and cross-correlation functions have been calculated for a Gaussian model where $\xi_0 = 100$ m, $V$ and $V_s$ each equal to 50 m/sec and $\xi_0$ varied, viz. 25, 50, 75 and 100 m. These are shown in Fig. 1 (a). It is clear that the time lag $\tau$ increases linearly with separation. The scale size for half correlation has been plotted for each case which remains constant with a value of 84 m [Fig. 1 (b)]. To introduce errors cross-correlation values have been decreased by 2% and 5% and recalculated values of scale size plotted again [Fig. 1 (b)]. The observed triangle size effect is clearly seen by introducing small errors of the order of 5%. Thus at low separation when cross-correlation exceeds 0.8 at zero time lag, a 25% decrease in scale size occurs in this particular case. From these calculations a typical isotropic pattern is derived as shown in Fig. 1 (c). Considering a small right-angled triangle the scale size along three directions is plotted and the points lie along a circle for a case when there is no error in correlation functions. For 5% error in correlation functions the two sides will underestimate scale size by about 25% and the points now lie along an ellipse with orientation along the hypotenuse. To further support these arguments it must be noted that the correlation values exceeding 0.9 were shown by Kumar and Rai in the example of correlograms at Udaipur in their report. Small equilateral triangle will make equal errors in all directions and would not give apparent alignment along the hypotenuse but would definitely underestimate the size of ground pattern.

4. Effects near Magnetic Equator

Near magnetic equator diffraction patterns are highly elongated. The correlation is extremely high along north-south and smaller along east-west. Errors will occur therefore along north-south direction only.
This will result in the underestimation of the major axis and axial ratio. Chandra et al.\textsuperscript{5} found that correlation along north-south is systematically decreasing with separation, the value of the major axis remaining independent of separation. However, for cases when correlations along north-south are high and remain same at different separations, value of major axis increases with separation. Underestimation of the correlation along north-south seems to be the reason why the average axial ratio of 3 during 1964 at Thumba\textsuperscript{10} was too low and increased to about 6 in 1967 with same separation (120 m). To support the argument, histograms of the maximum north-south correlation at Thumba for a separation of 120 m are compared for the years 1964 and 1967 (Fig. 2). The correlation during 1967 is mostly varying from 0·90 to 1·00 while it varies between 0·80 and 0·95 during 1964. The increase\textsuperscript{11} in 1968 to above 10 was, however, associated with the increased separation along north-south (480 m) adopted later. Hence the size of major axis as well as the axial ratio observed at Thumba represents only a lower limit due to inadequate north-south separation. North-south line must be increased further near magnetic equator for future experiments. Rastogi et al.\textsuperscript{12} reported north-south correlation exceeding 0·9 even at a separation of 480 m.

5. Conclusion

The apparent decrease of the major axis or the axial ratio of the ground patterns near equator for small triangle size is shown to be caused by the errors in correlation function due to instrumental limitations. Similarly the apparent alignment of the diffraction pattern observed at other latitudes with small right-angled triangle is due to such errors. Appropriate correlation rather than the geometry or separation is the important factor to choose. In selecting the separation, correlation greater than 0·8 (at zero time lag) must be avoided in performing correlation analysis.

References