On the Triangle Size Effect in Spaced Receiver Drift Experiments

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It is shown that small decreases in correlation values due to instrumental defects could account for the fact that the drift velocity determined by full correlation analysis of spaced receiver records increases when the separation of the receivers is increased.

1. Introduction
The amplitude fading of radio waves reflected from the ionosphere and recorded at three spaced receivers on the ground is widely used to measure the drift velocity of irregularities in the D-, E- and F-regions. The “true” drift velocity \( V \) as obtained by the full correlation method has been found by several authors to be low if the triangle formed by the receivers is small in size, and to increase as the receiver separation is increased. From an investigation of this “triangle size effect” using 89 aerials at Buckland Park, South Australia, Golley and Rossiter concluded that it could be due to instrumental limitation in the recording and digitizing processes. In this paper an attempt is made to investigate the effect of such errors theoretically. It is shown that they could explain the observed results.

2. Theory
For simplicity only a one-dimensional case is considered and the correlation functions are assumed to be Gaussian. Following Briggs the space-time correlation function will be of the general form

\[
\rho(\xi, \tau) = f\left( (\xi - V \tau)^2 + V_e^2 \tau^2 \right) \tag{1}
\]

which, for a Gaussian model, becomes

\[
\rho(\xi, \tau) = \exp\left[ -\frac{(\xi - V \tau)^2 - V_e^2 \tau^2}{\xi_1^2} \right] \tag{2}
\]

where \( \xi_1 \) is the scale of the ground pattern, \( V \) is the velocity with which the pattern moves and \( V_e \) is a parameter which is a measure of the random changes of the pattern as it moves.

The auto- and cross-correlation functions are therefore

\[
\rho(0, \tau) = \exp\left[ -\frac{(V^2 + V_e^2) \tau^2}{\xi_1^2} \right] \tag{3}
\]

and

\[
\rho(\xi_0, \tau) = \exp\left[ -\frac{(\xi_0 - V \tau)^2 - V_e^2 \tau^2}{\xi_1^2} \right] \tag{4}
\]

where \( \xi_0 \) is the receiver separation.

From Eq. (3) it can easily be shown that the shift for maximum cross-correlation is given by

\[
\tau = \frac{(\xi_0 V)/(V^2 + V_e^2)} \tag{5}
\]

and so the “apparent” velocity \( V' \) is

\[
V' = \frac{(V^2 + V_e^2)}{V} \tag{6}
\]

Also, the time shift \( \tau_e \) for which the auto- and cross-correlation functions are equal is given by

\[
\tau_e = \frac{\xi_0}{V} \tag{7}
\]

Thus, if \( \tau_e \) is known, the “true” velocity of full correlation analysis can be found from the equation

\[
V = \frac{\xi_0}{(2\tau_e)} \tag{8}
\]

Generally, the cross-correlation functions at zero time-lag are used to calculate true velocity. Briggs \textit{et al.} defined fading velocity \( V_e' \) as the ratio of space shift to time shift needed for equal change in the correlation function. Thus

\[
V_e' = \frac{\xi_0}{\tau_1} \tag{9}
\]

where \( \tau_1 \) is the time shift for equivalent auto-correlation.

\[
\rho(0, \tau_1) = \rho(\xi_0, 0) \tag{10}
\]

The value of \( V \) can be then obtained from the relation

\[
V = \frac{(V_e')^2}{V'} \tag{11}
\]
Fooks has suggested another alternative to calculate $\tau$ using the maximum value of cross-correlation function. If $\tau_m$ is the time-lag for equivalent autocorrelation then the following relation can be used to calculate $\tau$.

$$\tau^2 = (\tau)'^2 + \tau_m^2 \quad \ldots \quad (12)$$

If the assumptions of full-correlation analysis hold, and if there are no instrumental errors in determining the correlation functions, neither the apparent velocity $V'$ nor the true velocity $V$ should depend upon the receiver separation.

3. Effect of Instrumental Errors

Instrumental effects such as differences in the linearity of receivers, noise, and digitizing errors will always decrease the measured correlation coefficients. We are dealing here only with the differences between different receiver channels which will affect only the cross-correlation values. In order to investigate the possible influence of such errors, auto- and cross-correlation functions were first calculated from Eqs. (3) and (4) for particular values of the parameters $V$, $V_c$, $\xi_0$ and $\xi$. The cross-correlation values were then all decreased by some amount and the "true" velocity was recalculated from the new value of $\tau$, using Eq. (8). This was done for different values of $\xi_0$ the receiver separation, and the calculated velocity was plotted as a function of $\xi_0$.

Fig. 1 shows an auto-correlation curve and a set of cross-correlation curves for various values of $\xi_0$, calculated for the case $\xi = 100 \text{ m}$, $V = V_c = 50 \text{ m/sec}^{-1}$. As the separation is reduced, the maximum value of cross-correlation increases, and approaches unity for $\xi_0 = 25 \text{ m}$. The cross-over point $\tau_e$ of the auto- and cross-correlation functions varies linearly with receiver separation $\xi_0$, confirming that $V$ is independent of separation if the assumptions of full correlation analysis are valid, and no errors are present.

To simulate instrumental errors, the cross-correlation curves of Fig. 1 were moved downwards by amounts from one per cent to five per cent in steps of one per cent each, and recalculations of $V$ were made for each case. The change in $\tau$ (and hence in $V$) is largest when the auto-correlation curve is nonlinear, i.e., near the peak of the auto-correlation curve which is widely used for correlation analysis.

The variation of $V$ with $\xi$ is shown in Fig. 2 for three values of $V_c$: $V_c = 50 \text{ m/sec}^{-1}$, $V_c = 25 \text{ m/sec}^{-1}$ and $V_c = 0$. The numbers on each curve indicate the percentage amount by which the cross-correlation values were reduced. The increase of $V$ with $\xi$ reproduces the "triangle size effect" very well. The curves are not much affected by the value of $V_c$. In all cases, the velocity at $\xi_0 = 25 \text{ m}$ is reduced to about half its correct value when the cross-correlation values are decreased by five per cent. The "apparent" velocity is, of course, unchanged, because the position of the maximum of the cross-correlation function is unchanged. This agrees with the observations.

The upper curves of Fig. 2 show the original computed values of the correlation value $\rho (\xi_0, \tau_e)$ at the "cross-over" point ($\tau = \tau_e$) for different separations $\xi_0$. It will be noted that these values are smaller when $V_c$ is large. Since the variation of $V$ with $\xi$ is approximately the same for all values of $V_c$, this implies that higher correlation values can be used, for the same error in $V$, when $V_c/V$ is low than when $V_c/V$ is high. However, it is important to note that the "triangle size effect" still exists, even if $V_c = 0$.

Calculations of $V$ were also made using the zero time-lag method and the peak-correlation method to investigate which of the methods is most suitable.
The variation of $V$ with $\xi_0$ is shown in Fig. 3 for the three methods of calculations, i.e. the zero time-lag method (A), the peak-correlation method (B) and from $\tau_\sigma$ (C). Calculations were made for $V = V_c = 50$ m/sec$^{-1}$. At smaller separations all the methods give identical results; however, the zero time-lag method gives comparatively more error in $V$ at large separations.

The upper curves in Fig. 3 show the original computed values of the correlation function $\rho (\xi_0, 0)$ at zero time-lag (A), $\rho (\xi_0, \tau')$ at peak correlation (B), and $\rho (\xi_0, \tau_\sigma)$ at the cross-over point (C) at different receiver separations.

The examples shown here indicate that for about 15% reduction in $V$, the value $\rho (\xi_0, \tau_\sigma)$ at cross-over point or $\rho (\xi_0, \tau')$ at peak correlation should not exceed 0.80 while the value $\rho (\xi_0, 0)$ at zero time-lag should not exceed 0.60 when $V_c/V = 1$. In all cases a receiver separation equal to the scale of the pattern ($\xi_0 = \xi_i$) will be large enough to obtain acceptable values of velocity even for a correlation reduction of five per cent.

4 Discussion

Golley and Rossiter$^3$ found that different receivers recording from the same aerial did not give a computed correlation of unity. The observed correlation for different receivers varied from 0.98 to 0.86. They applied correlations of 2% and 4% to raise the values of the correlation functions, and found that the "triangle size effect" decreased.

The results of the present paper confirm the arguments of Golley and Rossiter$^3$ that small errors in the evaluation of correlation functions could lead to a decrease in velocity if the receiver separation is too small compared to the pattern scale. The errors could be due to differences between the receiver channels, limitations to the digitizing process, or of manual origin when manually digitizing records from films or paper charts. Caution must be taken to have proper receiver separation so that the correlation values are not too high. A limit of 0.8 for the maximum value of cross-correlation and of 0.60 for zero time-lag is suggested.

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