Diagnostic circulation model for the sensitivity of eddy viscosity coefficients in the western tropical Indian Ocean

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The sensitivity of a 18-level diagnostic model of circulation to two different values of horizontal eddy viscosity coefficients has been investigated. Vertical eddy viscosity coefficient has been kept constant in both the experiments. Model driven with January mean wind and thermohaline fields attained steady state within 15 days of model integration. Computed results at 10, 100 and 500 m depths for both the experiments are presented in this paper. Circulation features at 10 m depth in terms of both the magnitude and direction are identical for both the numerical experiments although the horizontal eddy viscosity coefficient was increased two-fold in the second experiment. However, the magnitude of circulation at 100 and 500 m depths was found to decrease marginally in those regions where strong currents were observed. Dynamics of circulation at the above mentioned depths is explained in terms of the forcing parameters and other dynamical characteristics of the model. It has been found that the surface circulation is mainly controlled by wind field and the subsurface circulation patterns by thermohaline forcing.

In large scale ocean circulation models, exchange coefficients such as horizontal and vertical eddy viscosity coefficients that represent the process of exchange of momentum in the horizontal and vertical directions respectively are either parameterised in terms of known measurable variables or supplied with appropriate constant values. There are no universally acceptable parameterisation formulae for the above mentioned coefficients. Values of these eddy coefficients vary widely in space and time in the ocean. So far, no systematic measurements of relevant physical variables in the Indian Ocean have been carried out to estimate the eddy coefficients. One approach is to use constant values of coefficients to be obtained through a series of sensitivity tests. By changing the values of coefficients in a phased manner in numerical models, the most appropriate coefficient that would compute the essential characteristics of circulation in that area can be obtained. Such sensitivity tests in relation to different coefficients are one of the essential components in the development of mathematical models for geophysical fluid system. In the present study, the sensitivity of an 18-level diagnostic model of circulation to two different values of horizontal eddy viscosity coefficients has been investigated. The vertical eddy viscosity coefficient is kept constant in both the numerical experiments.

Materials and Methods

Diagnostic model equations

The present model utilizes the basic hydrodynamical equations governing the large scale 3-dimensional flows in spherical co-ordinate system given by Gill. The model has 18 levels in the vertical with a maximum depth of 900 m. These 18 levels are standard oceanographic depths for data collection. The sign convention used in the model is different from the standard sign convention used in oceanography. A co-ordinate system wherein the longitude(λ), the colatitude(θ) and the vertical co-ordinate(z) increase
towards east, south and downward directions respectively are used. Assuming approximations of Boussinesq, hydrostatics and incompressibility, the final model equations can be written as follows:

\[
\frac{\partial u}{\partial t} + A u \cot \theta \frac{u^2}{R} = -\frac{1}{\rho_0 R \sin \theta} \frac{\partial p}{\partial \lambda} + F^a
\]

... (1)

\[
\frac{\partial v}{\partial t} + A v \cot \theta \frac{u^2}{R} = -\frac{1}{\rho_0 R \sin \theta} \frac{\partial p}{\partial \theta} + F^b
\]

... (2)

\[
\frac{\partial p}{\partial z} = g \rho
\]

... (3)

\[
\frac{1}{R \sin \theta} \left[ \frac{\partial u}{\partial \lambda} + \frac{\partial (v \sin \theta)}{\partial \theta} \right] + \frac{\partial w}{\partial z} = 0
\]

... (4)

where

- \( \theta \) - co-latitude = 90-\( \theta \), \( \theta \) is the latitude.
- \( u, v, w \) - the velocity components along the east(\( \lambda \)), south(\( \theta \)) and downward(\( z \)) directions respectively [LT\(^{-1}\)].
- \( p \) - hydrostatic pressure [MLT\(^{-1}\)].
- \( \rho \) - density of seawater [ML\(^{-3}\)].
- \( \rho_o \) - constant density [ML\(^{-3}\)].
- \( R \) - radius of the Earth [L].
- \( l \) - 2\( \Omega \) cos\( \theta \)[T\(^{-1}\)].
- \( \Omega \) - angular velocity of Earth's rotation [T\(^{-1}\)].
- \( A u, Av \) - advective acceleration terms [LT\(^{-2}\)].
- \( F^a, F^b \) - friction terms in the momentum equations [LT\(^{-2}\)].
- \( \mu, \nu \) - horizontal and vertical turbulent mixing coefficients respectively [LT\(^{-2}\)].

\[
A \rho = (A u, Av) = \frac{1}{R \sin \theta} \left[ \frac{\partial (u \cos \theta)}{\partial \lambda} + v \frac{\partial (v \sin \theta)}{\partial \theta} \right] + \frac{\partial w}{\partial z}
\]

\[
F^a = \frac{\partial (v \cos \theta)}{\partial z} + \mu \left( A u - \frac{1}{R^2 \sin^2 \theta} \frac{\partial (v \cos \theta)}{\partial \lambda} \right)
\]

The density of seawater '\( \rho \)' can be written as the sum of the average density '\( \rho_o \)' and the density anomaly '\( \rho(\lambda, \theta, z) \)'.

Integrating the hydrostatic Eq (3) in the vertical from free surface to depth, the expression for hydrostatic pressure can be written as:

\[
p = \rho_o g \zeta + g \int_0^z \rho' \, dz
\]

... (5)

where '\( \zeta \)' is the sea surface topography which is otherwise called sea level. The unknowns in the model equations are the velocity components (\( u, v, w \)), pressure (\( p \)) and sea level (\( \zeta \)). There are five unknowns in four equations. Density of seawater which is imposed on the model from observations of temperature and salinity is not a prognostic variable in diagnostic circulation models.

It is, therefore, necessary to derive an expression for sea surface topography (sea level) to have a closed system of five equations in five unknowns. A brief description of the procedure to derive sea surface topography equation and the final formula for the same are given below for completeness.

To derive sea surface topography equation, it is necessary to find out the solutions for \( u \) and \( v \). The equations (1-4), taking into consideration the equation for hydrostatic pressure (Eq. 5), are solved with the help of a leap-frog numerical scheme. Following conditions are used for the solution of equations:

At the sea surface (\( z=0 \)), a rigid lid boundary condition is employed for the vertical velocity field. In addition, the components of wind stress in the zonal and meridional directions are expressed in terms of the gradient of velocity in the vertical multiplied by the vertical turbulent mixing coefficient and mean density of seawater.
i.e.,
\[ \rho_o \frac{\partial \mathbf{u}}{\partial z} = -\tau_x; \quad \rho_o \frac{\partial \mathbf{v}}{\partial z} = -\tau_y; \quad \mathbf{w}|_{z=z_0} = 0 \]
\[ \text{(6)} \]

where \( \tau_x \) and \( \tau_y \) are the components of wind stress in the zonal and meridional directions respectively and \( u, v \) are the components of velocity fields. Since temperature and salinity equations are not included in the model equations, there is no need of giving surface boundary conditions for temperature and salinity.

At the bottom \( z=H(\lambda, \theta) \), non-slip boundary condition for velocity fields was applied.
\[ \text{(7)} \]
i.e. \( u = v = w = 0 \)

At the rigid lateral boundaries, the normal flux boundary conditions of zero velocity is prescribed.
\[ \text{(8)} \]
i.e. \( \mathbf{v}_n = 0 \)

where 'n' is the normal to the boundary.

At the open sea boundaries, the velocity components computed with the help of a quasi-geostrophic model were prescribed.

The solutions are obtained on a 46x51 staggered grid. After having found the solution for \( u \) and \( v \) at all levels, the integrated velocity components \( \mathbf{U} \) and \( \mathbf{V} \) are obtained by summing up the velocity components for all the levels. By applying the boundary conditions for vertical velocity at surface and bottom \( (w=0) \) and substituting the integral velocity components in the continuity equation, we obtain the two-dimensional integral continuity equation which is the equation for sea surface topography at the internal grid points. By applying the lateral boundary condition for velocity fields and substituting it in the two-dimesional integral continuity equation, the equation for sea surface topography at the boundary points are also obtained.

The two-dimensional integral continuity equation for sea surface topography at the internal grid points is written as follows:

\[ \frac{1}{R \sin \theta} \left[ \delta_x \mathbf{U}^{\theta} + \delta_\theta \mathbf{V} \sin \theta \right] = O \]
\[ \text{(9)} \]

where \( \delta_x \) and \( \delta_\theta \) are the finite difference operators. Substituting the values of \( \mathbf{U} \) and \( \mathbf{V} \) in the integral continuity equation, the final expression for sea surface topography is written as follows:

\[ \frac{1}{R \sin \theta} \left[ \delta_x \left( a \delta_x \mathbf{U}^{\theta} + b \delta_\theta \mathbf{U}^{\theta} \right) \right] = R_s \]
\[ \text{(10)} \]

where
\[ a = \frac{\Delta \tan H}{1 + \left( \frac{\Delta \tan H}{4} \right)} \]

\[ b = a \frac{\Delta t}{2} \]

\[ R_s = \frac{1}{R \sin \theta} \left[ \delta_x \mathbf{R}^{\theta} + \delta_\theta (R \sin \theta) \right] \]

\[ R_u = R_u - \left( \frac{\Delta t}{2} \mathbf{R} \right) \]
\[ R_v = R_v + \left( \frac{\Delta t}{2} \mathbf{R} \right) \]
\[ R_n = v^{n-1} + \Delta t \left[ \frac{1}{2} u^{n-1} \left( \frac{g}{\rho_0} \int_{-\infty}^{z} \frac{\partial \rho}{\partial \theta} \, dz \right) \right. \\
+ \left. \left( \tan \theta \right)^{n-1} \right] \\
- A v \cdot \cot \theta \cdot \frac{\partial \theta}{\partial z} \right) + \left( R^\theta \right)^{n-1} \]

where \( \Delta t \) is the time step and \( n \) is the time step number.

The sea surface topography (Eq.10) is solved by successive overrelaxation technique.

Results and Discussion

Two numerical experiments were conducted to study the sensitivity of the model solutions to two different values of horizontal eddy viscosity coefficients. The vertical friction coefficient is kept constant at 10 cm² sec⁻¹ in both the experiments. A comparative study of the results of experiment nos. 1 and 2 will give information on the effect of horizontal eddy viscosity coefficients on model solution. In the first numerical experiment, the horizontal eddy viscosity coefficient was kept at 5x10⁷ cm² sec⁻¹, while in the second experiment, the value of the horizontal eddy viscosity coefficient was increased to 10⁸ cm² sec⁻¹. The sensitivity experiments were performed for January, a representative month for the winter season.

The main forcing parameters used in the model are the monthly mean wind stress at the sea surface and thermohaline field at different levels. We have used the monthly mean wind data compiled by Helleman et al.⁷, the monthly mean temperature and seasonal mean salinity data compiled by Levitus⁸ to drive the model. The spatial distribution of mean resultant wind stress for the month of January is presented in Fig. 1. During the month of January, northeasterly and easterly winds with magnitudes up to 1 dyne cm⁻² are observed throughout Arabian Sea north of the equator. In the doldrum region near the equator, the wind field is generally weak except the regions off Somali and African coasts. Southeasterly winds with magnitudes greater than 1 dyne cm⁻² are observed in the southern hemisphere between 10⁰S and 20⁰S latitude. These southeasterly winds turn towards north and east on approaching the equatorial regions between 0⁰ and 10⁰S latitude.

The diagnostic model Eqs. (1-4), considering Eq.(5) were solved subject to the boundary and initial conditions mentioned above using a leap-frog numerical scheme. A time step of 1 hour was used for the integration of the model equations. To inhibit time-splitting instability normally encountered in leap-frog scheme, a Matsuno scheme which averages solution at two consecutive time steps was employed at every 24 time steps. The time evolution of velocity components at two selected points in the model is presented in Fig. 2. As the density is imposed on the model from observations, the spin up time in diagnostic models of ocean circulation is very short as compared to prognostic models where the density is computed on the basis of model dynamics and surface boundary conditions. It could be seen from Fig. 2 that solution reaches stationarity within 15 days of model integration and thereafter it remains almost constant even after 25 days of integration.

**Computed circulation at 10 m depth**

The computed circulation patterns at 10 m depth for the first and second numerical experiments are presented in Fig. 3a and b respectively. In terms of magnitude and direction of currents, the computed circulation features are identical for both the numerical.
experiments, although the horizontal eddy viscosity coefficient was increased twofold in the second numerical experiment. The flow fields in the entire Arabian Sea north of the equator are southwestward in response to northeasterly winds and thermohaline forcings. The current speeds along the equator, Somali and African coasts are high as compared to other regions and are of the order of 100 cm sec⁻¹. A fairly strong counter current with magnitude of 50 cm sec⁻¹ flows towards east between the equator and 10⁰S latitude. The magnitude of current between Madagascar and African coast is very weak because of weak winds prevailing over the region. In general, the current field in the entire model area at 10 m depth follows more or less the wind pattern which again confirms the existing knowledge that upper layer circulation is mainly controlled by the wind field, although thermohaline forcing modifies the circulation pattern. Results show that thermohaline forcing does strengthen or weaken or deflects the flow fields depending on the direction of density gradient. Southwestward flow at 10 m depth encountered in the entire Arabian Sea north of the equator is generated due to the combined effect of wind stress and density gradient. Northern Arabian Sea is characterized by low temperature and high saline waters as compared to
southern part resulting in the sloping down of density surfaces from north to south. This slope, which is oriented in a northeast-southwest direction, generates a geostrophic flow in the southwestward direction. Observed southwestward flow in the Arabian Sea is, therefore, due to the combined effect of both wind stress and density gradient forcing. Because of the influence of thermohaline forcing, the current vectors in the northern hemisphere are deflected to the left of the wind field instead of deflecting towards right. In the trade wind region of the southern hemisphere, the density surface slopes down from south to north, generating a westward northwestward flow along this region. Here also, both wind stress and density forcings reinforce each other, thereby strengthening the current.

The computed sea surface topography for the first and second numerical experiments are shown in Fig. 4a

![Fig. 4](image)

**Fig. 4**—Computed sea surface topography (cm) for the month of January during the first (a) and second (b) numerical experiments

![Fig. 5](image)

**Fig. 5**—Computed January mean current at 100 m depth during the first (a) and second (b) numerical experiments
and b respectively. It has been found that the sea level values are unchanged with the doubling of the horizontal eddy viscosity coefficient. Maximum sea level gradient is observed between 7.5°S and 20°S latitude where the meridional gradient is of the order of 35 cm. Sea level slopes down from 20°S to 7.5°S latitude. Sea level is found to be negative in the entire western Arabian Sea and equatorial Indian Ocean. Positive sea surface topography is found off the west coast of India and eastern Arabian Sea.

Computed circulation at 100 m depth

Computed circulation at 100 m depth for the first and second numerical experiments are shown in Fig. 5a and b respectively. The characteristics of circulation for both the experiments are almost identical in all the regions except off Somali and African coasts where strong currents are observed. Magnitude of currents along the Somali and African coasts are marginally decreased when the horizontal eddy viscosity coefficient was increased by two-fold. This observation is in accordance with the theory of friction; the greater the frictional effects the less will be the velocity fields. Frictional effects are seen more in strong current regions. In general, the current speeds at 100 m depth are very less compared to that at 10 m depth. The magnitude of current in the model area ranges between 10 to 15 cm sec⁻¹ except off equatorial regions where its magnitude ranges between 25 and 40 cm sec⁻¹. Impact of wind stress is not at all visible at 100 m depth which shows that the circulation at this depth is mainly controlled by thermohaline forcings.

Dynamics of circulation at 100 m depth can be explained in terms of the forcing parameters and other dynamical characteristics of the model. Temperature distribution at 100 m depth is presented in Fig. 6. In

Fig.6—Temperature distribution at 100 m depth

Fig.7—Computed January mean current at 500 m depth during the first (a) and second (b) numerical experiments
central Arabian Sea, temperature field is more or less uniform at 22°C. However, it decreases towards the western part near the Red Sea, Arabian coast and Persian Gulf. Temperature field decreases towards the northern extremity of Arabian Sea also. High dense waters from the Persian Gulf and Red Sea move towards the Arabian Sea and therefore the flows are offshore all along the coastlines. These features are well reflected in the model studies. High dense waters from the northern extremity of Arabian Sea move towards west and join with the dense waters from the Persian Gulf. Flow is westward in the northern Arabian Sea, north of 17°N latitude. Because of the intrusions of high density Red Sea waters into the Arabian Sea, the flow is eastward between 12.5°N and 17°N latitude. Another important characteristic of circulation in the western Arabian Sea is the strong northwestward and northward coastal undercurrent along the coast of Somalia between the equator and 7°N latitude which is mainly caused by the sinking of surface waters near the coast, generating a downward density slope towards the coast. Surface waters that sink off the Somali coast travel towards east up to 65°E as a strong equatorial undercurrent between the latitudes 3°N and 2.5°S. Westward current between the equator and 5°N is attributed to the combined effect of sea surface slope and density gradient that prevail over the region. Sea level is fairly very high off the southwest coast of India as compared to the western part of Arabian Sea (Fig. 4a). Westward flow between 20°S and 12.5°S latitude is mainly caused by the slope of the density surfaces which is downward from east to west (Fig. 6). Circulation pattern in the whole model area is very much consistent with the dynamics considered in the diagnostic model.

Computed circulation at 500 m depth

Computed circulation at 500 m depth for the first and second numerical experiments are presented in Fig. 7a and b respectively. When the horizontal eddy viscosity coefficient was increased by two-fold, the intensity of currents near the Somali and equatorial regions was reduced by 2 to 3 cm sec⁻¹. Direction of currents has not changed even when the value of horizontal eddy viscosity coefficient was increased. There are significant changes in the circulation pattern at this depth as compared to 100 m. Changes in the circulation pattern are mainly caused by the thermohaline forcing. Temperature distribution at 500 m depth is presented in Fig. 8. Isotherms at this depth are more or less zonally oriented with a gradual increase in temperature field on either side of the equator. Flow field in the entire northern Arabian Sea north of 10°N latitude is northward and northeastward because the density surface slope down from the south to north. Magnitude of circulation is still reduced as compared to 100 m depth with its magnitude ranges from 5 to 25 cm sec⁻¹. The northwestward Somali undercurrent found in the 100 m depth is observed at 500 m depth also. A westward flow with magnitude of 10 to 15 cm sec⁻¹ is found near the equator between 2°N and 7°N latitude. The eastward flow in equatorial undercurrent is observed between 2°N and 2°S latitude. The flow field is very weak between 10°S and 2.5°S because of low thermohaline gradient. The temperature field alone is not sufficient to infer the circulation pattern at this depth. Density calculation shows that there is a zonal gradient in the density field from east to west generating a zonal current towards west as shown in Fig. 7a and b respectively.

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References