Image Denoising Using Dual-tree Complex Wavelet Transform and Wiener Filter with Modified Thresholding

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This paper presents a new image denoising algorithm based on local variance estimation. In the process of denoising, the Wiener filter is used to remove the noise component of the dual-tree complex wavelet transform (DT-CWT) coefficients. The variances of noise-free coefficients are estimated by the DT-CWT coefficients transformed by modified thresholding. The tests show that the proposed method has better performance compared to the related algorithms.

Keywords: Image denoising, Dual-tree complex wavelet transform (DT-CWT), Wiener filter, Thresholding

Introduction

Image denoising is a key problem in the field of digital image processing. The developed transform domain methods\textsuperscript{1-6} have demonstrated the good denoising performance. Especially, the wavelet domain Wiener filter (WDWF) method proposed by Mihcak et al.\textsuperscript{3} is popular widely. Several modifications\textsuperscript{4-5} have been done on this method. For example, in\textsuperscript{4}, WDWF is improved by preprocessing images with a thresholding operation. However, wavelets fail to effectively capture the information such as edges, textures and etc. So, Xu et al.\textsuperscript{6} proposed a Wiener filter method in the shearlet domain (Shear-Wiener). But, the Shear-Wiener is not an ideal method on Wiener filter since the statistical characteristic of shearlet coefficients is not analyzed. Therefore, based on the modified thresholding operation and the dual-tree complex wavelet transform (DT-CWT)\textsuperscript{7}, this paper proposed a new Wiener filter method with excellent performance for image denoising.

Experimental

DT-CWT

The DT-CWT proposed by Kingsbury has been widely used in image processing\textsuperscript{7}. The DT-CWT is a combination of two separable discrete wavelet transforms (DWTs). It is computed using two critically-sampled DWTs in parallel to produce the real and imaginary parts of complex wavelet coefficients. In each decomposition level, the DT-CWT produces one low-pass sub-band and six high-pass sub-bands corresponding to the different directions. These directions are ±15°, ±45° and ±75°, respectively. Compared to DWT, the DT-CWT has shift-invariant property and improves directional resolution in two and higher dimensions. In addition, the computation is very efficient since the DT-CWT has limited redundancy and the filters used are separable compared to other redundant wavelet transform such as contourlets, shearlets and etc.

On the other hand, in this paper, the input image is corrupted with signal-independent additive white Gaussian noise of zero mean and variance $\sigma^2$. Since the DT-CWT is a linear transform, the DT-CWT coefficients of noisy image can be written as

$$y_{ij} = s_{ij} + n_{ij} \quad \ldots \quad (1)$$

where $y_{ij}$ is the noisy wavelet coefficient, $s_{ij}$ is the true coefficient, and $n_{ij}$ is the noise component.

Statistical Property of DT-CWT Coefficients of Noise

Mahbubur Rahman et al. have studied the statistical property of the DT-CWT carefully\textsuperscript{8}. They found that when a Gaussian distributed signal is decomposed by a two-dimensional DT-CWT, the real and imaginary components of the complex coefficients in the first level of decomposition become independent zero-mean Gaussian distributions with different variances. These variances of the real and imaginary components are $(1-p)\sigma_q^2$ and $(1+p)\sigma_q^2$, respectively. Here, the value of $p$ is a constant depending on the set of filter coefficients. And the $\sigma_q^2$...
depends on the transformation matrices of the DT-CWT. In the first level of decomposition, the DT-CWT uses the wavelet filters forming a Hilbert-pair approximately. Hence, the value of $p$ is nonzero for the first level decomposition.

This means that original Gaussian distributed signal is transformed to two independent zero-mean Gaussian distributed components with different variances in the first level of decomposition. The corresponding value of $p$ is close to zero since the second and higher-level decompositions of the DT-CWT use wavelet filters forming a Hilbert-pair almostly. For simplicity, assume that $p$ is zero in all the decompositions of the DT-CWT. In addition, in this paper, the Q-shift DC-DWT$^9$ is adopted in which all the filters beyond the first level are the perfect-reconstruction filter sets that can be orthonormal. So, in the second and higher-level decompositions, the $\sigma^2_q$ is equal to $\sigma^2_q$. In the first level, the $\sigma^2_q$ can be computed in term of $\sigma^2$. At this time, the $\sigma^2_q$ is also very close to $\sigma^2$. So in all decompositions, both $(1-p)\sigma^2_q$ and $(1+p)\sigma^2_q$ are $\sigma^2$ approximately. Therefore, it is assumed that the real and imaginary components of the complex coefficients of noise are still independent zero-mean Gaussian distribution with variances $\sigma^2$ in the proposed method.

Statistical Property of DT-CWT Coefficients of Noise-free Image

In the proposed method, the DT-CWT coefficients of original "clean" image are assumed as zero-mean Gaussian random variables. To motivate this model, some statistical simulations on the DT-CWT coefficients are done. Figs. 1 and 2 show that the histogram of those DT-CWT coefficients normalized by their estimated standard deviations from the real and imaginary components of the complex coefficients in $+15^\circ$, $+45^\circ$, $+75^\circ$, $-15^\circ$, $-45^\circ$, $-75^\circ$ in the first and second levels of decomposition of Lena (512×512) image, respectively. It is demonstrated that the normalized DT-CWT coefficients are fitted by Gaussian probability density function (p.d.f) with zero mean and unit variance.

Wiener Filter

From the statistical property of the DT-CWT coefficients of noise and "clean" image, it is believed that it will be feasible in practice and be sound in theory that Wiener filter operates in the DT-CWT domain. Unlike$^4$, the modified thresholding is used before the signal variance is computed.

The traditional WDWF$^3$ can be expressed as

$$ S_{ij} = \frac{E\{s_{ij}^2\}}{E\{s_{ij}^2\} + \sigma^2} y_{ij} $$

where the $S_{ij}$ denotes the denoised coefficient. The $E\{s_{ij}^2\}$ represents the expected value of $s_{ij}^2$ and is computed as $E\{s_{ij}^2\} = \max( q_{ij} - \sigma^2, 0 )$. The $q_{ij}$ is obtained by averaging the squared values of $y_{ij}$ in a window centered at $(i,j)$. Usually, the square window of size $R \times R$ is used.

Thresholding Operation

To improve the accuracy of the estimate of the expected value of $s_{ij}^2$, Kazubek proposed a Wiener filter with a thresholding operation$^4$. In this method, the

Fig. 1— Blue bar: histogram of the real (row 1) and imaginary (row 2) components of the DT-CWT coefficients scaled by estimated local standard deviations. Red line: unit-variance, zero-mean Gaussian p. d. f. Zoom into file for a better view. (a)–(f) shows the DT-CWT coefficients of the directions in $+15^\circ$, $+45^\circ$, $+75^\circ$, $-15^\circ$, $-45^\circ$, $-75^\circ$ in the first level of decomposition in turn for "clean" Lena (512×512) image.
local expected square error (LESE) is defined as
\[
\text{LESE} = E\left\{\left[ s_{i,j} - a_{i,j} y_{i,j} \right]^2 \right\}
\]
where
\[
a_{i,j} = E\{s_{i,j}^2\}/\left(E\{s_{i,j}^2\} + \sigma^2\right).
\]
Kazubek assumed that if \(\text{LESE} > E\{s_{i,j}^2\}\) then the linear estimate is bad so that the better solution should be presented. After a series of manipulations, Kazubek drew a conclusion that the coefficients with \(q < k\sigma^2\) \((k = 1 + [(2/R^2)]^{0.5})\) should be set to zero before computing \(E\{s_{i,j}^2\}\).

**Modified Thresholding**

In fact, the denoising result is ideal when LESE is zero. Therefore, \(\text{LESE} > E\{s_{i,j}^2\}\) enlarges the error. So, \(\text{LESE} > E\{s_{i,j}^2\}\) is modified as \(\text{LESE} > \tilde{r}E\{s_{i,j}^2\}\) \((0 < \tilde{r} < 1)\) where \(\tilde{r}\) is a constant which can be tuned in practice. So the proposed threshold \(k\) is equal to \((\tilde{r}^2 + 1)/(2\tilde{r}) + \{(2/R^2) + [(\tilde{r}^2 + 1)/(2\tilde{r})]^2 - 1\}^{0.5}\). In the test, the proper \(\tilde{r}\) is taken by hand. It is 1 - 0.008\sigma.

**Experimental Setup and Results**

The proposed method mainly consists of the following steps: 1) Perform a symmetric boundary extension on the noisy image. 2) Perform a DT-CWT on the extended noisy image. 3) In each high pass sub-band, computing \(q_{i,j}\) on \(y_{i,j}\) and the proposed \(k\), respectively. If \(q_{i,j} > k\sigma^2\) then \(\Sigma_{i,j} = y_{i,j}\) else \(\Sigma_{i,j} = 0\). And then computing \(E\{s_{i,j}^2\}\) on \(\Sigma_{i,j}\). At last, obtain the estimates \(S_{i,j}\) by using (2). 4) The denoised image is finally reconstructed by the inverse DT-CWT. The denoising performance is quantitatively evaluated by peak signal-to-noise ratio (PSNR) which is defined as
\[
\text{PSNR} = 10\log_{10}\left\{\frac{255^2MT}{\sum\sum (E_{m,t} - F_{m,t})^2} \right\} \quad \text{... (3)}
\]
where \(MT\) is the image size, \(E\) and \(F\) denote the original image and denoised image, and \(\sum\sum (E_{m,t} - F_{m,t})^2\) represents the sum of each \((E_{m,t} - F_{m,t})^2\) \((1 \leq m \leq M, 1 \leq t \leq T)\), respectively. Firstly, the two related techniques are used to compare. They are Locally adaptive window-based denoising using maximum likelihood (LAWML)\(^3\) and Wiener filter with threshold (Th-WF)\(^4\), respectively. For the proposed method, the DT-CWT with six decomposition levels is used. The size of the local window depends on the decomposition level. At the first decomposition level, local window of size 7×7 is used. The size of local window is 3×3 in the rest of decompositions. For LAWML, the orthogonal wavelet transform with five levels of decomposition and Daubechies 'symlet' with eight vanishing moments (Symmlet 8) is used. The sizes of the square window are 7×7, 5×5, 5×5, 3×3 and 3×3 in turn from the finest scale to the coarsest one. For Th-WF, the parameters are set according to the values given by its author in the corresponding refereed papers. In all the algorithms, it is assumed that the noise variance is known. Otherwise, it may be estimated from the finest resolution sub-band by the robust median estimator\(^1\).

Table 1 presents the PSNR results of several methods. It is observed that the proposed method dramatically outperforms LAWML and Th-WF in PSNR. The Th-WF only partly improves LAWML but the
The proposed method obtains the overall improvements on LAWML. Fig. 3 presents the visual comparisons of several methods. It is found that there is a lot of noise in the results with LAWML (Fig. 3c) and Th-WF (Fig. 3d). The proposed method achieves the best visual effect since the proposed method produces the least artifacts when removing noise well (Fig. 3e). To further verify the performance of the proposed method, the denoising algorithm via Wiener filter in the shearlet domain (Shear-Wiener) is used to compare. For the reliability of comparison, only the reported PSNR results are demonstrated since there is a lack of source code. From Table 2, it is seen that the proposed method significantly outperforms the Shear-Wiener in PSNR. Especially at $\sigma = 20$, the proposed method obtains 1.42 dB improvement compared to Shear-Wiener for Barbara image.

**Conclusion**

A modified threshold based Wiener filter is implemented with the DT-CWT for removing noise. It is based on Gaussian distribution modeling of sub-band coefficients. The test shows that the proposed method significantly and overall improves the performance of local Wiener filter.

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