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Temperature Anisotropy Instabilities of Obliquely Propagating Electromagnetic Modes in the Magnetosphere

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The nature of convective instability has been investigated for oblique propagation of the ordinary and extraordinary modes in the magnetosphere arising from temperature anisotropy. The dispersion equation is solved numerically using a TDC 316 computer for different values of the anisotropy ratio $T_\perp/T_\parallel(=\delta)$ of the perpendicular and parallel temperatures, the McIlwain parameter L and the lef-hand and right-hand cut-off frequencies in the ranges ω_1 to $4\omega_1$ and ω_2 to $4\omega_2$ and propagation angle θ. The dependence of instabilities of both the ordinary and the extraordinary electromagnetic modes on the above parameters is investigated. Computations are also made for perpendicular propagation of the wave.

1 Introduction

The characteristics of a magnetospheric instability are assessed by examining both the local plasma distribution function and the local plasma turbulence spectrum. Satellite observations on the particle structure in the magnetosphere has revealed that the magnetospheric plasma consists of several components having different temperatures. Temperature anisotropy instability in a plasma containing cold and hot species in the magnetosphere have been investigated by Renuka and Viswanathan[1]. The extraordinary mode electromagnetic instability associated with the plasma waves propagating perpendicular to the magnetic field is studied for the class of the plasma distributions of the Dory-Guest-Harris type by Lai[2].

2 Dispersion Relation

We shall assume that the electrons obey a distribution of the form given by Kito and Kaji[3]:

$$F_0(v_\parallel, v_\perp)=\frac{N_f}{\pi^{3/2}uw^2}\exp(-v_\parallel^2/u^2-v_\perp^2/w^2) \quad \dots (1)$$

where u and w are the thermal velocities and N_f is the normalization factor. This is an anisotropic velocity distribution function for the particles. Throughout this work the distribution function for a given species is normalized such that

$$\iiint_{\infty}^{\infty} F\,d^3v=1 \quad \dots (2)$$

The dielectric permittivity tensor of a plasma in a magnetic field is given by Akhiezer *et al.*[4] To study convective instability, let us write

$$k=k_r+ik_i,\ |k_i|\ll k_r \quad \dots (3)$$

For the distribution of the type shown in Eq. (1), the components of the dielectric permittivity tensor of the plasma in a magnetic field are worked out by Renuka and Viswanathan[5].

In the case of perpendicular propagation of the wave, the components of the dielectric permittivity tensor are given by

$$\varepsilon_{11}=1-\alpha^S(S_1+\mu S_2) \quad \dots (4)$$

$$\varepsilon_{22}=1-\alpha_a(\mu^2h_1+\mu h_2+S_1\omega) \quad \dots (5)$$

$$\varepsilon_{33}=1-\omega_p^2\omega^{-2} \quad \dots (6)$$

$$\varepsilon_{12}=\alpha_b(\mu^2h_3+\mu h_4+0.5\,S_1) \quad \dots (7)$$

$$\varepsilon_{13}=\varepsilon_{23}=0 \quad \dots (8)$$

where

$$\alpha_a=\alpha^S\omega \quad \dots (9)$$

$$\alpha_b=2i\alpha^S\omega_c/\omega \quad \dots (10)$$

$$h_1=3/2-S_1\omega/16+0.5\,S_2\omega+S_3\omega/8 \quad \dots (11)$$

$$h_2=1/2\,\omega+0.5\,S_2\omega \quad \dots (12)$$

$$h_3=3\,S_1/6-0.5\,S_2+9\,S_3/16 \quad \dots (13)$$

$$h_4=-0.5\,S_1+S_2 \quad \dots (14)$$

$$\alpha^S=\omega_p^2\exp(-\mu) \quad \dots (15)$$

$$S_1=1/(\omega^2-\omega_c^2) \quad \dots (16)$$

$$S_2=1/(\omega^2-4\,\omega_c^2) \quad \dots (17)$$

$$S_3=1/(\omega^2-9\,\omega_c^2) \quad \dots (18)$$

and

$$\mu=k^2\sin^2\theta\,w^2/2\,\omega_c^2 \quad \dots (19)$$

If n denotes the refractive index, the dispersion relation now becomes

$$An^4+Bn^2+C=0 \quad \dots (20)$$

where

$$A = \varepsilon_{11} \quad \dots (21)$$

$$B = -\varepsilon_{11}\varepsilon_{33} - \varepsilon_{11}\varepsilon_{22} - \varepsilon_{12}^2 \quad \dots (22)$$

and

$$C = \varepsilon_{33}(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{12}^2) \quad \dots (23)$$

where ε_{ij}s are given by Eqs. (4)-(19).

It is evident from Eqs. (4)-(19) that the components of the dielectric permittivity tensor are dependent on the parallel and perpendicular temperatures.

3 Discussion of the Analytical and Numerical Solutions

A number of satellite observations have been reported of electromagnetic emissions in the frequency range of $10\text{-}10^3$ Hz. These emissions are composed of waves propagating perpendicularly to the ambient magnetic field and confined within a few degrees of the magnetic equator in the earth's plasmasphere between 2 and 5 earth radii. The temperature anisotropy given by the bi-Maxwellian distribution of the plasma causes the electrostatic electron-cyclotron harmonic wave instability which was initially predicted by Harris[6] for the extreme temperature anisotropy $T_\perp/T_\parallel = \infty$. Normalized frequency bands of the VLF emissions observed by OGO 5 are calculated by Kennel *et al.*[7] for the conditions, $T_\perp/T_\parallel = 5$ and $T_\perp/T_\parallel = 10$. Gurnett and Shaw[8] reported that an electromagnetic noise band is frequently observed in the outer magnetosphere by the IMP 6 spacecraft at frequencies from about 5 to 20 kHz and this noise bands often contain a harmonic frequency structure which suggests that the radiation is associated with harmonics of the electron cyclotron frequency. A review of plasma wave observations made in the earth's magnetosphere was given by Shawhan[9].

The magnetic field at any point was represented by the dipole field given by

$$B = (B_0/L^3 \cos^6 \lambda)\,(1 + 3\sin^2 \lambda)^{1/2} \quad \dots (24)$$

where L is the McIlwain parameter, $B_0 \approx 0.31$ Gs is the equatorial magnitude of the magnetic field, λ is the geomagnetic latitude and is taken as zero in our numerical calculations.

The left-hand and right-hand cut-off frequencies are defined, respectively, as

$$\omega_1 = 0.5\,\omega_c\,[(1 + 4\,\omega_p^2/\omega_c^2)^{1/2} - 1] \quad \dots (25)$$

and

$$\omega_2 = 0.5\,\omega_c\,[(1 + 4\,\omega_p^2/\omega_c^2)^{1/2} + 1] \quad \dots (26)$$

The frequency ratio ω/ω_1 and ω/ω_2 are denoted by r_1 and r_2, respectively. The magnitudes of the growth rate (k_i) of ordinary (O) and extraordinary (EX) modes are plotted as a function of the anisotropy ratio $\delta (= T_\perp/T_\parallel)$, frequency ratio r_1 and r_2 and propagation angle θ in Figs 1-4.

Fig. 1 shows the effect of temperature anisotropy ratio δ on growth rate k_i at $L = 3$ for waves propagating at an arbitrary propagation angle of 60°. The curve shown in dotted and solid lines corresponds to wave with frequency ratio r_2 and r_1, respectively. For both frequencies the growth rate of ordinary mode is smaller compared to that of extraordinary mode. For extraordinary mode, the maximum growth is observed for waves with frequency ratio $r_1 = 1.8$ and anisotropy ratio $\delta = 5$. At this frequency the wave growth decreases as the anisotropy ratio increases and reaches a minimum and then starts growing. For the extraordinary mode with $r_1 = 2.8$, the maximum growth occurs at $\delta = 4$ and after that the wave growth decreases with increasing δ. At $r_2 = 2.8$, the extraordinary mode steadily increases as the anisotropy ratio increases from 2 to 5 and at $r_2 = 1.8$, the ordinary mode shows a minimum growth which occurs at $\delta = 4$ and thereafter the wave grows upto $\delta = 5$. Hence it is clear from Fig. 1 that for obliquely propagating waves also there exists a critical range of anisotropy parameter at which the growth of the wave is maximum or minimum. This result is in agreement with the investigations made by Renuka and Viswanathan[10] on the nature of convective instability for a VLF wave propagating along a magnetic line of force for an anisotropic and loss cone velocity distribution for the electrons.

In Fig. 2, the curve shown in solid lines corresponds to the plot of growth rate k_i versus r_2 for $\delta = 4$, $\theta = 60°$ and $L = 2$ and 3 and the corresponding wave number k_r is shown in broken lines. From the dotted curves shown it is clear that decreasing δ broadens the ranges of wave numbers for which instability develops. Hence our result verifies the results of Gaffey *et al.*[11] For $L = 3$, $\theta = 60°$, $\delta = 4$ and $r_2 = 1.8$, k_i is $0.80408\,E-06$ and k_r is $0.11063\,E-03$ which is in agreement with the assumption that $|k_i| \ll k_r$. This value of k_r corresponds to a wavelength of 0.5680 km and is in agreement with the observed magnetospheric signals[9]. It is also clear from the Fig. 2 that at distances close to the earth ($L = 2$) in the equatorial plane, both the ordinary and extraordinary modes show a large growth.

The angular dependence of growth rate can be visualized from Fig. 3. In Fig. 3, the curves for the ordinary mode at $L = 2$, $\delta = 2$ and $r_2 = 1.8$, 2.2, 2.5 and 2.8 are given. At $\theta = 90°$, the maximum growth is observed for waves propagating with frequency greater than $1.8\,\omega_2$. For waves propagating with frequency $1.8\,\omega_2$, the maximum growth occurs at $\theta = 80°$. The analysis of these results shows that at low propagation angles, the instability is low.

The curve shown in Fig. 4 is obtained by solving Eq. (20) numerically. It shows the variation of the

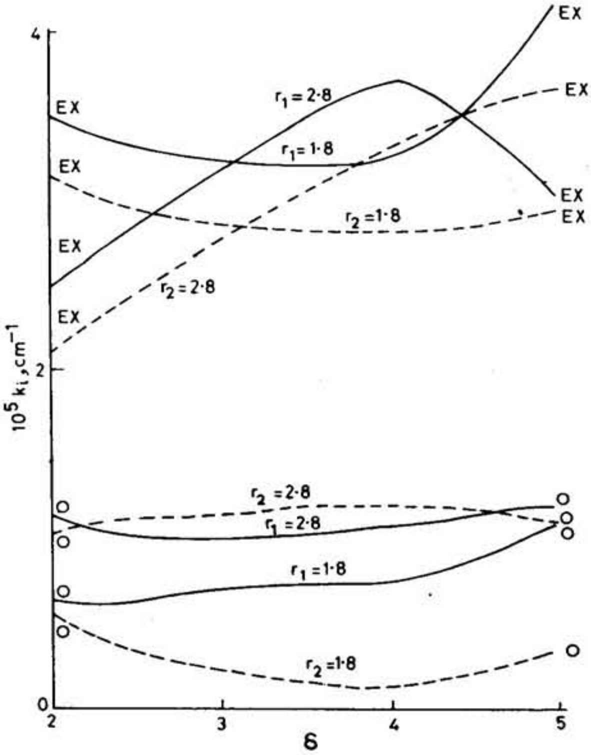

Fig. 1—Variation of growth rate k_i with the anisotropy ratio δ for ordinary (O) and extraordinary (EX) mode for the parameters $L=3$, $\theta=60$, r_1, $r_2=1.8$ and 2.8

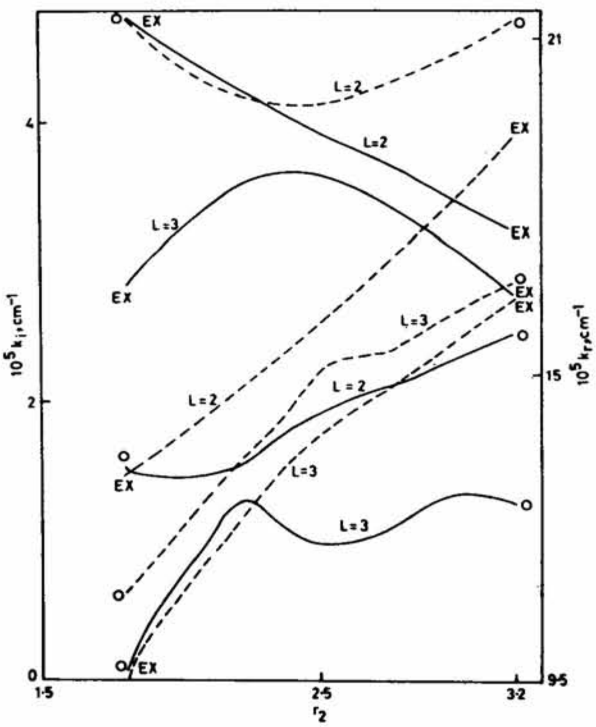

Fig. 2—Variation of growth rate k_i (solid line) and wave number k_r (broken line) with the frequency ratio r_2 for $\delta=4$, $\theta=60°$, $L=2$ and 3

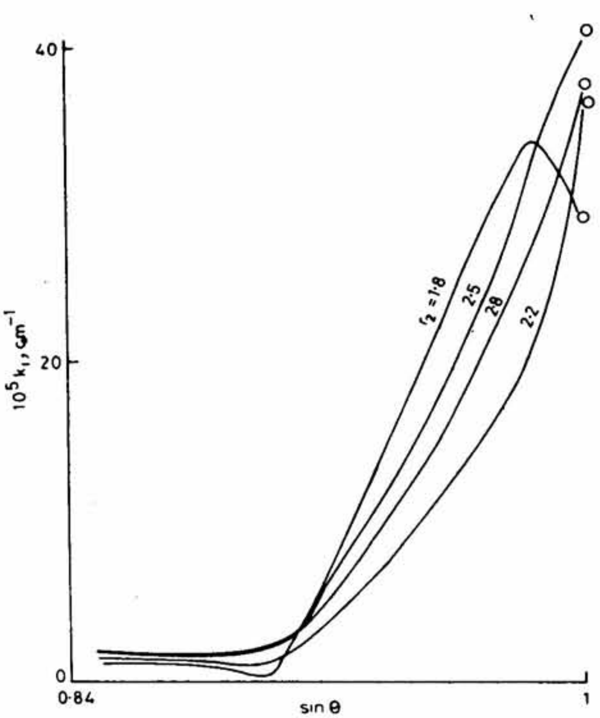

Fig. 3 Variation of growth rate k_i with $\sin\theta$ for $L=2$, $\delta=2$, $r_2=1.8$, 2.2, 2.5 and 2.8

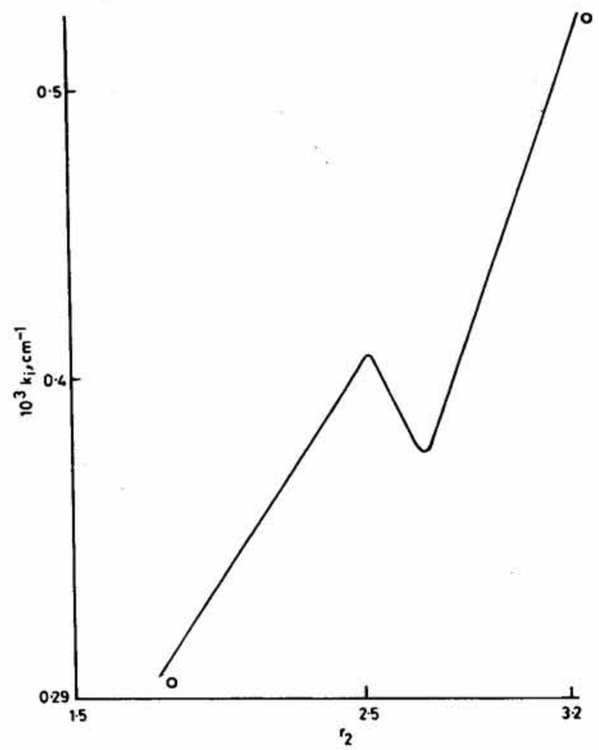

Fig. 4 –Variation of growth rate k_i with r_2 for propagation angle $\theta=90°$, $L=2$ and $\delta=2$

growth rate of the ordinary mode with frequency ratio r_2 at $\delta = 2$, $L = 2$ and for perpendicular propagation.

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