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Calculations of Electric Fields from Lightning above Finitely Conducting Ground

DIVYA & J RAI

Department of Physics, University of Roorkee,
Roorkee 247 667

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Expressions for the horizontal and the vertical electric field components due to an arbitrarily oriented lightning channel above a finitely conducting ground have been obtained. The theory has been applied to the vertical return stroke and the horizontal lightning. It has been found that the shapes of the vertical and horizontal electric fields versus frequency curves remain the same at all heights above the ground. The frequency spectrum of electric fields from horizontal lightning is, however, different from that of a return stroke. The ratio of horizontal to vertical electric field above ground is less than 1.0 for a return stroke, while it is greater than 1.0 for horizontal lightning for all ground conductivities (except $\sigma = \infty$).

1 Introduction

Although there exists an extensive amount of literature on lightning electromagnetic fields, most of the information is collected from ground-based observations. Since the lightning discharges take place at a height of about 5 km, the return stroke fields, that exist above ground and that are encountered by aircraft in flight, are of considerable interest, specially in the context of aircraft safety. In recent past, however, electric field measurements have been made from airplanes[1-4]. Pitts and Thomas[4] and Baum[3] studied the high frequency radiations emitted from lightning discharges. Hughes[1] measured the polarization of radiations from thunderstorms at a number of frequencies in the range 10-250 kHz from an airplane. He found that the horizontal field amplitudes were several times greater than those of the vertical field. Stanford[5], applying corrections for ground conductivity to the observations of Kohl[6], also showed that the horizontal components of atmospherics were several times larger than the vertical components.

Master *et al.*[7] gave the first detailed calculations of electric and magnetic fields (in time domain) due to lightning subsequent return strokes above a perfectly conducting ground. They calculated the fields using the model of Lin *et al.*[8] and also used a modified model in which the current waveform was a function of channel height. However, they argue that the fields in air are not much influenced by the height dependence of current beyond a distance of about 10 km. Hence, the validity of the modified model used by them, remains limited up to the distances of less than 10 km. Further, the finite ground conductivity is known to affect the electric and magnetic fields[9-11] and, therefore, its effect should also be taken into account in the calculations of electric and magnetic fields.

The purpose of the present paper is to calculate the parallel and perpendicular components of electric field above an imperfectly conducting ground due to an arbitrarily oriented lightning channel. The theory has been applied to different forms of lightning and the calculations have been made in the frequency domain. Since our calculations have been performed at distances greater than 10 km, a simple current model (Bruce and Golde model)[12] has been taken into account ignoring lightning tortuosity. The results have been compared with available, though limited, experimental observations.

2 Theory

The lightning channel is assumed to be straight and oriented in space arbitrarily from the vertical (Fig. 1). The radius of cross-section has been assumed to be very small compared to its length. The length of the channel increases with time and a time dependent current flows through it.

The vector potential, in general, at the observation point is given by:

$$\mathbf{A} = (\mu/4\pi)[\int I(t')\,\mathrm{d}z'/r] \quad \ldots (1)$$

where $t' = t - r/c$ is the retarded time, $\mathrm{d}\mathbf{z}'$ the elemental length of the channel, $I(t')$ the current flowing through the channel, and r the distance from the lightning channel to the point of observation. The functional dependence of both I and $\mathrm{d}z'$ on time can be combined into $\mathrm{F}(t')$ as:

$$\mathrm{A}_0\,\mathrm{F}(t') = \int I(t')\,\mathrm{d}z' \quad \ldots (2)$$

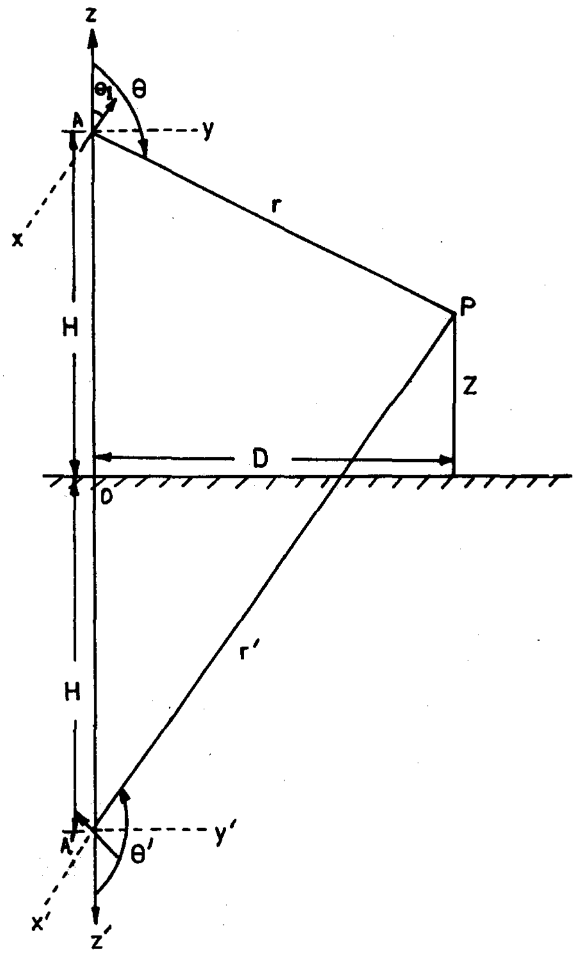

Fig. 1—Geometry of an arbitrarily oriented lightning channel (P is the point of observation in space)

where A_0 is a time independent factor having the dimension of electric dipole moment and $F(t')$ of s^{-1}.

Rai[9] showed that the vector potential **A** forms its image in the finitely conducting ground with the image vector potential given by:

$$|\mathbf{A}_m| = R_c|\mathbf{A}| \quad \ldots (3)$$

where R_c is the reflection coefficient.

For vertically polarized waves, one has[13]

$$R_c = R_v = [\{(\varepsilon+\sigma/j\omega)/\varepsilon_0\}\cos\psi - \{(\varepsilon+\sigma/j\omega)/\varepsilon_0 - \sin^2\psi\}^{1/2}]/ [\{(\varepsilon+\sigma/j\omega)/\varepsilon_0\}\cos\psi + \{(\varepsilon+\sigma/j\omega)/\varepsilon_0 - \sin^2\psi\}^{1/2}] \quad \ldots (4)$$

and for horizontally polarized waves,

$$R_c = R_h = [\cos\psi - \{(\varepsilon+\sigma/j\omega)/\varepsilon_0 - \sin^2\psi\}^{1/2}]/ \cos\psi + \{(\varepsilon+\sigma/j\omega)/\varepsilon_0 - \sin^2\psi\}^{1/2}] \quad \ldots (5)$$

where $\psi = \pi - \theta$, ε = permittivity of the medium, ε_0 = permittivity of the free space, σ = ground conductivity, and ω = angular frequency of waves.

We consider the spherical coordinate system in which the lightning dipole of finite length is situated at $(0,\theta_1,\varphi_1)$. Applying appropriate Maxwell equations, the vertical and horizontal electric field components are calculated from the vector potential **A** and $\mathbf{A}_m$, at a point of observation P (Fig. 1).

$$E_\perp(t') = (A_0/4\pi\varepsilon_0)[B_1(\theta,\varphi)\int F(t')\mathrm{d}t + B_2(\theta,\varphi)F(t') + B_3(\theta,\varphi)\partial F(t')/\partial t] \quad \ldots (6)$$

$$E_\parallel(t') = (A_0/4\pi\varepsilon_0)[\{B_4(\theta,\varphi)\int F(t')\mathrm{d}t + B_5(\theta,\varphi)F(t') + B_6(\theta,\varphi)\partial F(t')/\partial t\}^2 + \{B_7(\theta,\varphi)\int F(t')\mathrm{d}t + B_8(\theta,\varphi)F(t') + B_9(\theta,\varphi)\partial F(t')/\partial t\}^2]^{1/2} \quad \ldots (7)$$

The functions $B_1(\theta,\varphi)$, $B_2(\theta,\varphi)$, $\ldots B_9(\theta,\varphi)$ depend upon the orientation of the lightning source and the point of observation and also upon the ground conductivity. $B_1(\theta,\varphi)$, $B_4(\theta,\varphi)$ and $B_7(\theta,\varphi)$ correspond to electrostatic field, $B_2(\theta,\varphi)$, $B_5(\theta,\varphi)$ and $B_8(\theta,\varphi)$ to induction field, and $B_3(\theta,\varphi)$, $B_6(\theta,\varphi)$ and $B_9(\theta,\varphi)$ to radiation field components of atmospherics. These are given as:

$$B_1(\theta,\varphi) = f_1(\theta,\varphi)/r^3 + f_1' + (\theta',\varphi')/r'^3$$
$$B_2(\theta,\varphi) = f_1(\theta,\varphi)/r^2c + f_1'(\theta',\varphi')/r'^2c$$
$$B_3(\theta,\varphi) = f_2(\theta,\varphi)/rc^2 + f_2'(\theta',\varphi')/r'c^2$$
$$B_4(\theta,\varphi) = f_3(\theta,\varphi)/r^3 + f_3'(\theta',\varphi')/r'^3$$
$$B_5(\theta,\varphi) = f_3(\theta,\varphi)/r^2c + f_3'(\theta',\varphi')/r'^2c$$
$$B_6(\theta,\varphi) = f_4(\theta,\varphi)/rc^2 + f_4'(\theta',\varphi')/r'c^2$$
$$B_7(\theta,\varphi) = f_5(\theta,\varphi)/r^3 + f_5'(\theta',\varphi')/r'^3$$
$$B_8(\theta,\varphi) = f_5(\theta,\varphi)/r^2c + f_5'(\theta',\varphi')/r'^2c$$
$$B_9(\theta,\varphi) = f_5(\theta,\varphi)/rc^2 + f_5'(\theta',\varphi')/r'c^2$$

where

$$f_1(\theta,\varphi) = \cos\theta_1(2-3\sin^2\theta) + 3\sin\theta_1\sin\theta\cos\theta\cos(\varphi_1-\varphi)$$
$$f_1'(\theta',\varphi') = R_v[\cos\theta_1(2-3\sin^2\theta') + 3\sin\theta_1\sin\theta'\cos\theta'\cos(\varphi_1-\varphi')]$$
$$f_2(\theta,\varphi) = -\sin^2\theta\cos\theta_1 + \sin\theta_1\sin\theta\cos\theta\cos(\varphi_1-\varphi)$$
$$f_2'(\theta',\varphi') = R_v[-\sin^2\theta'\cos\theta_1 + \sin\theta_1\sin\theta'\cos\theta'\cos(\varphi_1-\varphi')]$$
$$f_3(\theta,\varphi) = 3\sin\theta\cos\theta\cos\theta_1 + \sin\theta_1\cos(\varphi_1-\varphi)(3\sin^2\theta-1)$$
$$f_3'(\theta',\varphi') = R_v[-3\sin\theta'\cos\theta'\cos\theta_1 + \sin\theta_1\cos(\varphi_1-\varphi')(1-3\sin^2\theta')]$$
$$f_4(\theta,\varphi) = \sin\theta\cos\theta\cos\theta_1 - \cos^2\theta\sin\theta_1\cos(\varphi_1-\varphi)$$

$f_4'(\theta',\varphi') = R_v[-\sin\theta'\cos\theta'\cos\theta_1 + \cos^2\theta'\sin\theta_1\cos(\varphi_1-\varphi')]$

$f_5(\theta,\varphi) = \sin\theta_1\sin(\varphi_1-\varphi)$

$f_5'(\theta',\varphi') = R_h\sin\theta_1\sin(\varphi_1-\varphi')$

(r,θ,φ) and (r',θ',φ') are the polar coordinates of the point of observation with respect to source and image respectively.

3 Electric Fields of a Return Stroke

The return stroke is generally taken as a vertical discharge extending from ground to the cloud base. Putting $\theta_1=0$, in the above equations, we get the vertical and horizontal electric field components due to a return stroke:

$$E_\perp^R = (I_0V_0/4\pi\varepsilon_0 ab)[-B_1(\theta)f_s(t') + B_2(\theta)f_i(t') - B_3(\theta)f_r(t')] \quad \dots (8)$$

$$E_\parallel^R = (I_0V_0/4\pi\varepsilon_0 ab)[-B_4(\theta)f_s(t') + B_5(\theta)f_i(t') - B_6(\theta)f_r(t')] \quad \dots (9)$$

where

$f_s(t') = \int_0^t F(t')\,dt,\; f_i(t') = F(t')$

$f_r(t') = -\partial F(t')/\partial t,\; A_0 = I_0V_0/ab$

where V_0, a and b are constants given by:

$V_0 = 9\times10^7$ m s^{-1}, $a = 3\times10^4$ s^{-1}

$b = 7\times10^5$ s^{-1}

The distances r and r' and the angles θ and θ' depend upon the point of observation. The geometry of Fig. 1 yields

$r=[D^2+(H-Z)^2]^{1/2},\; r'=[D^2+(H+Z)^2]^{1/2}$

$\sin\theta = D/r,\; \sin\theta' = D/r'$

$\cos\theta = (Z-H)/r,\; \cos\theta' = -(H+Z)/r'$

where H is the height of the cloud base, Z the altitude of the point of observation, and D the distance from the foot of the channel to the point on the ground just below the point of observation.

Fourier transforming the above equations and rearranging the terms, the perpendicular $|E_\perp^R(\omega)|$ and the parallel $|E_\parallel^R(\omega)|$ components of electric field due to a return stroke are given by:

$$|E_\perp^R(\omega)| = (I_0V_0/4\pi\varepsilon_0 ab)[(R_\perp^R)^2+(I_\perp^R)^2]^{1/2} \quad \dots (10)$$

$$|E_\parallel^R(\omega) = (I_0V_0/4\pi\varepsilon_0 ab)[(R_\parallel^R)^2+(I_\parallel^R)^2]^{1/2} \quad \dots (11)$$

where

$R_\perp^R = -B_1(\theta)R_s^R + B_2(\theta)R_i^R - B_3(\theta)R_r^R$

$I_\perp^R = -B_1(\theta)I_s^R + B_2(\theta)I_i^R - B_3(\theta)I_r^R$

$R_\parallel^R R = -B_4(\theta)R_s^R + B_5(\theta)R_i^R - B_6(\theta)R_r^R$

$I_\parallel^R = -B_4(\theta)I_s^R + B_5(\theta)I_i^R - B_6(\theta)I_r^R$

$R_s^R = k/p_1 - b/p_2 + a/p_3 - k/p_4 + b/p_5 - a/p_6$

$I_s^R = \omega[k/\alpha p_1 - b/(\alpha+a)p_2 + a/(\alpha+b)p_3 - k/\beta p_4 + b/(\beta+a)p_5 - a/(\beta+b)p_6 - A/\omega^2]$

$R_i^R = k\alpha/p_1 - b(\alpha+a)/p_2 + a(\alpha+b)/p_3 - k\beta/p_4 + b(\beta+a)/p_5 - a(\beta+b)/p_6$

$I_i^R = \omega[k/p_1 - b/p_2 + a/p_3 - k/p_4 + b/p_5 - a/p_6]$

$R_r^R = k\alpha^2/p_1 - b(\alpha+a)^2/p_2 + a(\alpha+b)^2/p_3 - k\beta^2/p_4 + b(\beta+a)^2/p_5 - a(\beta+b)^2/p_6$

$I_r^R = \omega[ka/p_1 - b(\alpha+a)/p_2 + a(\alpha+b)/p_3 - k\beta/p_4 + b(\beta+a)/p_5 - a(\beta+b)/p_6]$

$p_1 = \alpha^2+\omega^2,\; p_2 = (\alpha+a)^2+\omega^2$

$p_3 = (\alpha+b)^2+\omega^2,\; p_4 = \beta^2+\omega^2$

$p_5 = (\beta+a)^2+\omega^2,\; p_6 = (\beta+b)^2+\omega^2,\; k = b-a$

$A = (\beta-\alpha)[k/(\alpha+\beta) - b/[(\beta+a)(\alpha+a)] + a/[(\alpha+b)(\beta+b)]]$

For a fruitful computation of electric field intensities, one needs information regarding the flow of current in the return stroke and the velocity of upward propagation of the current waveform. The most widely used current expression is the one given by Bruce and Golde[12], which is derived analytically from the experimental observations. The physical interpretation for the double exponential function is given in Rai[14]. Rai[14] obtained the same expression theoretically and showed that the model is the most realistic one. The expression is given by

$$I_t = I_0(e^{-\alpha t} - e^{-\beta t}) \quad \dots (12)$$

where

I_0, α and β are constants given by

$I_0 = 22\times10^3$A, $\alpha = 1.6\times10^4$ s^{-1}

$\beta = 5\times10^5$ s^{-1}

Some have used a single exponential velocity expression for a return stroke. Srivastava[15] obtained a double exponential velocity expression from the photographs of the lightning channel by Schonland *et al.*[16] Iwata[17], Rai and Bhattacharya[18] showed that the double exponential velocity expression accounts well for the electric field observations on ground surface. Rai[14] obtained the double exponential expression theoretically, as

$$V_t = V_0(e^{-at} - e^{bt}) \quad \dots (13)$$

From the above velocity and current expressions, we obtain $F(t')$ using Eq.(2). The parallel and perpendicular components of the electric field are then calculated.

4 Electric Fields of Horizontal Lightning

Now it is believed that horizontal lightnings are in abundance in nature. Teer and Few[19] and Brantley *et al.*[20] found experimentally that the intracloud lightning discharges are predominantly horizontal. Even a return stroke becomes horizontal after entering into the cloud. However, very little is known about the current and velocity distributions of horizontal lightning. The double exponential expression for the velocity of a return stroke is due to the initial breakdown near the ground surface[10]. Such a breakdown in the case of horizontal lightning has not been reported in literature. In the absence of any practical information, we taken a single exponential velocity expression for the horizontal lightning.

The velocity expression[9,10]

$$V_t = V_0 \exp(-\delta t)$$

together with a double exponential current expression[10], as in the case of a return stroke given by Eq. (12), have been used for electric field calculations. The parameters used in the case of horizontal lightning are[10]

$I_0 = 22\times 10^3$ A, $V_0 = 9\times 10^7$ ms^{-1}
$\alpha = 1\times 10^3$ s^{-1}, $\beta = 5\times 10^5$ s^{-1}
$\delta = 1\times 10^4$ s^{-1}

We assume that the horizontal lightning channel is oriented along the Y-axis of the cartesian coordinate system. Therefore, $\theta_1 = \pi/2$ and $\varphi_1 = \pi/2$. Calculations have been made for the point of observation in the direction of the lightning channel ($\varphi = \varphi' = \pi/2$) and perpendicular to it ($\varphi = \varphi' = 0$) respectively. Thus, after Fourier transforming Eqs (6) and (7), we get the perpendicular and parallel electric field components.

$$|E_{\perp}^{h}(\omega)| = (I_0 V_0/4\pi\varepsilon_0\delta)\,[(R_{\perp}^{h})^2 + (I_{\perp}^{h})^2]^{1/2} \quad \dots (14)$$

$$|E_{\parallel}^{h}(\omega)| = (I_0 V_0/4\pi\varepsilon_0\delta)\,[(R_{\parallel}^{h})^2 + (I_{\parallel}^{h})^2]^{1/2} \quad \dots (15)$$

where

$$R_{\perp}^{h} = -B_1(\theta)R_s^h + B_2(\theta)R_i^h - B_3(\theta)R_r^h$$
$$I_{\perp}^{h} = -B_1(\theta)I_s^h + B_2(\theta)I_i^h - B_3(\theta)I_r^h$$
$$R_{\parallel}^{h} = -B_4(\theta)R_s^h + B_5(\theta)R_i^h - B_6(\theta)R_r^h$$
$$I_{\parallel}^{h} = -B_4(\theta)I_s^h + B_5(\theta)I_i^h - B_6(\theta)I_r^h$$
$$R_s^h = 1/p_1 - 1/p_4 - 1/p_7 + 1/p_8$$
$$I_s^h = \omega[1/\alpha p_1 - 1/\beta p_4 - 1/(\alpha+\delta)p_7 + 1/(\beta+\delta)p_8 + k/\omega^2]$$
$$R_i^h = \alpha/p_1 - \beta/p_4 - (\alpha+\delta)/p_7 + (\beta+\delta)/p_8$$
$$I_i^h = \omega[1/p_1 - 1/p_4 - 1/p_7 + 1/p_8]$$
$$R_r^h = \alpha^2/p_1 - \beta^2/p_4 - (\alpha+\delta)^2/p_7 + (\beta+\delta)^2/p_8$$
$$I_r^h = \omega[\alpha/p_1 - \beta/p_4 - (\alpha+\delta)/p_7 + (\beta+\delta)/p_8]$$
$$k = [\delta\beta(\beta+\delta) - \delta\alpha(\alpha+\delta)]/[\alpha\beta(\alpha+\delta)(\beta+\delta)]$$

where

$$p_7 = (\alpha+\delta)^2 + \omega^2,\quad p_8 = (\beta+\delta)^2 + \omega^2$$

The ratio of the parallel and perpendicular components of electric field, R, is given by

$$R = |E_{\parallel}(\omega)|/|E_{\perp}(\omega)| \quad \dots (16)$$

5 Results and Discussion

Figs 2a and 2b show the variations of $E_{\perp}$ and $E_{\parallel}$ with height for a return stroke at a distance of propagation $D = 100$ km and a frequency $f = 5$ kHz for different ground conductivities. The graphs are plotted for frequency 5 kHz, as this is the frequency of maximum radiation from lightning[14,21]. The vertical electric field is not a function of height, whereas the horizontal electric field increases almost linearly with increasing height for perfectly conducting earth. Master *et al.*[7] also found the similar results. They argued that in the

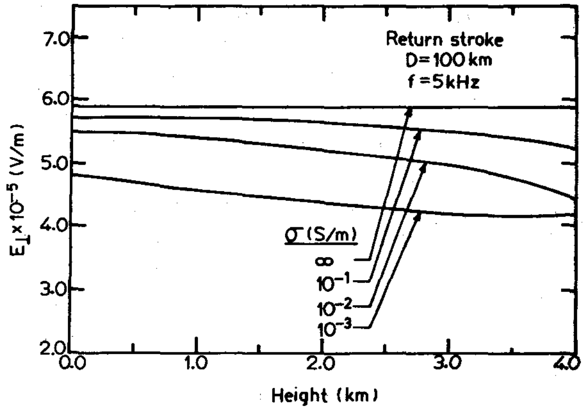

Fig. 2a—Variation of vertical electric field, $E_{\perp}$, with height for different ground conductivities for a return stroke

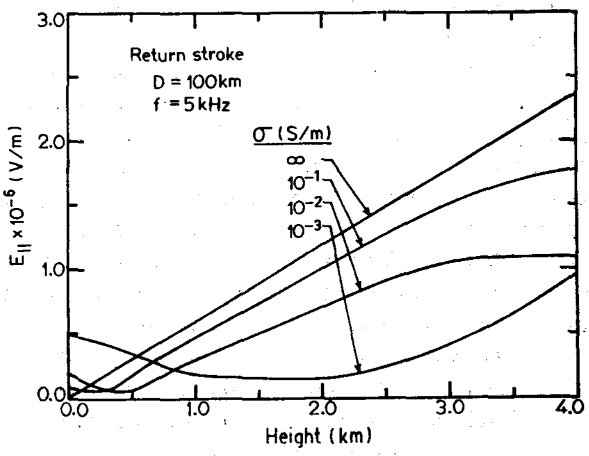

Fig. 2b—Variation of horizontal electric field, $E_{\parallel}$, with height for different ground conductivities for a return stroke

case of vertical polarization on perfectly conducting earth, the measurement of the fields taken on any point above the ground or on the ground should yield similar results and that this fact may be used to calibrate the airborne measurements, provided the horizontal distance from the foot of the channel to the point of observation remains the same. Master *et al.*[7], however, did not take into account the finite conductivity of the ground. For finitely conducting earth, the vertical electric field becomes a function of height. In Fig. 2a, as the height above the ground increases, the electric field $E_\perp$ decreases in general. This decrease is due to the decrease of the reflection coefficient with height. With increasing height, the amplitude of the ray reflected from the ground surface decreases and hence the contribution of the image current source also decreases. The reflection coefficient decreases with decreasing σ at a particular height, and hence $E_\perp$. Since no experimental or theoretical results of the vertical electric field above a finitely conducting ground are available, our results cannot be compared. However, our results are in excellent agreement with the experimental results of Johler and Lilley[22] and theoretical considerations of Divya and Rai[11], if the point of observation is taken on the ground surface ($Z = 0$). In such a case, the vertical electric field decreases with decreasing ground conductivity. Our calculations are limited to 4 km, because this was the altitude of the aircraft for measuring electric and magnetic fields[3].

The horizontal electrif field, $E_\parallel$, however, increases with height above about 2 km for all ground conductivities in general. For the point of observation on a perfectly conducting ground, the resultant parallel electric field is zero. As the ground conductivity decreases, the amplitude of the reflected ray also decreases and hence resultant parallel electric field increases. With increasing height, the contribution of the image dipole to the total parallel electric field decreases and hence the parallel electric field increases with height.

Baum[3] using a WC-130 aircraft obtained airborne data on electric and magnetic field characteristics and gave typical first and a typical subsequent return stroke waveform at a range of about 20 km. The data collected by Baum are high frequency (in MHz range) airborne recordings. The theory developed by us is applicable to the VLF radiations from lightning. No other experimental results are available, so our results cannot be compared with the experimental observations. However, the comparison has been made with the experimental observations taking the limiting case of $Z = 0$ (taking the point of observation at ground). If the ground has a relatively low conductivity, the high frequency components are strongly attenuated, whereas frequencies below 100 kHz are not much affected at a distance of 10 km or more over land[23-26]. We have, therefore, computed the frequency spectrum of the electric fields in the frequency range 1-100 kHz.

Figs 3a and 3b show the variations of $E_\perp$ and $E_\parallel$ with frequency for different heights ($Z = 0$ to $Z = 4$ km) at a ground conductivity of 10^{-2} S/m and a distance of

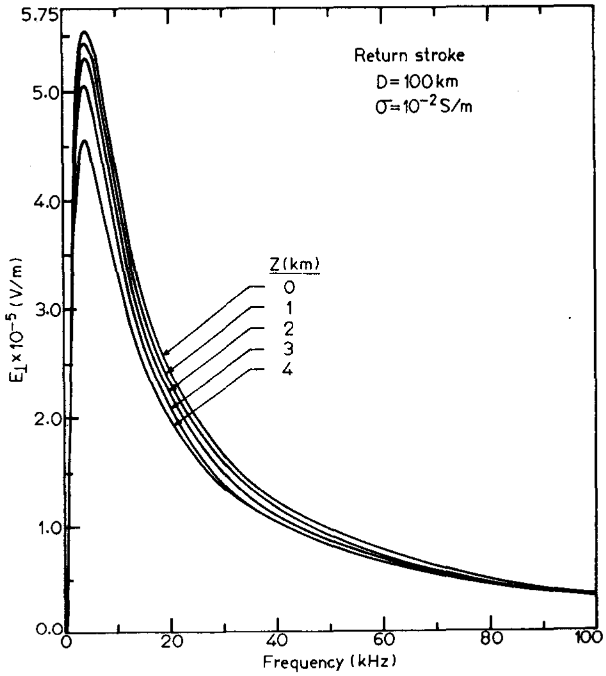

Fig. 3a—Variation of $E_\perp$ with frequency of radiation for different heights of point of observation Z ($\sigma = 10^{-2}$ S/m)

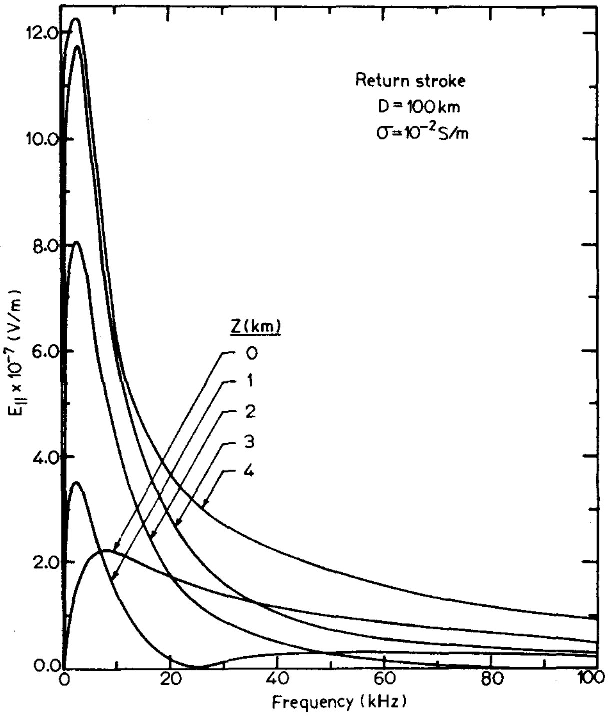

Fig. 3b—Variation of $E_\parallel$ with frequency for different Z ($\sigma = 10^{-2}$ S/m)

propagation equal to 100 km. The shape of the curve in Fig. 3a remains similar at all heights. The peak at $Z=0$ (ground) occurs at 4 kHz, which is in agreement with the values reported by different experimenters[27-29]. With the increasing height, the frequency of peak radiation remains the same, while electric field decreases. However, in the case of horizontal polarization, the peak occurs at 7 kHz for $Z=0$ and at 3 kHz for $Z=1$ onwards (Fig. 3b). The electric field at a given frequency increases with height.

The ratio R for a return stroke is shown in Fig. 4 as a function of height from the ground at a frequency of 5 kHz and ground conductivity 10^{-2} S/m for various distances. R increases with increasing height for all distances of propagation. The ratio also increases with decreasing distance of propagation at a given height. The horizontal electric field, however, is always smaller than the vertical electric field for all heights and distances of propagation of our interest. Master *et al.*[7] also found similar results.

The above results pertain to a vertical return stroke. Figs 5a and 5b show the frequency spectra of $E_\perp$ and $E_\parallel$ components of horizontal lightning for different heights at $D=100$ km, $\sigma=10^{-2}$ S/m and $\varphi=0$. $E_\perp$ decreases with increasing frequency, the decrease being more for greater heights in general (Fig. 5a). $E_\parallel$ also decreases with increasing frequency. The parallel electric field increases with height at frequencies below 30 kHz (Fig. 5b). For the heights of interest in our calculations, the $E_\perp$ and $E_\parallel$ components decrease monotonically with frequency for all heights. At a short distance from lightning, the electrostatic and induction fields dominate, whereas at a large distance, the radiation field dominates. For horizontal lightning, the radiation field becomes effective at distances around 140 km (Ref. 30). For $\varphi=\pi/2$, the vertical electric field is zero for all heights. This is in agreement with the theoretical considerations of Volland[31]. The horizontal electric field decreases with increasing frequency (Fig. 6), the amplitude being almost the same at all heights. The ratio R remains independent of frequency; for $\sigma=\infty$, its value is 0.18 at a height of 4 km and $D=100$ km (Fig. 7). For finitely conducting ground, R varies with frequency, but always remains above 1.0. This shows that for finite ground conductivities, $E_\parallel$ is always heighter than $E_\perp$.

The variation of R for horizontal lightning with distance at a height of 4 km for a frequency of 5 kHz and various values of σ is shown in Fig. 8. The value of R is larger than 1 for all ground conductivities except for perfectly conducting earth, which, however, corresponds to an ideal case. Further $R>1$ implies that the horizontal electric field components of electromagnetic radiation from lightning is larger than the vertical component.

Stanford[5], applying corrections for ground conductivity to the observations of Kohl[6], found that

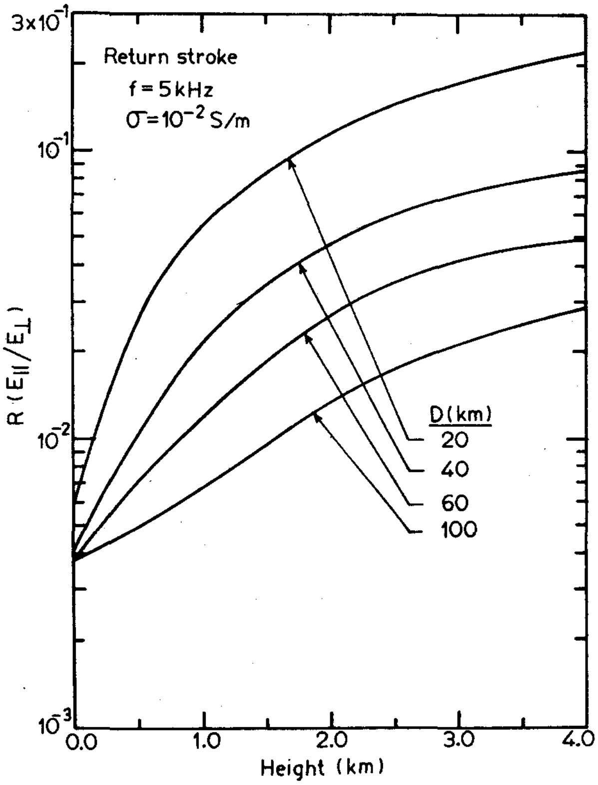

Fig. 4—Variation of R with height for a return stroke

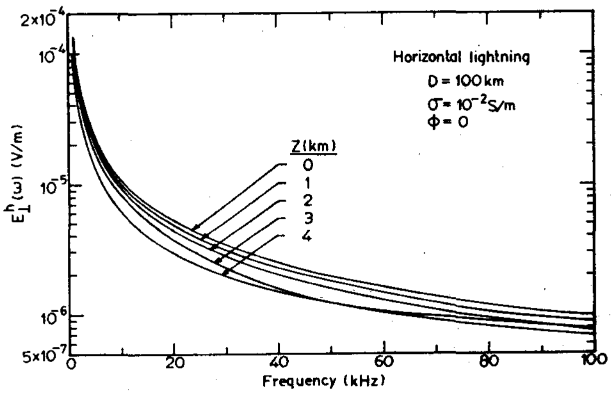

Fig. 5a—Variation of $E_\perp^h(\omega)$ with frequency for different Z ($\sigma=10^{-2}$ S/m and $\varphi=0$ for horizontal lightning)

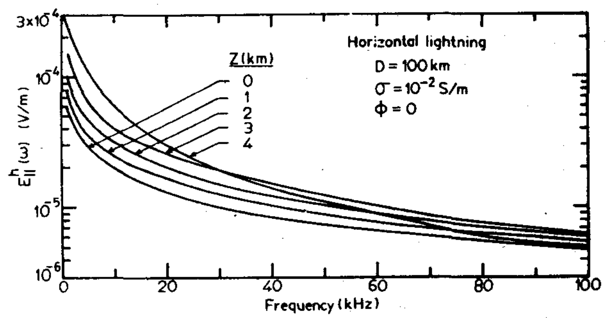

Fig. 5b—Variation of $E_\parallel^h(\omega)$ with frequency for different Z ($\sigma=10^{-2}$ S/m and $\varphi=0$ for horizontal lightning)

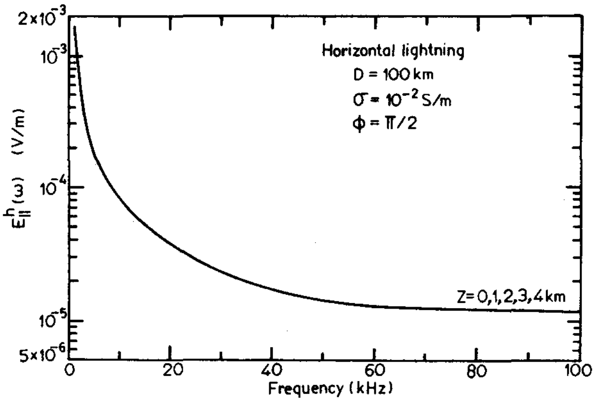

Fig. 6—Variation of $E^h_{\parallel}(\omega)$ with frequency for $\sigma = 10^{-2}$ S/m and $\varphi = \pi/2$ for a horizontal lightning

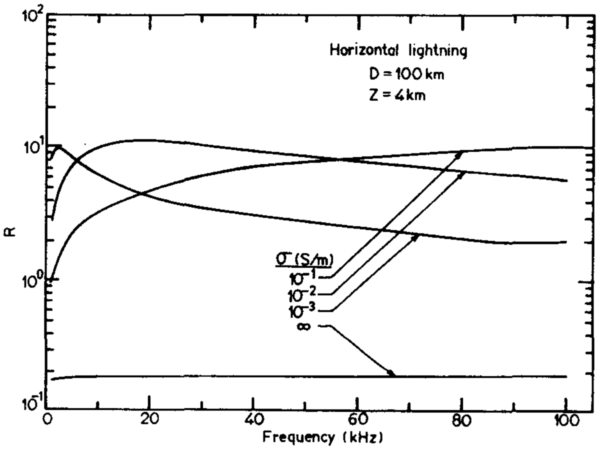

Fig. 7—Variation of R with frequency for horizontal lightning

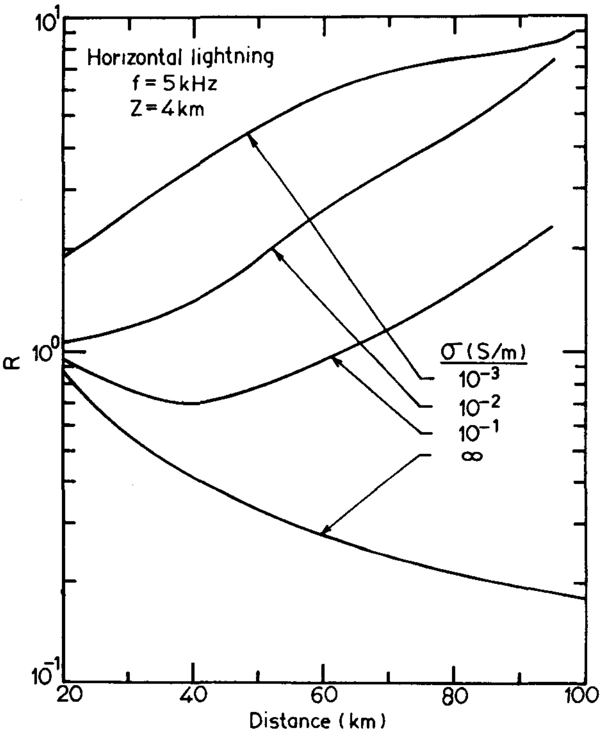

Fig. 8—Variation of R with distance

the radio frequency waves from thunderstorms have the true vertical electric field several times smaller than the horizontal component. Hughes[1], using an airplane, also found similar results. Pathak *et al.*[10] showed theoretically that all the characteristics of whistlers, which are caused by atmospherics, can be explained if we assume that the former are produced by horizontal lightning. Similarly, Madhu Bala *et al.*[32] showed that the tweeks are probably generated from horizontal lightning. If we assume that the lightning are predominantly horizontal (as discussed above), our results are in excellent agreement with those of Stanford[5] and Hughes[1].

6 Conclusions

Since the ground is known to be finitely conducting, the vertical electric field is expected to vary with height. Thus, the measurements of distant vertical electric fields on the ground and above ground cannot be used for the calibration of airborne measurements. The shapes of the vertical and the horizontal electric fields versus frequency curves remain the same on and above the ground at all heights. Ratio R for a return stroke has been found to be less than 1, in conformity with the theoretical predictions of Master *et al.*[7]

The frequency spectrum of the electric fields due to a horizontal lightning is quite different from that of a return stroke. In the former case the ratio R is always greater than 1 except for $\sigma = \infty$, which is in conformity with the observations of Hughes[1] and the theoretical considerations of Pathak *et al.*[10] and Madhu Bala *et al.*[32]

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