Critical Angle Effect on o- and x-Appearance-time Separation for Short Waves at Oblique Incidence

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The time necessary to maintain ordinary (o-) and extraordinary (x-)wave components of short waves to be of maximum usable frequency at oblique incidence is estimated. Concept of critical angle is assumed as a cause for the reflection of radio waves from the ionosphere. The results obtained with the new form of relation between the distance between transmitter and receiver and the angle of incidence are in general agreement with the experimental observations.

For a clear understanding of important factors which govern high frequency radio communication, direct observation of ionospheric propagation is essential. The sweep-frequency pulse technique1-3 and the method of fading of short wave radio signals near maximum usable frequency (MUF)4,5 are two well-known methods adopted by a number of workers for the study of the ionosphere. On the other hand, various electron density distribution models provide some methods of determining optimum distance between transmitter and receiver for possible reception6-12. The derivations, which have been put forward in this direction, utilize the idea that as the angle of refraction of the wave in the ionosphere becomes 90°, the wave is reflected6-8,10. Later Tiwari11 and Agarwal and Tiwari12 derived a new form of relation between distance between transmitter and receiver (D) and angle of incidence (i) in which the well-known behaviour of electromagnetic waves for reflection (the wave passing from a continuously changing denser medium to a rarer medium is reflected, where its angle of refraction, which is angle of incidence for the forwarding region, becomes equal to or greater than the critical angle) has been used. The aim of the present study is to estimate the 'time' necessary to maintain ordinary (o-) and extraordinary (x-)components of the wave to be MUF using the 'new' and the 'old' sets of (D-i) relations.

For long distance propagation, as the obliquity of the received signal is more, the waves are usually reflected from the lower part of the ionosphere. The value of refractive index in the lower part of ionosphere is larger and therefore the possibility of quasi-longitudinal mode is more \[ Y_T / 2 Y_L \propto (1 - X) \] in Appleton Hartree formula for refractive index. At such stages, the general (D-i) relation for x-wave may be used to investigate the problem of o- and x-wave appearance-time separation. It is given by12

\[
D = 2 y_m \left( \frac{f^2 - f f_H}{f_c} \right)^{1/2} - \sin i_0 \\
\times \log_e \frac{A - B}{f_c - (f^2 - f f_H)^{1/2} \cos i_0} + 2 h_0 \tan i_0
\]  

... (1)

where

\[
A = [f^2 - (f^2 - f f_H)(\cos^2 i_0 - \sin^2 i_0 \cot^2 r_{cx})]^{1/2}
\]

\[
B = (f^2 - f f_H)^{1/2} \sin i_0 \cot r_{cx}
\]

\[ y_m \] Semi-thickness of the layer

\[ f_c \] Critical frequency

\[ h_0 \] Height of the bottom of the layer

\[ f \] Frequency of transmission

\[ f_H \] Gyro frequency

\[ i_0 \] Angle of incidence

\[ r_{cx} \] Critical angle for x-wave

\[ D \] Distance between transmitter and receiver

In the morning hours due to increase in elec-
tron density, the o-wave appears short after the appearance of the x-wave. During this time, \( h_0 \) and \( y_m \) are assumed to remain constant and the appearance of the o-wave after the appearance of the x-wave is considered to be due to change in critical frequency of the reflecting layer. The change in critical frequency, necessary to maintain o- and x-wave components to be MUF is known as MUF separation, as discussed earlier. The MUF separation may be used as a source of measurement of the time separation of o- and x-wave appearances and is achieved with Eq. (1) using previously adopted method for both the frequencies of the transmitter.

Most prominent values of \( y_m = 60 \text{ km} \) and \( h_0 = 180 \text{ km} \) are introduced in Eq. (1) and different values of MUF separations are achieved for properly selected values of \( r_{cx} \) and \( r_{co} \) for \( D_{\text{skip}} = 1100 \text{ km} \) (distance between Karachi and Baraut) for 12 MHz and 15.79 MHz waves. Adopting the above procedure again and using \( r_{cx} = r_{co} = 90^\circ \), the MUF separation for old set is obtained. The ratios of o- and x-wave appearance-time separations for 12 MHz and 15.79 MHz waves are obtained with the ratios of their MUF separations and are given in Table 1.

The experimental verification of the o- and x-wave appearance-time separations for 12 MHz and 15.79 MHz waves may be made with the observations taken by Mehrotra and Garg and Mehrotra et al. Such information regarding the appearance for 12 MHz and 15.79 MHz waves indicates in general: "At the same time of observation, 12 MHz wave is received nearly 20 min earlier than 15.79 MHz wave." However, the appearance-time separation for o- and x-waves does not follow any systematic law. Some times \( (\Delta t_{o-x})_{12\text{MHz}} \) is about 3 min and \( (\Delta t_{o-x})_{15.79\text{MHz}} \) is 2 min or vice-versa; \( (\Delta t_{o-x})_{12\text{MHz}} \) and \( (\Delta t_{o-x})_{15.79\text{MHz}} \) are the o- and x-waves appearance-time separations for 12 MHz and 15.79 MHz waves; \( t \) and \( \Delta t \) represent time and small time interval respectively. At few stages such parameters maintain identical values. A comparison of the o- and x-waves appearance-time separations obtained with the new and the old sets of \( (D-i) \) relations with the experimental observations offers a significant overall improvement of the new set in reducing the errors. An examination of Table 1 reveals the following.

(i) Critical angle with varied values is the dominating parameter, which may convert the possible values of \( (\Delta t_{o-x})_{12\text{MHz}} \) and \( (\Delta t_{o-x})_{15.79\text{MHz}} \) for satisfying all the positions as

\[
(\Delta t_{o-x})_{12\text{MHz}} \geq (\Delta t_{o-x})_{15.79\text{MHz}}
\]

(ii) The old set of \( (D-i) \) relation always gives

\[
(\Delta t_{o-x})_{15.79\text{MHz}} < (\Delta t_{o-x})_{12\text{MHz}}
\]

(iii) The vertical columns (a) and (b) of Table 1 indicate

\[
(\Delta t_{o-x})_{15.79\text{MHz},12\text{MHz}} \leq (\Delta t_{o-x})_{15.79\text{MHz},12\text{MHz}}
\]

The results obtained for MUF separation and for o- and x-wave appearance-time separation in general indicate that the critical angle has its specific role in short wave radio communication problems.

### Table 1 — Ratios of o- and x-Wave Appearance-Time Separations for 12 MHz and 15.79 MHz Waves

<table>
<thead>
<tr>
<th>( f = 12\text{MHz} )</th>
<th>( f = 15.79\text{MHz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{cx} )</td>
<td>( r_{co} )</td>
</tr>
<tr>
<td>( 87^6' )</td>
<td>5.2</td>
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<tr>
<td>( 87^6' )</td>
<td>5.2</td>
</tr>
<tr>
<td>( 90^6' )</td>
<td>5.412</td>
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References