Indian Journal of Radio & Space Physics
Vol. 16, August 1987, pp. 318-320

Plasma β-Parameter & Instabilities of Obliquely Propagating Electromagnetic Modes in the Earth's Magnetosphere

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Received 3 *January* 1986; *accepted* 19 *May* 1987

The dependence of convective instabilities of both obliquely propagating ordinary and extraordinary modes on plasma β-parameter, the McIlwain parameter L, the propagation angle θ, the left-hand and right-hand cut-off frequencies ω_1 and ω_2, respectively, is investigated. Computations are also made for perpendicular propagation of the wave.

1 Introduction

Temperature anisotropy instability in a plasma containing cold and hot species in the magnetosphere has been investigated by Renuka and Viswanathan[1]. Cuperman *et al.*[2] calculated the maximum enhancement occurring in high-β thermally anisotropic plasmas upon addition of a cold plasma. The extraordinary mode of electromagnetic instability of perpendicularly propagating plasma waves is studied for Dory-Guest-Harris type plasma distributions by Lai[3]. The oblique propagation choosing a bi-Maxwellian distribution function of hot electrons permeating a cold plasma is studied by Hashimoto and Kimura[4] and by Lembege and Dawson[5]. The stability of near perpendicular propagation of electromagnetic ion cyclotron wave is investigated by Venugopal[6] for a cold ion component. The effects of plasma β-parameter, the McIlwain parameter (L), the propagation angle (θ) and the frequency of the wave on obliquely and parallel propagation for ordinary and extraordinary mode instabilities are studied earlier[7]. Now in this paper, a similar investigation for perpendicular propagation is done.

2 The Dispersion Relation

The dielectric permittivity tensor of a plasma in a magnetic field is given by Akhiezer *et al.*[8] The dispersion equation for ordinary and extraordinary modes have been given by Cap[9] for a cold magnetized plasma. To study convective instability, let us write

$$k = k_r + \mathrm{i}\, k_i, \quad |k_i| \ll k_r \qquad \ldots (1)$$

For electrons obeying a distribution of the form given by Kito and Kaji[10], Renuka and Viswanathan[11] worked out the components of the dielectric permittivity tensor of the plasma in a magnetic field [Ref. 10, Eqs (22)-(59)] and the dispersion relation is given by

$$(a_r + \mathrm{i}\, a_i) n^4 + (b_r + \mathrm{i}\, b_i) n^2 + (c_r + \mathrm{i}\, c_i) = 0 \qquad \ldots (2)$$

where n denotes the refractive index and

$$a_r = \varepsilon_{11}^R \sin^2\theta + \varepsilon_{33}^R \cos^2\theta + \varepsilon_{13}^R \sin 2\theta \qquad \ldots (3)$$

$$a_i = \varepsilon_{11}^I \sin^2\theta + \varepsilon_{33}^I \cos^2\theta + \varepsilon_{13}^I \sin 2\theta \qquad \ldots (4)$$

$$\begin{aligned} b_r = {} & \sin^2\theta\{\varepsilon_{22}^I \varepsilon_{11}^I - (\varepsilon_{12}^2)^R - \varepsilon_{11}^R \varepsilon_{22}^R\} + (\varepsilon_{13}^2)^R \\ & + \cos^2\theta(\varepsilon_{33}^I \varepsilon_{22}^I - \varepsilon_{22}^R \varepsilon_{33}^R) - (\varepsilon_{13}^2)^I - \varepsilon_{11}^R \varepsilon_{33}^R \\ & + \sin 2\theta(\varepsilon_{13}^I \varepsilon_{22}^I - \varepsilon_{22}^R \varepsilon_{13}^R) + \varepsilon_{33}^I \varepsilon_{11}^I \end{aligned} \qquad \ldots (5)$$

$$\begin{aligned} b_i = {} & -\sin^2\theta(\varepsilon_{22}^R \varepsilon_{11}^I + \varepsilon_{11}^R \varepsilon_{22}^I + 2\varepsilon_{12}^R \varepsilon_{12}^I) - \varepsilon_{11}^R \varepsilon_{33}^I \\ & - \cos^2\theta(\varepsilon_{22}^R \varepsilon_{33}^I + \varepsilon_{33}^R \varepsilon_{22}^I) - \varepsilon_{11}^I \varepsilon_{33}^R + 2\varepsilon_{13}^I \varepsilon_{13}^R \\ & - \sin 2\theta(\varepsilon_{13}^R \varepsilon_{22}^I + \varepsilon_{22}^R \varepsilon_{13}^I) \end{aligned} \qquad \ldots (6)$$

$$\begin{aligned} c_r = {} & \varepsilon_{33}^R\{\varepsilon_{11}^R \varepsilon_{22}^R + (\varepsilon_{12}^2)^R - (\varepsilon_{12}^2)^I - \varepsilon_{22}^I \varepsilon_{11}^I\} \\ & - \varepsilon_{33}^I(2\varepsilon_{12}^R \varepsilon_{12}^I + \varepsilon_{11}^I \varepsilon_{22}^R + \varepsilon_{22}^I \varepsilon_{11}^R) \\ & - \varepsilon_{22}^R\{\varepsilon_{13}^2)^R - \varepsilon_{22}^R(\varepsilon_{13}^2)^I\} + 2\varepsilon_{22}^I \varepsilon_{13}^R \varepsilon_{13}^I \end{aligned} \qquad \ldots (7)$$

$$\begin{aligned} c_i = {} & \varepsilon_{33}^R(\varepsilon_{11}^I \varepsilon_{22}^R + \varepsilon_{11}^R \varepsilon_{22}^I + 2\varepsilon_{12}^R \varepsilon_{12}^I) \\ & + \varepsilon_{33}^I\{\varepsilon_{11}^R \varepsilon_{22}^R + (\varepsilon_{12}^2)^R - (\varepsilon_{12}^2)^I - \varepsilon_{11}^I \varepsilon_{22}^I\} \\ & - \varepsilon_{22}^I\{(\varepsilon_{13}^2)^R - (\varepsilon_{13}^2)^I\} - 2\varepsilon_{22}^R \varepsilon_{13}^R \varepsilon_{13}^I \end{aligned} \qquad \ldots (8)$$

Solving Eq. (2) we get k_i and k_r and hence we can study the growth and decay of the wave. If k_i is negative, it signifies growth; otherwise it signifies damping.

3 Discussion of the Analytical and Numerical Solutions

The left-hand and right-hand cut-off frequencies are denoted by ω_1 and ω_2 and the ratios ω/ω_1 and ω/ω_2 by r_1 and r_2, respectively. The magnetic field at any point **B** and the plasma β-parameter (β) are given by

$$\mathbf{B} = (B_0/L^3 \cos^6 \lambda)(1+3\sin^2\lambda)^{1/2} \qquad \ldots (9)$$

$$\beta = 8\pi n_0 k_B T_{\parallel} L^6 \cos^{12}\lambda / \{B_0^2(1+3\sin^2\lambda)\} \qquad \ldots (10)$$

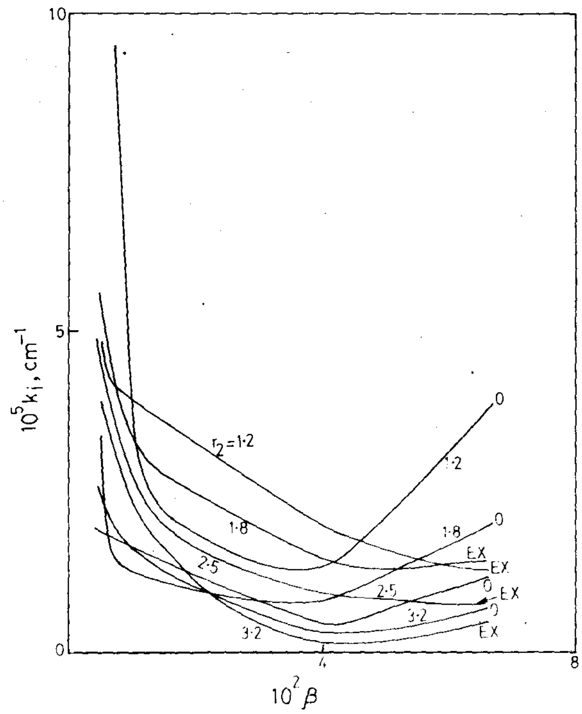

Fig. 1—Plots of growth rate k_i versus β for ordinary (O) and extraordinary (EX) mode for L=2, θ=60°, r_2=1.2, 1.8, 2.5 and 3.2

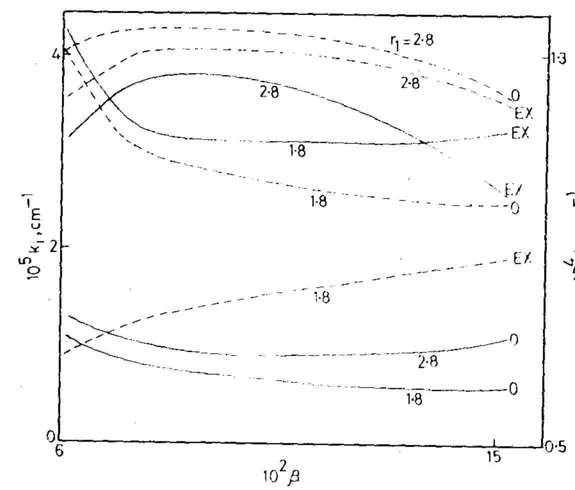

Fig. 2—Plot of k_i (solid line) and wave number k_r (broken line) versus β for L=3, θ=60°, r_1=1.8 and 2.8

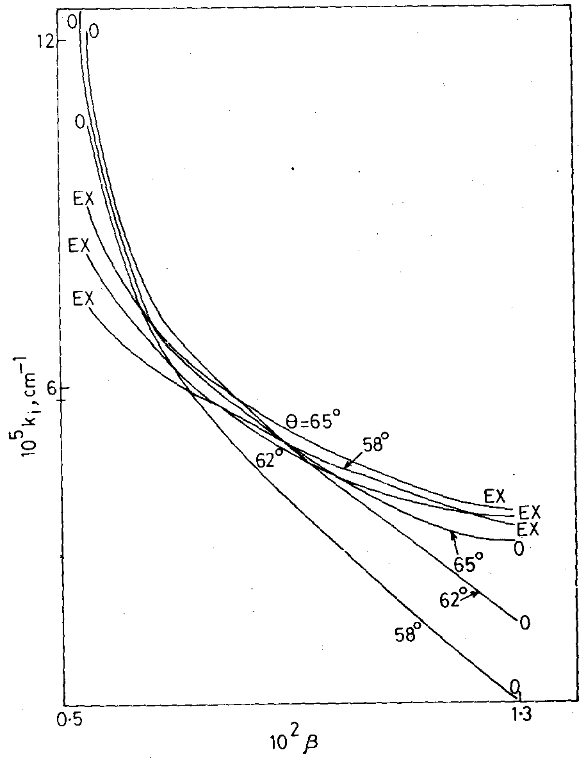

Fig. 3—Plot of k_i versus β for L=2, r_1=2.2, θ=58°, 62° and 65°

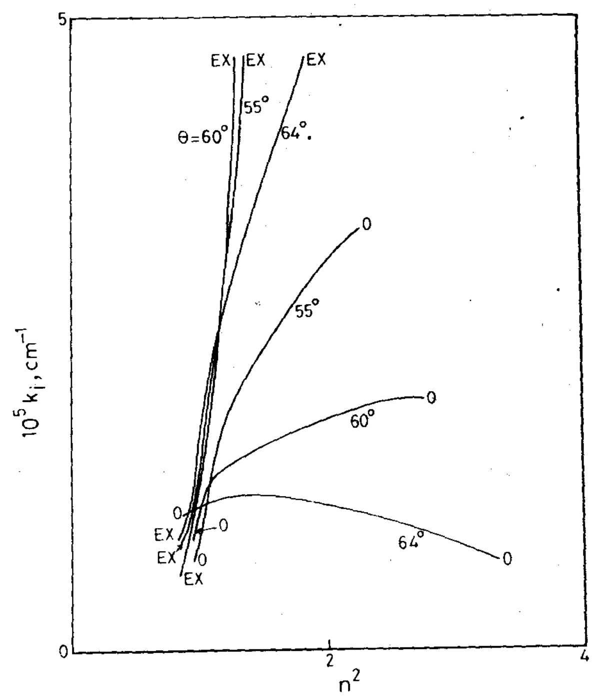

Fig. 4—Plot of k_i versus n^2 for L=2, r_2=2.5, θ=55°, 60° and 64°

where k_B is the Boltzmann's constant, λ the geomagnetic latitude and is taken as zero in our numerical calculations (equatorial plane), and the particle density n_0 chosen for $L=2$ and 3 is 500 particles per cm^3.

The dispersion Eq. (2) is solved numerically for different parameters as follows:

L = 2 and 3

B_0 = 0.31 Gs

$T_\parallel$ = 0.4-20 keV

θ = 55°-85°

ω/ω_1, ω/ω_2 = 1.2-3.2

Results of the numerical computations done using TDC 316 computer are shown in Figs 1-4. It is found that for the above chosen values of L, r_1, r_2, β and θ, both ordinary (O) and extraordinary (EX) modes are growing (k_i is negative). Fig. 1 is intended primarily to bring out the variation of the growth rate, k_i, with the plasma β-parameter. For ordinary and extraordinary modes propagating at an arbitrary angle of 60°, the growth rate is plotted against β at $L=2$; and $r_1 = 1.2$, 1.8, 2.5 and 3.2 Fig. 1 shows a minimum growth for a wide range of values of β.

In Fig. 2, the wave number k_r (broken line) and growth rate k_i (solid line) are plotted against β for waves propagating at an angle of 60° and frequency ratio $r_1 = 1.8$ and 2.8 at $L=3$. The wave number values obtained from computer calculations and hence the wave frequency are in agreement with the observed magnetospheric signals reported by Matthews and Yearby[12]. The growth rate is found to be greater for ordinary mode propagating with frequency higher and extraordinary mode propagating with lower frequency.

The dependence of propagating angle of the waves on the plasma β-parameter at $L=2$ and $r_1 = 2.2$ can be seen from Fig. 3. The amplitude of both ordinary and extraordinary mode decreases with increase in the value of β. The growth rate is found to be high for extraordinary mode propagating at high propagation angles.

The growth rate is plotted against n^2 for propagation angle $\theta = 55°$, 60° and 64° at $L=2$ and propagating frequency $\omega = 2.5\,\omega_2$ in Fig. 4. For the extraordinary mode, the growth rate shows a sharp increase with n^2. The dependence is significant in the case of ordinary mode, the growth rate being higher at lower propagation angle.

Table 1—Numerical Values of k_i and Wave Number k_r of Perpendicularly Propagating Ordinary Wave for $L=2$ and $\beta = 0.027$

r_2	k_r cm^{-1}	$k_i \times 10^4$ cm^{-1}
1.8	0.34682	−2.9551
2.2	0.41830	−3.6118
2.5	0.47916	−4.1043
2.8	0.53082	−3.7533
3.2	0.60075	−5.2536

In the case of perpendicular propagation of the wave, the components of the dielectric permittivity tensor are worked out by Renuka[13] and the dispersion relation is given as

$$\varepsilon_{11} n^4 - n^2\{\varepsilon_{12}^2 + \varepsilon_{11}(\varepsilon_{33} + \varepsilon_{22})\} + \varepsilon_{33}(\varepsilon_{12}^2 + \varepsilon_{11}\varepsilon_{22}) = 0 \quad \ldots (11)$$

Using a TDC 316 computer, Eq. (11) is solved and the results are reproduced in Table 1. The wave is found to be growing (k_i is negative) and amplitude of the wave shows an oscillatory nature with increase in the propagation frequency ratio r_2. The basic assumption that $|k_i| \ll k_r$ can also be verified from the data given in Table 1.

Acknowledgement

The author appreciates the stimulating discussions with Prof K S Viswanathan of the department.

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