A dynamical 2-D wind model for middle atmosphere

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A two-dimensional zonal wind model has been developed by solving basic wind equations under quasi-geostrophic and quasi-hydrostatic approximations. A realistic value of mean Kelvin wave flux observed experimentally is used to reproduce the westerly phase of semi-annual oscillation (SAO) in the model. The computed winds are qualitatively in good agreement with the observations. Further efforts to incorporate quasi-biennial oscillation (QBO) by way of model refinement are underway.

1 Introduction

Zonal mean circulation of the tropical middle atmosphere is dominated by long-period oscillations, namely, the quasi-biennial oscillation (QBO) between 18 and 35 km and semi-annual oscillation (SAO) between 35 and 60 km. The dynamics of these low-latitude zonal circulation is quite intriguing and different from what is observed in extra-tropics. While annual cycle dominates in middle and high latitudes and can be accounted for on the basis of simple atmospheric response to seasonal thermal driving, the circulation in the low-latitudes cannot be explained on the basis of thermal forcing as eddy motions play a fundamental role. The major eddy motions observed in the tropical middle atmosphere are the equatorial Kelvin and the mixed Rossby-gravity waves. Studies show that these are induced by diabatic heat sources near tropopause as a response of the atmosphere to the existing thermal excitations\(^{1-4}\). They propagate vertically in the atmosphere and contribute substantially in the evolution and modification of the mean wind field by imparting westerly (by Kelvin waves) and easterly (by mixed Rossby-gravity waves) acceleration to the prevailing winds by their thermodynamic and mechanical dissipation. This process has been suggested to be the main causative mechanism for the generation of corresponding phases of semi-annual and quasi-biennial oscillations in the tropics\(^{2,5-8}\).

Last two decades have witnessed a steady growth in the understanding of these atmospheric processes. Seasonal variations and other disturbances in the middle atmosphere have been explored by analysing balloon and rocket wind data\(^{9-17}\) and our knowledge in this respect has now improved substantially. Hirota\(^{18}\) and Murgatroyd\(^{19}\) have given excellent accounts of the present status in this regard. Additionally, attempts to investigate the tropical circulation over Indian longitudes have also been made by a number of workers\(^{20-27}\) during the last few years.

Apart from these observational studies of the dynamics of the tropical middle atmosphere, efforts to gain conceptual knowledge of various related processes on theoretical basis are also being pursued simultaneously. Modelling techniques have been employed to parameterize the average picture of mid-latitude general circulation\(^{28-32}\), stratospheric sudden warmings\(^{33,34}\), and wave-mean flow interaction\(^{31-35}\). However, simulation of tropical wind features in a numerical wind model has still not been very successful due to larger variability and complex behaviour of wind in this region. However, the theoretical interpretations of some of the atmospheric phenomena\(^{26,35-39}\) have shed enough light over the mechanism involved in the determination of circulation system in this region and have now paved the way for further investigations by incorporation of SAO and QBO, etc. in the theoretical model for the tropics. Few models developed recently\(^{40,41}\), though reproduce the average features of tropical middle atmosphere circulation qualitatively, still have some deficiencies as either they do not represent the observed features very successfully or they are not able to incorporate some of the seasonal or quasi-seasonal variations. The other debatable point is the feasibility of the approximations assumed therein\(^{42}\). It is so because the comparison of wind values obtained from these
models with the observed ones for similar latitudes has not been very convincing, which is one of the deciding factors of the model efficiency. The present effort is an extension of the modelling works mentioned above with special emphasis on tropical circulation. A realistic mean value of Kelvin wave flux observed experimentally at Trivandrum (lat. 8.5°N), India, has been introduced in the model and is caused to dissipate with altitude due to Newtonian cooling and eddy viscosity processes. Phase velocity of the Kelvin wave has also been varied from 40 to 60 m/s and fixed at a value which gives more satisfactory wind values besides reproducing SAO. The gross features of the calculated winds are in good agreement with the observations. The model and the results obtained therefrom are elaborated and critically examined in the following sections. Further attempts to refine the model by incorporating QBO and other atmospheric phenomena are underway.

2 Model

Basic equations governing the circulation of the middle atmosphere have been simplified using quasi-geostrophic and quasi-hydrostatic approximations. In addition, the time derivative and nonlinear advection of temperature in the zonal mean thermodynamic equation are also assumed to be negligible. The longitudinally averaged forms of the model equations are

\[ \frac{\partial \tilde{u}}{\partial t} + \tilde{v} \frac{\partial \tilde{u}}{\partial y} + \tilde{w} \frac{\partial \tilde{u}}{\partial z} \left( 2\Omega + \frac{\tilde{u}}{\cos \theta} \right) \tilde{v} \sin \theta = \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho \tilde{v} \frac{\partial \tilde{u}}{\partial z} \right) + F_K - \alpha_R \tilde{u} \]

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\[ \frac{2\Omega + \frac{\tilde{u}}{\cos \theta}}{\cos \theta} \tilde{v} \sin \theta = -\frac{\partial \tilde{\phi}}{\partial y} \]  

\[ N^2 \tilde{w} = \frac{\kappa J}{H} \left( Q_K + \alpha_N \frac{\partial \tilde{\phi}}{\partial z} \right) \]  

\[ \frac{1}{\cos \theta} \frac{\partial}{\partial y} \left( \tilde{v} \cos \theta \right) + \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho \tilde{w} \right) = 0 \]

where

\[ \tilde{u} \quad \text{Mean zonal wind} \]
\[ \tilde{v} \quad \text{Mean meridional wind} \]
\[ \tilde{w} \quad \text{Mean vertical wind} \]
\[ t \quad \text{Time} \]

The functional values of \( \nu, \alpha_R \) and \( \alpha_N \) are the same as used by Takahashi in his model.

The following function is used to calculate the momentum flux due to Kelvin waves

\[ \rho \tilde{u} \tilde{w} = \frac{B}{\left( c - \tilde{u}_{eq}(z,t) \right)^{12}} \exp \left[ -\int_0^z \frac{N\alpha_N}{\left( c - \tilde{u}_{eq} \right)^2} \right] \]

\[ + \frac{N^3 \nu}{\left( c - \tilde{u}_{eq} \right)^4} \frac{k}{dz} \exp \left[ -\frac{2\Omega a}{\left( c - \tilde{u}_{eq} \right)^2} \right] \]

where \( c \) is the phase velocity of the Kelvin waves, \( \kappa \) the zonal wave number of the Kelvin waves, \( \tilde{u}_{eq}(z,t) \) the mean zonal wind at the equator, and \( B \) a constant which corresponds to the average flux of the Kelvin waves at the base of the model.

3 Numerical solution

The computation is started from a motionless state taking \( \tilde{u} = 0 \) at \( t = 0 \) which is the northern hemisphere vernal equinox. Wind values are obtained for both the hemispheres in 20-100 km altitude region. The boundary conditions used at the lower and upper limits of the model are

\[ \tilde{u} = 0 \text{ at } z = 20 \text{ km} \]

\[ \frac{\partial \tilde{u}}{\partial z} = 0 \text{ at } z = 100 \text{ km} \] (\( z \) is a measure of height)

Further, we have used Eq. (4) to define a meridional mass stream function \( \tilde{x} \) such that

\[ \frac{\partial \tilde{x}}{\partial z} = -\rho \tilde{v} \cos \theta \]
and put a latitudinal boundary condition for $\hat{x}$ as

$$\hat{x} = \text{constant}$$  \hspace{1cm} (8)

Next we combine Eqs (2) and (3) with the help of Eq. (7) and get the following equation

$$N^2 \frac{\partial}{\partial y} \left( \frac{1}{\rho \cos \theta} \frac{\partial \hat{x}}{\partial y} \right) = \frac{\partial}{\partial y} \left( \frac{\rho J}{\rho n^2} \right) + \frac{\partial}{\partial z} \left( \frac{\rho a_N}{\rho n^2} \right)$$

$$\times \left( 2 \Omega \sin \theta \hat{u} + \frac{\hat{w}^2 \tan \theta}{a} \right)$$  \hspace{1cm} (9)

This equation is solved for $\hat{x}$ at all grid points and the values of $\hat{x}$ thus obtained are put in Eq. (7) to calculate $\hat{v}$ and $\hat{w}$. These values of $\hat{v}$ and $\hat{w}$ together with the values of acceleration due to Kelvin waves ($F_K$) are then used in Eq. (1) to integrate it numerically for $\hat{u}$ at the next time step. The backward type of implicit scheme is adopted in this process.

The value of $J$ is calculated separately using the algorithm developed by Holton and Wehrbein. All the equations are transformed into their respective finite differences form with grid intervals: $\Delta \theta = 5^\circ$, $\Delta z = 5$ km, and $\Delta t = 1.5$ hr. The model run is extended for a period of 450 days and the first 90 days during which model develops and stabilizes are discarded. The year is taken as comprising 360 days for convenience.

### 3.1 Calculation of Kelvin wave flux

The momentum flux associated with Kelvin waves is given by

$$F_K = \rho_0 \langle u' \rangle \langle w' \rangle$$  \hspace{1cm} (10)

where $\rho_0$ is the mean air density and $u'$ and $w'$ are perturbations in zonal and vertical wind fields respectively. The bar over $u'$ and $w'$ represents long-term time averaging. In the present study the average value of $u'$ is obtained by analysing the time-series at 20 km for the duration June to September of the years 1984 to 1987. These time-series are formed using zonal winds obtained from high altitude balloons launched three times a week from the equatorial station Trivandrum. The duration June to September is chosen as during these months wind circulation is mostly easterly in the lower stratosphere (<20 km) which prohibits the passage of mixed Rossby-gravity waves and only Kelvin waves are supposed to be present in the stratosphere. The waves of the periods greater than 40 days are filtered out from the time series using a high pass filter keeping in view the predominance of Kelvin wave modes of shorter periods (<40 days) in the observational data. The average amplitude of Kelvin waves ($\langle u' \rangle$) is obtained from these filtered time-series separately for all the four years (1984 to 1987). The mean of these values ($\langle \bar{u}u' \rangle$) at this height level (20 km) comes out to be 4.65 m/s which is substituted in Eq. (10) along with the values of $\rho_0$ and $w'$ ($= 1.5$ mm/s, the approximate value) to calculate the value of $F_K$ which is used in the model.

### 4 Results

#### 4.1 Sensitivity test: Effects of $\alpha_N$, $\nu$ and $\alpha_R$ on the model

The model is seen to be very sensitive to radiative heating, and wind values are almost directly proportional to it. A reverse trend is seen with Newtonian cooling coefficient ($\alpha_N$) where an increase of 5 per cent in its value reduces the wind speed with almost the same amount of magnitude. This reduction is substantial in 40-55 km region specially when wind flow is westerly (where a change of 10-12 per cent in the wind values is observed). However, a large change (10 per cent) in $\alpha_N$ causes instability in the model calculation mainly because of its nonlinear involvement in the model. In the case of eddy viscosity ($\nu$) and Rayleigh friction ($\alpha_R$) coefficients, the model is found to be less sensitive and even a 10 per cent increase in their values causes a variation of only about 2 per cent in the winds. In addition, the effect of these parameters is visible around 55 km only where winds are stronger.

#### 4.2 Calculated winds

The rocket observed mean winds at Thumba (lat. 8.5°N, average of 16 years) and Balasore (lat. 21.5°N, average of 9 years) are shown in Figs 1 and 2. In these figures the time-height cross-sections of the mean zonal winds are described for a complete year in 20-60 km altitude range. The calculated model winds at 7.5°N and 22.5°N are compared with the winds at Thumba and Balasore respectively to assess the efficiency of the model and also with a view to improving it further by varying few model parameters (pertaining to Kelvin waves which play a significant role in the equatorial atmosphere).

A graphical representation of calculated winds at 7.5°N when the contribution of Kelvin waves is excluded from the calculations is given in Fig. 3. Fig. 4 depicts the computed winds at the same latitude obtained using the Kelvin wave flux suggest-
ed by Dunkerton \[1.33 \times 10^{-3} \text{m}^2/\text{s}^2 \times \rho_0 (17 \text{ km})\] (Ref. 8). The phase velocity of Kelvin wave is kept constant at 60 m/s. In the first case (Fig. 3) the winds are observed to be always easterly with a maximum of \(-50 \text{ m/s}\) in the month of July. However, after the inclusion of the Kelvin wave acceleration (Fig. 4) the easterly winds during winter show a trend of slowing down. Further, these calculated winds, though easterlies for most part of the year, are seen to turn to weak westerlies (\(<10 \text{ m/s}\)) and flow for some limited span of time during winter. As these winds, specially the westerlies, differ significantly from the observed mean winds at Thumba (lat. 8.5°N) (Fig. 1), we have made an improvement and substituted the observed value of Kelvin wave flux [described in Sec. 3, which comes out to be \(6.98 \times 10^{-3} \text{ m}^2/\text{s}^2 \times \rho_0 (20 \text{ km})\)] in place of Dunkerton's value. This is to provide a larger westerly acceleration to the wind flow. The winds obtained after this modification (Fig. 5) show clearly the easterly and westerly regimes of winds, one during summer (easterly re-
regime) and the other during winter (westerly regime). The obtained winter westerlies in this case are still seen to be very weak (max. 15 m/s in October-November at around 55 km altitude) compared to the observed winds but their strengthening is a clear indication of the role of Kelvin waves in affecting the wind circulation system near equator.

In Figs 6 and 7, the calculated wind patterns at 22.5°N (near Balasore, lat. 21.5°N) are shown without and with inclusion of acceleration due to Kelvin waves. The figures depict two clear wind regimes; westerlies during winter (October-March) and easterlies during summer (April-September) months. Winter westerlies are stronger and attain the maximum value of about 70 m/s during December/January at around 50 km altitude when the acceleration due to Kelvin waves is included (Fig. 7). Summer easterlies are not affected much in this case and continue to have a maximum value of about 40 m/s in the months of June/July at similar heights. However, when the acceleration is not taken into account the calculated westerly wind values are smaller in magnitude by about 20 m/s (maximum westerlies are of 50 m/s) as expected. The reversal from winter westerlies to summer easterlies takes place during March/April and from summer easterlies to winter westerlies during September/October in conformity with the observations.

As the obtained winds at 7.5°N (Fig. 3) do not agree satisfactorily with the observational values at Thumba, (Fig. 1), a further modification in the model has been made by reducing the phase velocity of Kelvin waves from 60 m/s to 45 m/s with the idea that this would provide enough time to the waves to dissipate and impart westerly momentum to the mean flow. As a result of this, westerlies of Fig. 5 get strengthened substantially and attain the maximum value of 35 m/s in the month of April at 45 km and 25 m/s in mid-November near 60 km altitude (Fig. 8). One more noteworthy feature in this case is the existence of westerly wind regimes one during January-April and the other during October-December in addition to an easterly wind regime during May-September. The reversal from winter westerlies to summer easterlies is seen to take place during the month of June, and easterlies with maximum value of -45 m/s are observed at around 60 km altitude in the monsoon month of July. Then after the winds start slowing down, reversal from easterlies to westerlies is seen during September/October. These wind values are now in better agreement with the observed mean winds at Thumba (Fig. 1), despite the approximations and simplifications used in the calculation. It is also noteworthy here that the mean wind values are more sensitive to
the phase velocity of Kelvin waves than the flux described in the preceding paragraph. Further, these parameters have little impact upon the prevailing winds at higher latitudes such as 22.5°N near Balasore.

4.3 Semi-annual and annual oscillations

The Kelvin wave flux described earlier has been incorporated to produce SAO in the model. Fig. 9 shows clearly the two phases of this oscillation in the zonal wind at 7.5°N when prevailing component and the annual oscillation are removed from the calculated raw winds. We see that the time of westerly maxima occurs in the months of March/April. The amplitude of SAO maximizes at 50 km with a value of 23 m/s and then decreases rapidly at higher heights. The annual oscillation (AO) is also observed to be substantial at this latitude (≈ 21 m/s at 65 km). However, AO increases rapidly with latitude and has a magnitude of 70 m/s (height 55 km) at 22.5°N. The phase (time of westerly maxima) of this oscillation occurs during the months of January/February.

5 Discussion

Comparison of model zonal winds obtained at 7.5°N (Fig. 8) with the mean observed winds (average of 16 years) at Thumba (Fig. 1) shows a satisfactory agreement. The westerlies obtained from the model (35 m/s at around 45 km) are of the same order as of observed values (30 m/s, same height) at this station and flow during the winter months (October-May) above 30 km or so. In real observations also these westerlies are seen to persist for the same duration. Descent of SAO phase with height is, however, not clearly discernible in the model winds although it is conspicuous in the observational values. On the other hand, calculated easterlies during summer months are somewhat stronger (max. 40 m/s) than the observed monthly mean wind values (max. 20 m/s) at 50 km altitude.

The calculated zonal winds at 22.5°N (Fig. 7) are also compared with mean winds at Balasore (Fig. 2) obtained from rockets and are found to be in good agreement. Although the calculated winds are a little larger than the mean observed winds but during some particular years the latter are seen to be of the same magnitude as there is significant year to year variability in the wind flow.

The experimentally obtained Kelvin wave flux \( F_K \) used in the model to impart westerly momentum is about five times greater than the value used in earlier modelling works\(^6,41\). This value may be overestimated as the presence of other waves in the time-series (described earlier and used in the calculation of the flux) cannot be totally ruled out. However, the westerly winds and the SAO produced due to inclusion of the flux are in conformity with the real observations. It is also notable that without inclusion of Kelvin wave flux the winds obtained at 7.5°N are mostly easterlies (Fig. 3) which is expected due to strong Coriolis acceleration near equator. It is only after the introduction of the Kelvin waves that the winds get accelerated in eastward direction significantly and westerlies of substantial strength are observed (Fig. 8). The SAO amplitude obtained at 7.5°N from the model (23 m/s) is also roughly the same as observed at 8.5°N at Thumba (≈ 20 m/s) at 50 km altitude. Similarly the amplitude of annual oscillation (AO) obtained at the same latitude is seen to be in fair agreement with the value experimentally observed at Thumba (model value 21 m/s observed value 25 m/s). However, at 22.5°N the model value of SAO amplitude (8 m/s) is much smaller than the observed value at 21.5°N (20 m/s). The AO amplitude, in contrast to this, is larger (≈ 70 m/s) than the experimentally observed value (≈ 40
m/s). This discrepancy may be attributed to the model equations and the approximations which are well suited to extra-tropical regions and should be improved further for more accurate representation of tropical wind features. This requires a better understanding of the low latitude atmospheric phenomena and their suitable parameterization in the model.

6 Conclusion

The model dealt with in the preceding sections provides wind values which are qualitatively in fair agreement with the observed wind values for the corresponding latitudes. The difference in the magnitudes of the calculated winds and the experimental values can be attributed to the approximations used during the simplification of the basic equations which control the middle atmospheric circulation. As the geostrophic approximation is not outright valid in the tropical region, some further refinement is necessary in the model. Inclusion of some other important processes (related to radiative transfer, etc.) and employment of more realistic boundary conditions may prove helpful in improving the model efficiency. Additionally, incorporation of the effect of mixed Rossby-gravity waves in producing quasi-biennial oscillation (QBO) is also needed.

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