Signal processing research at IIT, Delhi

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This paper presents a comprehensive up-to-date account of signal processing activity at the Indian Institute of Technology (IIT), Delhi, which encompasses two broad categories, viz. digital signal processing and statistical signal processing. Attention has been focussed on work which has already appeared in professional technical journals. A brief account of current activities and future plans are also given at the end.

1 Introduction

Serious and concentrated efforts in research on signal processing (which includes filtering) started at the Indian Institute of Technology (IIT), Delhi in the late sixties and during the last two decades, some important developments have taken place in frontier areas in the field as a result of the dedicated work of a varying group of faculty and research scholars of the institute. Work has been carried out in almost all aspects of signal processing, viz. analog, digital and statistical. This article, however, presents a comprehensive account of the latter two aspects only; partial information on analog signal processing work is available elsewhere and the preparation of a comprehensive account is planned for the future. Most of our work on digital and statistical signal processing have appeared in professional technical journals, and we focus our attention on them only. Work reported in conference proceedings and technical reports will not be easily available to the readers, and hence are omitted from the list of references.

The field of digital signal processing (DSP) has seen spectacular advances in the last three decades. Although it has its roots in the work of Newton, Gauss and other ancient mathematicians, the real impetus to its development came from the availability of high speed digital computers and high quality digital hardware in the form of integrated circuit chips. The disclosure of an efficient algorithm for fast computation of the discrete Fourier transform in 1965 added a new dimension to DSP. On the hardware side, the recently marketed versatile and compact DSP chips have added further excitement. The group at IIT, Delhi, has tried to keep pace with the development of this exciting and ever expanding field, and the results of their work are described in Sections 2 through 12 of this article.

Similarly, the field of statistical signal processing, which forms the backbone of many modern applications, has made tremendous advances since the days of Wiener filtering. Its many diverse applications include, today, radar and sonar, communications, geophysics, biomedical, speech and image processing, to mention just a few. Over the last two decades it has gone through a state of flux, intermingling with and deriving benefits of developments in system identification, adaptive filtering, spectral estimation, image processing and, of course, the original areas of estimation and detection. It has now acquired an identity of its own, as evidenced by a host of books, journals and other publications in this area. There has been a significant research effort at IIT, Delhi, in various aspects of statistical signal processing over the last two decades. This work is summarized in Sections 13 through 22 of this paper.

2 Digital filter design by transformation/analogy

In view of our experience with passive and active filters, our early work naturally concentrated on digital filter design starting from well documented analog filters. One method of designing digital filters is to start with the magnitude squared function of an analog low-pass filter (LPF) and replace \( \omega / \omega_c \) where \( \omega_c \) is the cut-off frequency, by either \( \tan(\omega T/2) \cot(\omega T/2) \) or \( \sin(\omega T/2) \cosec(\omega T/2) \), \( T \) being the sampling period. This results in the so-called digital tan or sine filter. We made a critical comparison of these two types of filters with the generating analog filter in respect of performance in the transition as well as stop bands, using Butterworth, Chebyshev and Papoulis optimum filters as specific examples. We found that the tan filter is superior to both analog and sine filters. The cut-off slope and asymptotic
loss depend on \( \omega_c \), and both of them increase in the case of the tan filter, and decrease in the case of the sine filter, as \( \omega_c \) approaches the Nyquist frequency.

Stimulated by the work of Budak and Aronhime\(^4\), we had worked on sharpening the cut-off of maximally flat (MF) analog filters by introducing multiple \( j\omega \)-axis zeros\(^5\). We looked into the possibility of extending this technique to digital filters, and succeeded. An interesting result emerging from this study is that the Butterworth tan filter is a special case of the maximally flat sine filter with coincident transmission zeros at the Nyquist frequency, whose multiplicity equals the order of the filter\(^6\). This technique was later extended to filters with equiripple passband characteristic and it was shown\(^7\) that the performance of the resulting filter was better than that of other similar filters reported earlier in the literature.

Achieving linear phase in infinite impulse response (IIR) filters proved to be a difficult task. We derived a simple minded solution to this problem by starting from a well established result in analog filters, viz. that an infinite array of poles at equal intervals on a line parallel to the \( j\omega \)-axis in the left half of the \( s \)-plane leads to an equal ripple delay characteristic in the entire frequency range\(^8,9\). We, therefore, argued that if all the poles of a digital filter are distributed uniformly on a circle of radius \( r < 1 \) in the \( z \)-plane, there should result an approximation to linear phase. Indeed, the transfer function having this distribution of poles is
\[
H(z) = 1/(rm + z^m)
\]  
and its phase is given by
\[
\phi(\omega) = -m\omega T + rm \sin(m\omega T)
\]
where, the approximation is valid for large \( m \). The error in linear phase is seen to be periodic with a maximum value of \( rm \), which can be made arbitrarily small by choosing \( r \) and \( m \) suitably. The magnitude response of Eq. (1) is
\[
M(\omega) = 1 - rm \cos(m\omega T)
\]
and has an equal ripple all-pass characteristic. The filter can be made low-pass by cancelling the stopband poles by introducing zeros at these locations. Retaining linear phase requires their mirror image zeros also to be introduced; using this simple idea, we worked out an analytical design procedure\(^10,11\) for a low-pass filter satisfying prescribed specifications on delay, maximum delay error and asymptotic slope.

A simple technique of introducing transmission zeros into a low-pass all-pole filter function has been developed\(^12\) so as to make its stopband transmission equiripple. The zeros are introduced through a transformation which maps the stopband of the filter onto the unit circle and the passband onto a segment of the real axis in the transformed plane. The technique has been successfully used to improve the loss characteristics of all-pole constant delay filters. The method has been shown\(^13\) to give the same filter function as that obtained by Unbehaun\(^14\), although the approaches are different.

The skirt selectivities at the two ends of the passband of an usual digital band-pass filter (BPF), obtained by transformation of an analog LPF, may differ considerably if the passband is not symmetrically located in the baseband. For such cases, a new magnitude function having improved characteristics has been presented\(^15\).

Broom\(^16\) formulated a low-pass to band-pass transformation which generates, from a prototype LPF, a realizable BPF with approximate arithmetic symmetry and nearly equal roll-off rate at both ends of the passband. The degree of the transformed BPF is the same as that of the prototype LPF, if the latter has finite impulse response (FIR); however, if the prototype is linear phase, the transformed filter fails to retain this characteristic. We proposed a simple modification\(^17\) of Broom's transformation such that linear phase low-pass FIR filter transforms to an approximately linear phase BPF with a constant phase shift of \( \Theta \), where \( \Theta \) can be chosen as desired. When \( \Theta \) can be chosen as zero or \( \pi/2 \), the transformed BPF becomes strictly linear phase.

A simplified procedure has been developed\(^18\) for determining a digital transfer function from a given real part on the unit circle.

3 Discrete Hilbert transform and its applications

The discrete Hilbert transform (DHT) plays an important role in signal processing and communication systems. A matrix formulation of the DHT is known\(^19,20\) in which the vector \( \Phi = [\phi(0), \phi(1), ..., \phi(N-1)]^T \), representing the DHT of the periodic sequence \( \{a(n)\} \), \( n = 0 \) to \( N-1 \), is expressed as \( [C_{m,n}]_{N \times N} \), where \( [A] = [A(0) A(1) ... A(N-1)]^T \), \( [A(k)] \) being the discrete Fourier transform (DFT) of \( \{a(n)\} \). By exploiting certain properties of the coefficients \( c_{m,n} \) we obtained\(^21\) an alternative formulation in the form \( \Phi = [\hat{A}] [\hat{C}] \), where \( [\hat{C}] \) is a column vector and \( [\hat{A}] \) has elements of the form \( A(i) - A(j) \), in which the number of multiplications is reduced by a factor of two. We applied this formulation to a digital LPF, and showed that this leads to reduced storage requirement and improved noise performance, in addition to reduced computations\(^22\).

4 Round-off noise in digital filters

In a digital signal processor, the signal and the co-
coefficients are stored as binary numbers in registers whose lengths are, of necessity, finite; this gives rise to coefficient quantization and multiplication round-off errors, which can lead to very undesirable effects like limit cycles and overflow oscillations. These errors are intimately connected with the structure of the processor, as revealed in a study we conducted on the second order digital notch filter. For the general digital filter, the multiplication round-off error variance at the output due to a noise source is given by

\[ \sigma_0^2 = \left( \frac{q^2}{12} \right) \int \left| \frac{1}{2\pi j} \right| \frac{H(z)}{H(z^{-1})} z^{-1} \, dz \cdots (4) \]

where, \( q \) is the quantization step size, \( U \) the unit circle and \( H(z) \) the noise transfer function (NTF). We obtained two new methods for evaluating Eq. (4), both of which are based on the decomposition of the given NTF in terms of digital all-pass transfer functions. The motivation for such decomposition is that for each component, Eq. (4) simply reduces to \( q^2/12 \). The first method, based on first order component NTFs, requires somewhat complex arithmetic calculations, but takes less computation time than the best existing method. The second method utilizes the concept of polyphase digital filtering for decomposition into first and second order components; the accuracy obtained in this method is not as good as in the first, but it has the advantage of a drastic reduction in computation time, especially for higher order NTFs. Both of these methods, we believe, offer a new insight into the round-off noise problem in digital filters.

5 Quantized coefficient design of digital filters

As to the minimization of coefficient quantization error, we investigated the possibility of optimally choosing the coefficients from a discrete parameter space, once the wordlength is decided by considerations of cost. The existing techniques for this purpose use non-linear optimization methods, in which a global optimum is not guaranteed. We introduced an entirely different approach, based on the pseudo-Boolean (PB) technique, which consists of a combination of dynamic programming and Boolean methods, for solving the problem. We found that the problem of quantized coefficient design of digital filters essentially reduces to that of solving, repeatedly a set of PB inequalities which are linear in the FIR case and non-linear in the IIR case. For the former case, the solution can be made non-iterative and consequently takes much less computer time as compared to the existing techniques; at the same time, it has much better chances of converging to a global optimum. For the IIR case, we formulated an alternative method of solving non-linear PB inequalities to reduce the computational load significantly; with this modification, we obtained the same advantage as in the FIR case over existing methods.

6 Variable digital filters

In many applications such as speech processing and spectrum analysis, it is often required to change the cut-off frequency of a digital LPF or the bandwidth or centre frequency of a digital BPF, by controlling, preferably, a single parameter in the filter structure. The usual technique for achieving this is to use a suitable frequency transformation. For example, if each delay element in a prototype is replaced by an all-pass network, a variable filter results, the control being affected by the all-pass parameter(s). If the prototype filter is IIR one, then this approach leads to delay free loops in the structure which cannot be implemented digitally. We used a very simple computational trick to get over this problem. As compared to an earlier method, our procedure offers, besides simplicity, a reduction in computations by a factor greater than two; this can be of great advantage in reducing the settling time in adaptive IIR filters.

For variable cut-off linear phase FIR filters, we investigated the use of the cosine transformation, given by

\[ \cos \omega = \sum_{k=0}^{p} A_k (\cos \Omega)^k \cdots (5) \]

(where \( \omega \) refers to the prototype and \( \Omega \) to the transformed filter frequency) in details, and obtained the following new and useful results:

(i) In the first order (1°) case, the variable parameter \( A_0 \) (which equals \( 1 - A_1 \) for invariant d.c. gain and is zero for the prototype) is restricted to lie in the range \( 0 \) to \( \cos^2(\omega/2) \), where the subscript c denotes cut-off condition. Also, in the transformed filter, \( \Omega_c \) increases, while the cut-off slope decreases monotonically, with increasing \( A_0 \). These drawbacks can be overcome by the following optimum 2° transformation

\[ \cos \omega = A_0 + \cos \Omega - A_0 \cos^2 \Omega \cdots (6) \]

which retains single parameter control; also, the computational load is approximately half of that of a double length prototype with 1° transformation, the performance in the two cases being comparable. We also extended this technique to the design of two-dimensional filters.

(ii) The 1° transformation can be used to design a BPF with fixed centre frequency \( \Omega_0 \) and variable bandwidth \( \Delta \), again with single parameter control, but not for constant \( \Delta \Omega = \Delta \)
width) and variable $\Omega_0$. However, if $\Delta \Omega > \Delta \omega$, then it is possible\textsuperscript{31} to maintain $\Delta \Omega$ fixed and vary $\Omega_0$ above or below $\omega_0$ (prototype centre frequency); this increases the filter order.

(iii) The difficulty mentioned under (ii) can be overcome by a 3\textsuperscript{rd} transformation

$$\cos \omega = A_0 + A_1 \cos \omega - A_3 \cos^3 \omega \quad \ldots (7)$$

where, $A_0$ and $A_1$ are related to $A_1$ through $\omega_{1,2}$ and $\Omega_{1,2}$ and the subscripts 1 and 2 refer to the lower and upper cut-off frequencies. By appropriately constraining the parameters and prescribing $A_3$ as $2^{-n}$, $n$ being an integer (so that multiplication by $A_3$ amounts to a simple shifting operation only), it is possible to achieve constant $\Delta \Omega = \Delta \omega$ and single element controlled variable $\Omega_0$ without increasing the computational load as compared to the 1\textsuperscript{st} transformation\textsuperscript{32}.

Another low-pass to band-pass transformation for linear phase FIR filters has been obtained\textsuperscript{33} which allows implementation in a network structure in which the cut-off frequencies can be controlled through two parameters. The range of variation depends on the cut-off frequency and the stopband edge frequency of the prototype low-pass filter. For a given prototype, the range is obtained from a set of simple graphs.

7 Digital filter structures

It has already been mentioned that the finite word-length effects are intimately related to the structure of the digital filter. While many alternative structures are known for IIR filters, much less is known about the FIR case. We investigated a special FIR filter, viz., the discrete Hilbert transformer (DHT), having a transfer function

$$H(z) = z^{-M}[(a_0 + a_1(z + z^{-1}) + \ldots + a_M(z^M + z^{-M})] \quad \ldots (10)$$

where, $b_n = a_n/a_{n-1}$, $n = 1$ to $M$ and $b_0 = a_0$. $b_n$'s should be all comparable in magnitude, and the effects of quantization should be less pronounced. We carried out a deterministic as well as a statistical analysis of the errors in the new structure, which we called the 'nested structure', and found that, compared to the direct form implementation, it gives less coefficient quantization as well as multiplication round-off errors\textsuperscript{36}.

Later, we made a comprehensive investigation of the generalized nested structures, in which the coefficients to be implemented have been made to lie in the range 1/2-1, by suitable decomposition and scaling. We showed that using fixed point arithmetic, these structures achieve the same round-off noise as those obtained in the floating point implementation. The optimum structure to achieve the minimum round-off noise can be found; however, the search in this direction is not as involved as in the cascade form, and in most cases, near-optimum results are easily obtained by mere inspection of the impulse response coefficients\textsuperscript{37}.

For IIR filters, it is advantageous to have the realization as a cascade of second order sections and a possible first order section. There have been various attempts at deriving second order structures with low sensitivity and round-off noise, the most well known being that of Agarwal and Burrus\textsuperscript{38} who considered all-pole structures. This was modified by Munson and Liu\textsuperscript{39} for second order sections with zeros, which were realized separately, and low noise performance was achieved through a cleverly formulated scaling scheme. We investigated this problem thoroughly and showed that alternative structures are possible which, while retaining the scaling scheme of Agarwal and Burrus\textsuperscript{38}, give better signal-to-noise ratio, and the same low coefficient sensitiv-
ity characteristics. The basic technique utilized is that of combining zeros with poles in poles preceding zeros form.

We also considered cascade realizations for FIR structures in terms of fourth order elemental sections having reciprocal pairs of complex conjugate zero pairs and derived new low sensitivity structures for various locations of zeros. Using such structures, along with appropriate section ordering, it has been possible to obtain cascade realizations with less wordlength requirements than the direct form realization.

In sharp cut-off low or high-pass IIR filters, pole and zeros are clustered around band edges and in close proximity of the unit circle. Consequently, the sensitivity performances of such filters are linked with their bandwidth. Analysis of the sensitivity characteristics of some well known low sensitivity structures revealed their inadequacy to encompass the entire range of bandwidth. We derived some new structures to fill the gaps, by using the previously mentioned strategy of combining poles with zeros in the pole preceding zero form.

We next considered the design of low sensitivity band-pass and band-stop IIR filters using a new approach, viz. that of transforming the sensitivity characteristics of a low sensitivity low-pass structure to the appropriate angular region on the unit circle. It was found that a fourth order section of band-pass/band-stop filter can be realized thereby, whose level of sensitivity is of the same order as that of the prototype low-pass second order section, and yet using fewer multiplications than in the cascade approach.

8 Interpolation in frequency and time domains

The fast Fourier transform (FFT), which plays a central role in many signal processing situations, finds the DFT at uniformly spaced frequencies. It is not, therefore, suitable in some important spectral analysis situations, e.g., constant Q, where the spectrum has to be estimated at nonuniform frequencies. Available techniques for these purposes are either computationally inefficient or applicable to some restricted situations only. We evolved a new and general method for this purpose which retains the order of computation at FFT levels. It is based on the fact that the FT at any arbitrary frequency can be expressed as a weighted sum of its DFT coefficients. These weights are suitably approximated such that the FT is the sum of (i) a few directly computed dominant terms of the weighted sum, the number being determined by the acceptable error, and (ii) the DFT of a derived sequence, formed by multiplying the original sequence with a sawtooth function. The number of directly computed terms is so chosen that the error does not exceed the desired limits.

Simultaneously, we also investigated the time domain interpolation problem, and presented a general scheme of interpolation of a uniformly sampled signal at arbitrary time instants. This is an effective alternative to the existing multistage low-pass filtering scheme. Starting from the fact that the interpolated value can be expressed in terms of a Taylor series around the nearest sample, suitable approximations are incorporated such that the evaluation of the interpolated value does not require any derivative higher than the second. The differentiations are then performed through FIR filtering. The method has been found to work satisfactorily even with approximate differentiators obtained by truncating the coefficients to simple negative powers of two, thereby simplifying the multiplication operations to mere binary shifts. The error of approximation can be kept within specified limits by adding a few correction terms. The error behavior of the algorithm and its computational aspects have also been thoroughly investigated.

9 Digital filtering with identical blocks

Kaiser and Hamming introduced the technique of 'sharpening' for improving the performance of a symmetrical FIR filter by multiple use of the same filter. The method is based on the amplitude change function (ACF), \( H_0 = f(H) \), where \( f \) is a polynomial relationship between the amplitudes \( H \) and \( H_0 \) of the prototype and transformed filters, respectively. We used the artifices of scaling and/or shifting of the ACF for further improvement in the passband and/or stop-band response of the transformed filter. We also obtained a new generalized ACF whose characteristics are similar to those of the Kaiser-Hamming-Chebyshev ACF, but has the advantage of not requiring additional correction filters for its implementation. Shifting and scaling of the new ACF are shown to result in further improvement of the response.

Continuing this investigation, we showed that the cut-off frequency of such filters can be varied without modifying the coefficients of the prototype filter and adjusting a few gain factors. A modular hardware implementation scheme has been suggested. Further, the concept of ACF has been used for transforming a low-pass prototype into a high-pass, band-pass or band-stop filter. The method has also been extended to obtain variable cut-off and variable centre frequency band-pass/band-stop filters.

The ACF approach to variable filtering has been further explored for single parameter control.
limits of variation have been found, and it has been shown that the modular implementation can be used to achieve the largest variation for a given order of transformation.

10 Maximally flat FIR filters

Linear phase maximally flat (MF) FIR filters are needed in many practical situations, particularly in applications requiring extremely high stopband loss. Such filters have also been used as building blocks for improving the performance of optimal (equiripple) filters\textsuperscript{50,51}. Extensive research on the design of such filters has been carried out at IIT, Delhi, starting with Miller’s work\textsuperscript{52}. An explicit formula has been obtained for the coefficients of Miller’s designs, and a simple computation scheme has been presented\textsuperscript{53}. General recursive relations have also been obtained\textsuperscript{54} for the coefficients in which the amount of computation required is uniformly the same for any combination of permissible degrees of flatness at $\omega = 0$ and $\omega = \pi$.

In some applications, such as variable cut-off filtering, it is convenient to express the MF transfer function as a series in $\cos\omega$. An explicit formula has been obtained\textsuperscript{55} for the coefficients of this series by exploiting the properties of the Kaiser-Hamming ACF mentioned earlier. Later, we developed a matrix for the purpose\textsuperscript{56} using the properties of the Bernstein polynomial and showed that the required transformation matrix is a product of the well-known $Q$ matrix of Jury and Chan\textsuperscript{57} and a diagonal matrix. This also resulted in a simple relationship between two $Q$ matrices of successive orders. The approach was reformulated later to develop a direct and a recursive method for generating the transformation matrix\textsuperscript{58}. It has been shown that the problem reduces to that of matrix multiplication with integer entries in the matrices. The use of integer arithmetic enables one to determine the exact wordlength required for implementing this class of filters on any binary machine.

The design methods so far are based only on the specified degrees of flatness at $\omega = 0$ and $\omega = \pi$. If in addition, the cut-off frequency ($\omega_c$) of a low- or high-pass MF filter is specified, no direct and general methods are available for determining the coefficients of the required filter. We examined this problem in detail and derived a simple and general method\textsuperscript{59} for direct computation of these coefficients.

The properties of the Bernstein polynomial have been utilized further to establish an equivalence and link between various existing methods of MF FIR filter design\textsuperscript{60}, and to provide an analytical support for Herrmann’s empirical relation\textsuperscript{61}. The Bernstein polynomial approach has also been used to develop a new design procedure which is computationally more efficient than the previous methods, and to provide additional insight into the physical significance of the order of flatness.

Further investigations of MF filters were directed to the use of Bernstein polynomials for developing an optimal design procedure for such filters\textsuperscript{62}. Using a set of recurrence relations, the method searches for the optimum order of filter and the orders of tangency at $\omega = 0$ and $\omega = \pi$ in a ‘staircase-like’ manner. A detailed comparison with the earlier methods shows that the approach is indeed superior on the following two counts: (i) none of the earlier methods could cater to the specifications exactly and (ii) being an optimal method, it results in the minimum order of the filter for the given specifications, and the reduction in order is significant. With slight modifications, it has been possible to extend the method for the design of monotonic FIR filters with arbitrary magnitude specifications\textsuperscript{63} for which, presently, no method exists. Further, the method has also been used to design optimal MF quadrature mirror filters with extremely low reconstruction errors.

11 Design of two-dimensional digital filters

A structure for realizing a first order two-dimensional (2-D) all-pass transfer function has been derived\textsuperscript{64,65} which uses five multipliers and two delays; this has been achieved by modifying the signal flow graph of an existing structure which uses six multipliers and two delays. The multipliers of the new structure are shown to be real for stable filters.

A simple analytical technique has been evolved\textsuperscript{66} for determining the coefficients of the first order McClellan transformation for the design of circularly symmetric 2-D FIR filters from 1-D ones. A similar investigation\textsuperscript{67} was carried out successfully for the design of elliptically symmetrical 2-D FIR filters; the results obtained were found to be nearly identical to those obtained by an existing optimization procedure. The technique was later extended\textsuperscript{68} to approximate an arbitrarily oriented elliptical passband boundary, with better results than obtained with existing techniques.

A known technique for designing 2-D FIR filters using a separable transformation was modified\textsuperscript{69} to achieve circular symmetry with better results than known earlier.

A method using Remez algorithm has been developed\textsuperscript{70} for determining the coefficients of the first order McClellan transformation so that the cut-off frequency of the 1-D prototype filter maps onto a contour in the 2-D frequency plane which approxi-
mates a specified contour in the equiripple sense. The initial estimate of the transformation coefficients and the cut-off frequency of the 1-D filter are obtained analytically, and then refined by using Remez algorithm, which converges very rapidly.

12 Digital differentiator design

Digital differentiators (DDs) form integral parts of many computational systems, and find extensive use in a host of practical systems including radar and sonar. For obvious advantages, FIR rather than the IIR configurations are used for DD, and the most popular designs are based on the well known minimax relative error (MRE) criterion\(^{71-75}\); beyond formulating the minimization problem, such designs are totally algorithmic. Motivated by the general aim of establishing DSP on the same analytical rigour as their analog counterparts, we investigated DDs with a different criterion of approximation, viz. that of forming the minimization problem, such designs are indeed possible to obtain. Besides analytical rigour, such designs have the advantage of achieving good differentiation over a limited frequency band with much less order as compared to MRE designs, which, basically are suitable for wideband performance.

An ideal DD has the frequency response

\[
\hat{H}_i(\omega) = j\omega \hat{H}_i(\omega), \quad -\pi \leq \omega \leq \pi \tag{12}
\]

where, \(\hat{H}(\omega) = \omega\) is purely real. Our designs approximate \(\hat{H}(\omega)\) by \(H(\omega)\) which has maximal linearity at \(\omega = 0, \pi/2\) or \(\pi\) for low, mid and high frequency bands, respectively (the subscripts \(i, m\) or \(M\) and \(h\) will be used to indicate the corresponding frequency responses), and we characterize the effectiveness of the approximation by the relative error, defined by

\[
RE = \frac{100}{\pi} \left| H(\omega) - \hat{H}(\omega) \right| / \omega \tag{13}
\]

For the low frequency range, the simplest approximation to \(j\omega\) is \(j\sin\omega\), which can be realized by the transfer function \((1 - z^{-2})/2\), with an additional linear phase equal to \(-\omega\). This simple example motivates us to write

\[
H_i(\omega) = \sum_{i=1}^{n} d_i \sin(i\omega) \tag{14}
\]

where, \(n = (N-1)/2\), \(N\) being the length of the filter which is assumed to be odd, for well known reasons. We now force the following condition at \(\omega = 0:\)

\[
H_i(0) = 0; \quad dH_i(\omega)/d\omega|_{\omega=0} = 1 \tag{15a}
\]

and

\[
d^uH_i(\omega)/d\omega^u|_{\omega=0} = 0, \quad u = 2, 3, ..., 2n-1 \tag{15b}
\]

The corresponding set of equations in \(d_i\) are then solved to obtain a recursive formula\(^{74}\) for \(d_i\). Another route to obtain \(d_i\)'s would be to use the fact that such a maximally linear DD can be obtained by integrating an MF LPF function from 0 to \(\omega\). This resulted in a recursive\(^{75}\) as well as an explicit formula for the \(d_i\)'s. As an example of performance of such differentiators, let \(RE \leq -200\) dB be required in the range \(0 \leq \omega \leq 0.20\pi\). Our design achieves this with \(N=21\) as contrasted to \(N=127\) required in the MRE design.

For the midband frequencies, MRE designs require the use of half sample delay/advance together with an even \(N\), both of which are undesirable. By starting with

\[
H_m(\omega) = d_0 + \sum_{i=1}^{n} d_i \cos(i\omega) \tag{16}
\]

where, \(n = (N-1)/2\), \(N\) odd, and imposing the condition of maximal linearity at \(\omega = \pi/2\), we have obtained a recursive formula\(^{76}\) for computing the \(d_i\)’s. To illustrate the performance, let \(RE \leq -100\) dB for \(0.45\pi \leq \omega \leq 0.55\pi\); MRE designs for such specifications require 8 multiplications per sample, whereas our design reduces it to 3. Also because of odd \(N\), no non-integral delay is needed.

For the midband range, we have also shown\(^77\) that it is possible to realize the 90° phase shift exactly by choosing the magnitude as

\[
H_m = (\pi/2) \sum_{i=1, i=odd}^{n-1} d_i \sin(i\omega) - (1/2) \sum_{i=2, i=even}^{n} d_i \sin(i\omega), \quad n = (N-1)/2; \quad n \text{ even, } N \text{ odd} \tag{17}
\]

The necessary recursive formula for \(d_i\) has been derived and it has been demonstrated that designs superior to MRE can be obtained over ±25 per cent of the band around \(\pi/2\). For \(RE \leq -140\) dB for \(0.37\pi \leq \omega \leq 0.63\pi\), our design requires \(N=29\) as compared to \(37\) for MRE design. The approximation \(H_m(\omega)\) can be easily modified\(^77\) to obtain a Hilbert transformer with maximum flatness at \(\pi/2\).

For the high frequency design, there is a contradiction. If we wish to realize the factor \(j\) exactly, then \(H_i(\omega)\) should be of the form \(\Sigma d_i \sin(i\omega)\); this, however, is identically zero at \(\omega = \pi!\) This can only be overcome by introducing a half sample delay as required in the MRE design. Admitting this necessity, we start with the function

\[
H_h = \pi \sum_{i=1}^{n} c_i \sin((i-1/2)\omega) - \sum_{i=1}^{n} d_i \sin(i\omega),
\]

\(n = N/2; \quad N \text{ even} \tag{18}\)
and impose the condition of maximal linearity at \( \omega = \pi \). Solving the resulting set of equations gives recursive formulas\(^77\) for \( c_i \) and \( d_i \). As an example of performance, for \( RE = -60 \) dB in the range \( 0.5\pi \leq \omega \leq \pi \) (half of the baseband!) our design requires \( N = 16 \), as compared to \( N = 128 \) for the MRE design.

A remarkable by-product of our investigations is a novel and efficient architecture for DDs for variable frequency range of operation, where, simply by changing the tap, one can obtain a desired range. It is also possible to have a single architecture for low as well as midband operation. These findings have opened up the possibility of fabricating a universal differentiator chip, and has led to patent application\(^26\), which is currently in the final stages of processing. Some details of this innovation are available in two review papers\(^89,81\).

Another by-product of these investigations, which is of great academic as well as practical interest, is that we have been able to derive a complete picture\(^62\) of the interrelationships amongst the DD, the DHT and the half band LPF. Using these, we have shown that the design of one member of the family can be transferred to that of another in a very simple manner.

13 Optimum receivers for analog communication

The problem of optimization of receivers for various kinds of analog modulation schemes, may perhaps be considered to be amongst the earliest areas of work at IIT, Delhi, in the broad area of statistical signal processing. The motivation for this research problem was provided by some interesting developments that were taking place in the estimation theory literature at that time. First, a state variable based Markovian representation of signals had been shown to be eminently powerful for the development of optimal, recursive filtering algorithms for processing of noisy signals. Simultaneously, it provided a unified framework for the development of recursive algorithms not only for prediction and filtering, but also for smoothing, which, for the price of a nominal but fixed lag between the observed and processed (estimated) signals, offered the possibility of a significantly improved performance in terms of mean squared value of the estimation error\(^43,84\).

Based on this realization, we proceeded to carry out a formal derivation of a new class of analog demodulators, called by us “fixed-lag demodulators” for the various linear and non-linear modulation schemes used in analog communication\(^85-89\). It was shown that it is possible to obtain stable, realizable structures, which through the introduction of a nominal delay in the receiver, could approximately yield the ideal performance of the unrealizable infinite delay receiver. While this by itself is no surprise, the novelty of this approach was in the design of the specific processor which could do this job. An in-depth performance analysis further showed that the required lag is of the order of the time constant of the message signal of interest for the case of linear modulation schemes like AM and PAM, but could be considerably smaller for non-linear modulation schemes like FM and PM, if associated with a large modulation index\(^87\).

Later, we went on to apply this theory to a variety of communication systems. In particular, we looked into the design of optimum demodulators for “direct detection optical communication receivers”, for the demodulation of AM and FM signals in ‘fading channels’, and finally for the demodulation of more complex, composite modulations like PAM/FM, PPM/FM, etc., used in the some telemetry applications\(^90\). In another related work, we looked at the problem of optimizing the receiver design for situations when the modulators/transmitters are associated with a class of undesirable but known saturation-type non-linear distortions. It was found that such optimization can be effected via the introduction of similar non-linearities in appropriate places in the receiver\(^90\).

In yet another study, we investigated the case of the so-called impulsive noise, which is usually fatal in the sense, that it completely masks the message during the interval of its occurrence while also saturating the receiver. One of the practical solutions is to blank out the receiver whenever such a noise impulse occurs, thus saving the receiver from going into saturation. However, there did not exist any known formal mechanism either for the detection of the occurrence of these impulses (required for blanking purposes), or more importantly for the estimation of the message in this interval. We were able to propose a framework for the solution of this problem, and developed an optimum receiver structure for doing this based on a state variable formalism\(^91\).

14 MTI filters

An MTI filter is used in a radar system to eliminate “clutter” arising from undesired echoes from stationary objects. The classical MTI filters consist of simple, single delay line or double delay line cancellers. The performance of MTI filters is usually measured in terms of the ‘improvement factor’, defined as the ratio of the signal to clutter ratios at the input and output, respectively. While, the conventional MTI filters are able to reject stationary clutter very well, they are not very effective against certain mov-
ing sources of clutter such as clouds and trees, which cause a spread in the clutter spectrum. In some excellent papers, Capon\textsuperscript{92} and later Hsiao\textsuperscript{93}, suggested an optimization procedure which optimized MTI filters of arbitrary orders against this class of clutter with a known power spectrum, with only a marginal deterioration in the performance for stationary clutter.

In our work, we were concerned with two important related problems, viz., the optimization of performance against moving clutter without sacrificing performance against stationary clutter, which in most situations is likely to constitute the major component of the total clutter power and secondly, optimization of MTI filters against moving clutter without making detailed assumptions regarding the clutter power spectrum in order to make them robust to variations in the nature of the clutter. For the first objective, we formulated a constrained optimization problem which, while carrying out the required optimization of the MTI filter against moving clutter, would also guarantee a complete elimination of the stationary clutter. It was shown that the design of the new MTI filters could be carried out via the solution of “a modified eigenvalue problem”, involving the clutter covariance matrix A, an appropriate projection matrix P ensuring the null at the fixed clutter frequency, and a “visibility region” matrix B specifying the visible region of interest for the target Doppler. We showed that the optimum filter coefficients are given by the eigenvector corresponding to the smallest eigenvalue of the modified eigenvalue problem\textsuperscript{94} as follows.

\[
\text{PAP}_x = \delta \text{PBP}_x
\]  

On the basis of this theory, we designed a variety of MTI filters having these features and yielding large improvement factors in the presence of both moving and stationary sources of clutter.

For many practical situations, where assumptions regarding the knowledge of the clutter power spectrum are not warranted due to its nonstationary nature, we were looking for an analytical approach which required only some broad specifications like the ‘bandwidth’ of the clutter spectrum. We could eventually obtain such a solution based on a worst-case design approach, in the sense that it tries to maximize the improvement factor corresponding to the most unfavourable clutter characteristics satisfying the bandwidth constraint. For analytical tractability, we proposed the use of a Gabor-like notion of bandwidth for discrete-time sequence \( \{ a_j \} \) for setting up the bandwidth constraint:

\[
\text{BW} = \frac{1}{2\pi} \left| \sum_{i=1}^{n-1} (a_i - a_{i+1})^2 / \sum_{i=1}^{n} a_i \right|^{1/2} \leq B \quad \ldots (20)
\]

It was then possible to formulate the worst-case design problem again in terms of the eigenvectors of a simply constructed tridiagonal matrix \( G \) obtained from rewriting Eq. (20) above as a ratio of quadratic forms. It was shown that the approach could be used for both the regular and staggered PRF (pulse repetition frequency) radars. Further the worst-case improvement factors were found to be comparable with those obtained for a specifically optimized solution\textsuperscript{95}.

15 CFAR detection

Another problem arising in the detection of radar signals is the variability in the properties of noise and clutter in terms of its power distribution. System design in such situations usually requires the maintenance of a constant false alarm rate (CFAR) in the automatic detection of a target. A number of ingenious ideas had previously been suggested for the regulation of false alarm rate for some classes of clutter distributions such as the log-normal or Weibull. An important contribution to this problem was made by us by way of developing an adaptive detection procedure by which the detection threshold is so adjusted as to provide an asymptotic false alarm rate that is approximately invariant with changes in radar clutter return amplitude pdf’s (probability density functions) in a broad class. The usefulness of the method was demonstrated for the Rayleigh, the Weibull and the log-normal pdf’s\textsuperscript{96}.

16 Automatic equalization

Automatic equalization for dispersive channels is crucial in obtaining reliable performance from a digital communication system. Dispersion in the channel causes intersymbol interference, which causes a reduction in the noise margin and a consequent increase in the error probability. Although the ultimate performance criterion of interest here is the error probability, equalizers are usually optimized using a minimum mean squared error or peak distortion criterion, defined respectively as

\[
\text{MSE} = \sum_{i \neq 0} |h_i|^2 \quad \ldots (21)
\]

and

\[
\text{PD} = \sum_{i \neq 0} |h_i| \quad \ldots (22)
\]

where \( |h_i| \) is the sampled impulse response sequence of the channel after equalization.

A comprehensive theory of such equalizers was introduced by Lucky\textsuperscript{97} in the late sixties, who showed
that a transversal equalizer of length \((2N+1)\) which minimizes the worst-case or peak distortion essentially does so by forcing \(2N\) zeros in the sampled impulse response of the equalized channel. Such zero-forcing equalization is usually done via an adaptive algorithm which is a gradient-descent type optimization algorithm for the equalizer weights. Our initial work in equalization essentially suggested a mechanism for the implementation of such a “zero-forcing equalizer”.

More recently, we proposed an interesting new state space model\(^9\) for the equalization problem, in which the transmitted data are modelled as a white noise input. Equalizer structures are then developed using the theory of white noise sequence estimation from the noise corrupted output of a linear system. This involves the use of smoothing techniques for estimating the input data\(^9,10\). An important advantage of this approach of equalization is shown to be that the order of the required state space model can be made significantly smaller than the length of the impulse response of the channel, thus resulting in saving in computations.

17 Seismic signal processing

Signal processing techniques play an important role in making sense out of the enormous amount of data collected in the form of stacks of so-called reflection seismograms by the exploration geophysicists. A seismic trace is obtained by exciting a section of the earth’s surface by a finite duration seismic wavelet (via an explosion or an airgun) and recording the reflections from the subsurface layer structures, in order to reconstruct pictures of these layers and identify structures which can trap oil or other resources. The recorded seismic trace \(y_k\), however, comprises a superposition of the reflected seismic wavelets from various layers, along with noise. Thus, we can write

\[ y_k = z_k + n_k \] ...

(23)

where \(n_k\) is the noise component of the measurements and \(z_k\) is the true reflection signal. A simple model for \(z_k\) is the following convolution relation

\[ z_k = \sum_{j=1}^{k} a_j b_{k-j} \] ...

(24)

where, \(a_j\) represents the reflection coefficient of the \(j\)th layer and \(b_k\) represents the sampled value of the seismic wavelet used.

Identification of the reflectivity sequence comprises the deconvolution problem, which is usually an ill-conditioned one in the best of situations, and complicated further in offshore explorations due to the presence of multiples and reverberant reflections.

The standard method of deconvolution has been based on the use of an autoregressive model pioneered by Robinson and Treitel, which has a number of limitations, particularly for no-stationary data. We proposed the use of modern adaptive filtering techniques to eliminate many of these limitations. The simplest form of adaptive filtering comprises a continuously adaptive linear prediction operator in which the operator coefficients are updated using a simple adaptive algorithm. Motivated by these views, we considered it desirable to investigate the effectiveness of an adaptive deconvolution approach to seismic data. To start with, we investigated the effectiveness of three specific adaptive structures, viz., the adaptive tapped delay line filter, the adaptive lattice filter and the adaptive Kalman filter identifier for deconvolution of seismic data. It turned out that all the three methods exhibited superior performance to the ‘fixed structure’ predictor. In addition, we showed that the adaptive lattice filter, which has a faster convergence rate than the adaptive tapped delay line filter, also exhibited superior deconvolution performance. The adaptive Kalman filter identifier was found to be better than both the others, but is computationally more complex\(^1\)1. Later we devised deconvolution algorithms using the Fast Kalman and the least squares lattice filters to overcome this limitation of the Kalman filter identifier\(^102,103\).

The other algorithms proposed by us include the one based on the generation of a minimal order innovations model\(^10\), another based on a constant coefficient ARMA model, leading also to a recursive ARMA algorithm and a ‘Maximum A-posteriori Probability’ (MAP) criterion based detection/estimation algorithm\(^105\) using an interesting formulation of the deconvolution problem by Mendel as that of estimating the white noise input sequence of a linear system from its noisy output. Space limitations, however, do not permit a detailed description of these algorithms and their important features. In summary, we could demonstrate that one can use the modern developments in the estimation theory and system identification areas to the advantage for the deconvolution problem.

18 Array pattern synthesis and adaptive arrays

The notion of arrays of transducers for the purpose of realizing large apertures as well as flexibility in signal processing has been in existence for a long time, in areas as diverse as communications and radars (antenna arrays), sonar systems, radio-astronomy and seismic exploration systems. However,
there has been a remarkable surge of renewed interest over the last decade or so in the area of array processing for a wide range of engineering applications, both old and new such as satellite communications, anti-jam radar and communication arrays, sonar beam-forming and towed arrays (for sonar and offshore seismic explorations). The potential for real time complex signal processing has also increased greatly with the advent of LSI and VLSI devices and processors. These two factors have resulted in the need for formulating and solving a variety of new problems in array processing. These include, among others, optimization of beam patterns for irregular array geometries (as in conformal and towed arrays, for example), imposition of constraints (like directions of pattern nulls and sidelobes etc.), to eliminate known sources of interference as, for example, self noise of the propeller for a ship mounted sonar array), automatic steering of the nulls in directions of strong, unknown interfering sources like jammers etc., and a high resolution analysis of the field via short arrays in order to distinguish objects of small angles of separation.

We took up a number of above problems for investigation. First, realizing the need for a broad minimum of the array response in certain applications (such as needed in channels with angular spreading, caused by multipath) to suppress some undesirable sources, we presented a simple and elegant theory for a family of such patterns which could be easily realized via a linear array. The ideas suggested here were also shown to be applicable for the realization of an adaptive array, when the direction of the desired signal is known with some uncertainty. Later, somewhat in the same vein, we proposed the formulation of a new class of array optimization problems, in which we seek to optimize the response in a specified angular sector. The optimization of the array directivity was shown to be a special limiting case of this class of problems, as the width of the specified angular sector approaches zero. The optimum array patterns were also shown to be related to the well known prolate-spheroidal functions.

Next, motivated by the need in many applications, to design arrays with nulls and sidelobes in certain prescribed directions, we proposed a geometrical formulation of this problem for arrays of arbitrary shapes, which permitted a simple geometric method (called the method of alternating orthogonal projections) for the iterative solution of this synthesis problem. Later we generalized this work further by developing a new class of array synthesis problems whereby the array pattern is optimized in some desirable sense (like low sidelobe level or beam-width) subject to some linear and non-linear constraints say, for obtaining nulls and sidelobes in certain prescribed directions. The novel techniques introduced here were used to obtain useful optimum designs having a low equiripple sidelobe behaviour, while yielding the prescribed nulls etc., for the class of circular and arc arrays for the first time, for applications in sonar beam-forming.

19 High resolution bearing and spectrum estimation

In all the above works, the central theme of our approach was based on the premise the matrix theory based techniques provide general and rather elegant solutions to a large class of problems of interest in array processing. This is due to the fact that linear algebra is emerging as a powerful tool of analysis and synthesis with a lot of structure to it that is well understood. It was, therefore, hardly surprising to see the next major breakthrough in array processing and spectral estimation taking place with the help of this tool. Schmidt and Reddi while addressing the problem of bearing estimation of closely spaced multiple emitters from a rank deficient array covariance matrix showed via a geometric-algebraic approach, that the space spanned by the columns of the covariance matrix can be decomposed into two orthogonal subspaces, viz., that spanned by the array manifold (comprised the steering vectors of the sources whose bearing is sought) and that spanned by noise. This decomposition led, independently, to the celebrated algorithm now known by the name of MUSIC (Multiple Signal Classification), which under high SNR conditions, is capable of yielding unlimited resolution. The approach was of special significance to us, since it was reminiscent of some of the optimum MTI filtering approaches used by us, outlined earlier.

Since the original publication of the MUSIC algorithm, a lot of interest has been generated in making it more useful from a number of different points of view, such as making it applicable to a larger class of problems than those originally addressed and making it computationally efficient. The array covariance matrix is given by the expression

\[ R = AA^T + Q \]  \hspace{1cm} (25)

where, \( A \) is the array manifold of steering vectors of the \( d \) sources, \( S \) is the matrix of mutual correlations of these sources and \( Q \) is the covariance matrix of the additive noise present in the sensor signals. In the original MUSIC algorithm, the matrix \( Q \) is generally assumed to be known, except for a scaling factor. This, however, is not warranted in practice, being generally an unknown entity.
suggested, therefore, the use of a transform based covariance differencing approach, in which, the subspace analysis is done on an appropriately constructed difference matrix, such that it eliminates Q, while retaining the features of the array manifold A (Refs 112 and 113). We showed this approach to be useful for a number of other problems as well, such as the estimation of time of arrivals of superimposed wavelets and the estimation of pole locations of a system function from measured transient response data, when the additive noise is nonwhite and unknown. Later, we obtained a method for identifying the signal and noise subspaces, without the use of the eigendecomposition and involving only simple householder transformations114. This led to a significant increase in the computational efficiency of MUSIC for this class of problems.

Another related problem that we have investigated in detail, is concerned with the effect of multipath propagation on the high resolution bearing estimation algorithms. For this, we have suggested a new spatial smoothing algorithm, which improves upon the previously proposed technique, by reducing the loss in the array aperture inherent in spatial smoothing. The new technique, by virtue of its application of smoothing in both the forward and backward directions, makes more efficient use of the aperture, as clearly demonstrated by us. Later we developed an approach for the identification of the coherency structure of the sources based on this smoothing technique, and showed that by looking at the ranks of a sequence of smoothed covariance matrices, we can determine the source coherency structure in an efficient manner115. Finally, we have also suggested a class of spatial smoothing techniques for the recently proposed and powerful ESPRIT algorithm which considerably enhances the scope of its application116.

Most of the above work carried out by us for the bearing estimation problem is applicable with equal ease to the high resolution estimation of frequencies of sinusoids embedded in white Gaussian noise, since the two problems are essentially similar117. A particularly interesting method of frequency estimation, which requires very few computations, was proposed by us based on an iterative auto-correlation method. The method was shown to work well even for low SNR situations118.

20 Sonar signal processing

Over the years, we have also been motivated to look into several important problems concerning the active and passive sonar signal processing. The problem of time-delay estimation has attracted a great deal of attention in recent years119. The accurate estimation of time-delay between signal wavefronts arriving from a source at two or more geographically separated sensors is very important for passive sonar methods of source localization. We investigated the effect of medium scattering (such as occurring in acoustic propagation of the oceans), due to its practical significance. We showed that there exists an optimal separation of the sensors, for which the performance of both the range and bearing estimates based on time-delay estimates is optimum, and also provided a method for calculating this optimal separation119.

Most recently, we have taken up investigations into various aspects of towed array signal processing, with a view to helping in the indigenous efforts being made to develop towed array systems in the country. Some of the problems that arise specially in the signal processing of towed arrays include (i) distortion of the array shape in an unknown manner due to the hydrodynamical forces of the ocean and the motion of the towing vessel, (ii) presence of a type of a high wavenumber noise caused by the turbulence around the array, called flow noise, and (iii) loss of spatial coherence of the signal over the length of the array due to multipath. All these effects cause loss in signal deductibility as well as in the accuracy of bearing estimation. In the work that we have done, we have suggested some algorithms for the estimation of the array shape for the purpose of accurate beam-forming. We have also developed a general technique for the study of the spatial coherence loss across the array aperture for a specified velocity profile in the ocean, and analysed its effect on the performance loss of the conventional and optimal array processors. Regarding flow noise, we have investigated its reduction via the use of special signal processing techniques, whereby each element of the array is realised as a cluster of sensors. Finally, we have investigated the robustness of the conventional beam-former and of the eigenvalue based methods for the detection of low SNR signals via towed arrays120-122.

21 Statistical modelling and adaptive signal processing

In many applications concerning time-series analysis and stochastic signal processing (as in the deconvolution problem discussed earlier), one of the fundamental issues of interest is that of modelling a given stochastic sequence. This requires finite parametrization of the infinite auto-correlation of a given time series. The autoregressive moving average (ARMA) models are an important class of such parsimonious approximants. The literature is rich with a host of techniques for the identification of
ARMA models, most of which, unfortunately, are either too complex or yield only a suboptimal solution. This is essentially due to the non-linearity of the identification problem.

In some very recent work, we have attempted to explore the structural properties of an ARMA process in depth, starting from first principles. By regarding the random variables as vectors in a Hilbert space, some interesting and useful results have been obtained concerning the nature of multiple-step ahead predictors of a time series. In the course of these investigations, an entirely new representation for these processes is obtained, called by us a predictor space representation (PSR), which has a number of useful and interesting properties\textsuperscript{123,124}. Unlike the parameter space associated with the conventional ARMA representation, that associated with this representation is a linear one, which helps us in obtaining a new, nearly optimal recursive procedure for the estimation of ARMA parameters and solving the so-called ARMA filtering problem\textsuperscript{124}.

22 Digital communications

Signal design and signal processing for digital communications have been a problem of recurrent interest for us. In addition to some new results in the area of equalization via new state space approaches as detailed in Sec. 17, we looked at the problems of carrier recovery in the presence of phase jitter, and the possibility of combining the tasks of adaptive equalization, carrier recovery/phase jitter compensation and echo signal cancellation on the basis of optimization of a single global error criterion. We showed that there are several alternatives for positioning the various components of the receiver in the global structure\textsuperscript{125}. A new receiver structure involving passband decision feedback equalization and carrier recovery/phase jitter compensation is proposed and shown to be a viable structure giving some performance gains over the other structures\textsuperscript{126}.

We also considered the problem of mitigating intersymbol interference when associated with non-Gaussian noise. A basic theory for this subject has been developed by considering several approaches for the design of what are called 'locally optimum receiver structures'\textsuperscript{127}. We proposed several alternative structures, including among them, a decision feedback receiver, a composite-hypothesis testing MAP receiver and a maximum likelihood receiver. We also derived an efficient, though suboptimal, Viterbi-algorithm implementation of the maximum likelihood receiver. Finally, we derived performance bounds for the non-linear receivers for some typical non-Gaussian noise distributions and channels\textsuperscript{127}.

There has been a lot of interest, of late, in code division multiple access systems such as in satellite and spread spectrum applications. For these applications, it is required to use binary sequences, which have a good (spiky) auto-correlation function, small cross-correlation values with codes of other users in the system and which must be difficult to decipher from a partial knowledge of the sequence for the purpose of secrecy. Some of the well known sequences having the first two of these properties are Gold sequences, Kasami sequences etc. Unfortunately, these are not good sequences from the point of view of communication secrecy, since they are known to have a very small 'equivalent linear span', which is a measure of the ease with which these can be completely determined from a partial knowledge of the sequence.

We obtained some very exciting results in the design of such binary cipher sequences with good auto-correlation function for cryptographic applications, i.e., which simultaneously have a large value for the 'equivalent linear span'\textsuperscript{127-130}. Later, we obtained, using a non-linear construction procedure based on interleaved \textit{m}-sequences, another class of sequences which have good auto- and cross-correlation properties over the set, and also have a large value of linear complexity, thus making them difficult to break.

23 Current research and future plans

Current DSP activities at IIT, Delhi, are concentrated on two specific problems viz., that of efficient designs of second and higher order digital differentiators, and that of polynomial computation and interpolation in a computationally efficient manner. Future plans of the DSP group include work on neural computation in the context of DSP; further work on digital differentiators having maximal linearity at an arbitrary frequency; alternative optimization criteria; and work on digital integrator design.

The current research activities in the statistical signal processing include, among others, array processing, speech and image processing, spectral estimation and adaptive signal processing. In array processing, we are presently in the process of developing a class of spatial state space techniques for the bearing estimation problem, which promise a number of interesting features such as computational efficiency and choice of spatial smoothing operations at several levels suitable for coherent sources environment.

In speech processing, we are looking into the problem of segmenting a speech waveform into its elementary units, which may be at the phonetic or subphonetic level. Several new approaches are be-
ing attempted to carry out this segmentation, including the one based on the use of statistical distances between adjacent blocks of speech data, and another based on the use of an artificial neural network. In image processing, we are interested in investigations into the use of morphological operations for eliciting desired information subset (such as a network of roads) from a given map.

The current effort in spectral estimation and time series modeling is devoted to the cases of multichannel and multidimensional signals. Our main concern here is to devise techniques for multichannel parameter estimation which are amenable to parallel processing. We are interested in the problems associated with higher order spectra.

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