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Strong diffusion of resonant electrons by VLF waves

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Various criteria for strong diffusion of magnetospheric electrons under the influence of mid-latitude whistler mode VLF emissions are analysed in detail under the assumption that the waves propagate along the external magnetic field. It is pointed out that strong diffusion is unlikely below 59° geomagnetic latitude ($L = 3.8$) and diffusion is strongest at $L = 4.5$.

1 Introduction

The cyclotron resonance of energetic electrons with whistler mode waves propagating parallel to geomagnetic field plays an important role in diffusion of these particles into the lower ionosphere. During the process of resonant interaction the pitch angle (α) of an electron may become lower than a critical value (α_0, the equatorial loss cone pitch angle) and these electrons may precipitate into the lower ionosphere causing D and E region perturbations, X-ray emission, light and heating[1-3]. In cyclotron resonance the Doppler-shifted wave frequency seen by the electrons is equal to the electron gyrofrequency $[2\pi(f_H - f) = k_\parallel \, v_\parallel]$, where f_H is the electron gyrofrequency, f the wave frequency, $k_\parallel$ the component of wave vector parallel to magnetic field, and $v_\parallel$ the particle's velocity component parallel to the earth's field. In this case the electron experiences an approximately stationary wave field for an extended period of time resulting in energy exchange with wave fields and thereby causing diffusion in pitch angle and energy.

Various workers have put forth different theories (and formulae) to measure the diffusion of electrons. These can be summarized into four: (1) loss cone filling criterion, (2) integral flux criterion, (3) comparison of electron life times, and (4) comparison of wave magnetic intensity with a standard intensity. It may be pointed out that since all these theories are based on different definitions of 'strength of diffusion', these may contradict each other in some regions[4].

According to the criteria of Kennel[5] and Kennel and Petschek[6], the strength of diffusion is based on population of loss cone. In a weak diffusion regime the loss cone is filled only near its boundaries while the rest of the loss cone is empty. In this case pitch angle distribution increases exponentially from $\alpha = 0$ to $\alpha = \alpha_0$. In strong diffusion case, pitch angle distribution remains almost isotropic. To measure the diffusion of the electrons, Kennel and Petschek[6] introduced a parameter Z which is expressed as

$$Z = \alpha_0 / \sqrt{D \, t_E} \qquad \ldots (1)$$

where D is the diffusion coefficient and t_E the escape time of electron. In this case if $Z \gg 1$, the diffusion is said to be weak and if $Z \ll 1$, the diffusion is said to be strong.

Etcheto *et al.*[7] considered integral flux (J) responsible for strong diffusion instead of wave magnetic intensity. They defined strong diffusion as a state when J is approximately a linear function of source intensity S, i.e. $J = J_0 \equiv a\,S$, where a is a constant. The source intensity S will possess higher value for strong diffusion and lower value for weak diffusion. They introduced a parameter b (known as limiting intensity of particle source) to measure the diffusion and assumed a dynamic equilibrium in which the waves are continuously generated and particles are continuously injected and lost in the atmosphere due to pitch angle diffusion into the loss cone. The injection of particles is accounted for by a term dn_2/dt (known as source intensity) which is the rate of entry of new particles; here, n_2 is the density (el. m^{-3}) of injected (hot) particles. The concept of diffusion was, then, classified as under:

$$b > \frac{dn_2}{dt} \quad \text{weak diffusion}$$

$$b < \frac{dn_2}{dt} \quad \text{strong diffusion}$$

The diffuison of electrons can be measured by comparing their life times T_L with $T_{L\,min}$, $T_{L\,min}$ being their life time in strong diffusion limit[8]. If $T_L/T_{L\,min} > 1$, it is weak diffusion, and if $T_L/T_{L\,min} < 1$, it will be strong diffusion.

Recently, Prakash[9] introduced a new criterion based on the comparison between the actually observed wave spectral density B_f^2 and the limiting spectral density (i.e. that required for strong diffusion) b_f^2. He concluded that if $b_f^2 > B_f^2$, the diffusion is weak, and if $B_f^2 > b_f^2$, the diffusion is strong. He computed b_f^2 by replacing dn_2/dt by b (limiting rate of electron injection at equator) in Etcheto *et al.*'s expression for B_f^2 (see Section 2).

The aim of this paper is to analyse various criteria for strong diffusion in some more detail based on the actual observations of VLF emissions at middle latitudes.

2 Method of calculation and ionospheric model

The equatorial loss cone pitch angle (α_0) is calculated at mirror height 110 km, the base of ionosphere (see Ref. 3 for details). For normalized frequency $x(=f/f_H)$, the wave energy spectral density formula can be written as[7]

$$B_f^2(x) = \frac{10.62 \times 10^{-6}}{\sqrt{n_0}} \cdot \frac{\mathscr{L}}{\log(R^{-1})} \cdot H(x) \cdot \frac{dn_2}{dt} \quad \ldots (2)$$

Limiting source intensity (particle injection) is given by

$$b = \frac{0.818 \times 10^{12} \cdot \log(R^{-1})}{\mathscr{L} \cdot L^4} \quad \ldots (3)$$

For $x \ll 1$, $H(x)$ is written as[9]

$$H(x) = 2.54 \times 10^{-14} \cdot n_0^{3/2} \cdot L^{9/2} \quad \ldots (4)$$

Here R is the wave power reflection coefficient, $\mathscr{L}$ the length of the interacting region, and n_0 the background plasma density. If we replace dn_2/dt by b, limiting wave spectral density will be

$$b_f^2\,(pT^2/Hz) = 22.07 \times 10^{-8} \cdot L^{1/2} \cdot n_0\,(el.\,m^{-3}) \ldots (5)$$

Assuming $R = 0.1$ and $\mathscr{L} = L\ R_E$ (R_E being the earth's radius), Etcheto *et al.*[7] simplified expression for b to

$$b = 3 \times 10^5/L^5\,(el.\,m^{-3}\,s^{-1}) \quad \ldots (6)$$

The cold plasma density is taken from experimental profiles obtained by Angerami and Carpenter[10] (see Fig. 11 of Ref. 10), employing the whistler technique during moderate storm periods ($K_p = 2$-4). The densities are 4000, 1000 and 400 el. cm^{-3} for L values of 2.42, 3.0 and 4.0 respectively, and 16 el. cm^{-3} for $L = 5$ (plasmapause boundary). Prakash and Singh[11] have used these values in their mid-latitude VLF intensity calculations. As most of the VLF events are observed in 4-6 kHz range, we use 5 kHz as interacting wave frequency[12].

3 Results and discussion

Based on Eq. (5) and the observed electron densities, we calculated limiting wave intensity (b_f^2) for 5 kHz frequency at $L = 2.42$-5.00. The values are found to be in the range 1368-106 mγ^2/Hz (1 pT^2/Hz = 1 mγ^2/Hz) and indicate that strong diffusion based on the criterion by Prakash[9] (and Etcheto *et al.*[7] as well) is unlikely to take place. It may be mentioned here that improved models by Sazhin[13,14] produce almost similar expression for B_f^2. Hence b_f^2 value will remain almost unaffected. In the subsequent discussion we consider other criteria for strong diffusion.

Bullough *et al.*[15-17] have made a detailed study on the global occurrence and intensity of VLF emissions based on Ariel 3/4 data. Three main zones of intense emissions have been observed in both hemispheres, the first of which was located at invariant latitude above 70°, the second near 50° and the third below 30°. They predicted that the middle latitude (second zone) emissions were generated by the cyclotron instability. Such intense emissions in the same zone are also observed by other satellite such as OGO 2/6 (Refs 18 and 19), Injun 3/5 (Refs 20 and 21), and Allouette 2 (Ref. 22). The intensities of VLF emissions observed by all these satellites were found to be in the range of 10^{-13}-10^{-11} W m^{-2} Hz^{-1}. These values correspond to $1.14 \times 10^{-6}\ \gamma^2$/Hz at $L = 4$ and $4.75 \times 10^{-7}\ \gamma^2$/Hz at $L = 5$ if we apply the following conversion formula[15]

$$1\ \gamma^2/Hz = 3 \times 10^{-3}/4\pi\mu\ W\,m^{-2}\,Hz^{-1} \quad \ldots (7)$$

Here μ is the refractive index. Sazhin[13], like us, assumed a peak power of $1.00 \times 10^{-6}\ \gamma^2$/Hz at $L = 3.6$ for 5 kHz waves in the same zone.

Like various theories of strong diffusion, there exist following different formulae for the calculation of diffusion coefficient D depending on various factors (L, B_f^2, f_H and f): (1) Fokker-Planck's expression[23], (2) Sazhin's expression[13], (3) Roux and Solomon's expression[24], (4) Torkar *et al.*'s expression[25], (5) Kennel and Petschek's expression[6], and (6) Tsurutani *et al.*'s expression[26].

We first calculated parameter Z by applying Eq. (1) and the well known Fokker-Planck's expression which can be written as[23,27]

$$D(rad^2/s) = 1.03 \times 10^{-2}\ B_f^2\,(m\gamma^2/Hz) \quad \ldots (8)$$

The escape time of electron is computed from the following formula[9]

$$t_E = L\ R_E/V_r \quad \ldots (9)$$

where V_r is the resonant velocity of the electron and R_E the earth's radius (6370 km).

As discussed above we have a maximal wave intensity $1.14\times10^{-6}\ \gamma^2$/Hz at $L=4$. Resonant velocity $[V_r=2\pi(f_H-f)/k]$ at $L=4$ is 0.19×10^8 m/s; here, k is the wave number. These values give Z parameter at $L=4$ to be 0.766, indicating strong diffusion. The Z value at $L=3.6$ comes out to be 1.33 which makes the diffusion less likely in this region, but it is 0.578 at $L=5$ suggesting strong diffusion. Variation of Z with L parameter is shown in Fig. 1. It is clear from the figure that diffusion increases with L and that the strong diffusion is less likely to take place at $L\leqslant3.8$.

The diffusion coefficient expression of Fokker-Planck depends upon B_f^2 only. Now we use D expression of Sazhin[13] which is a function of f, f_H and B_f^2. The expression is

$$D=4\,\pi^2 f\cdot f_H\cdot B_f^2/B_0^2 \qquad \ldots (10)$$

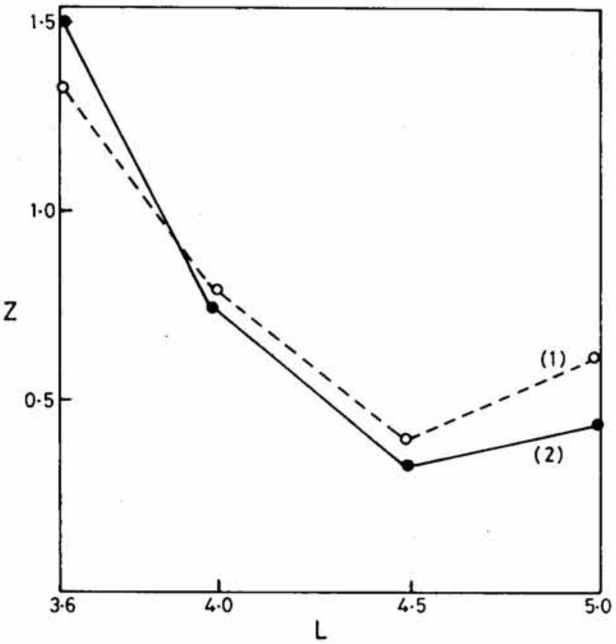

Fig. 1—Variation of Z with L by applying (1) Fokker-Planck's expression, and (2) Sazhin's diffusion coefficient expression

Here B_0 is the induction of the magnetospheric field. Eq. (10) yields Z values to be 1.494 and 0.735 at $L=3.6$ and 4.0 respectively, supporting the result that strong diffusion is possible above $L=4$ only. The variation of Z values from Eq. (10) with L is also depicted in Fig. 1. The graph here too gives impossibility of strong diffusion below $L=3.8$.

Roux and Solomon's diffusion coefficient expression is a function of not only B_f^2 but also of wave group velocity (V_g) and V_r (Ref. 24). This expression gives D values at considered values of L to be 1.7% greater than those given by Eq. (10). Hence Z values are nearly the same, supporting our result. Diffusion coefficient expressions of Kennel and Petschek[6], Tsurutani *et al.*[26] and Torkar *et al.*[25] produce the same result.

The $T_L/T_{L\ min}$ ratios also favour our conclusion. The T_L values are calculated by equating them with $1/D$ (Refs 26 and 27). The value of $T_{L\ min}\,(=2\,t_E/\alpha_0^2)$ (Ref. 9) in these cases are 291 and 1321 at $L=4$ and 5 respectively giving $T_L/T_{L\ min}$ ratio to be 0.290 at $L=4$ and 0.155 at $L=5$, indicating strong diffusion at these L values. The values of D, T_L, $T_{L\ min}$ used here are given in Table 1.

Tsurutani and Smith[28] gave the following expression for wave power (B_w) required to produce strong diffusion

$$B_w^{SD}=25\,E^{1/4}/L^{7/2} \qquad \ldots (11)$$

Here SD stands for strong diffusion and E is the electron energy in keV. The calculated wave amplitude (B_w) comes out directly in gammas (γ). Resonant energy (E) of 3 keV at $L=3.6$ requires 27.6 mγ^2/Hz intensity for strong diffusion, whereas resonant energy of 1 keV at $L=4$ needs 7 mγ^2/Hz intensity. These values clearly reject strong diffusion at $L=3.6$.

We have shown that "comparison of lifetimes of electron" and 'loss cone filling' criteria clearly favour strong diffusion at $L=4$. It may be questioned why models of Prakash[9] and Etcheto *et al.*[7] do not produce strong diffusion at considered L values. The model of Prakash[9] has one serious drawback in addition to those pointed out by Sazhin[4]. Etcheto *et*

Table 1—Values of D, T_L, α_0 and $T_{L\ min}$ used in the present study

L	α_0 rad	D $10^{-2}\times$ rad^2.s^{-1}	T_L s	$T_{L\ min}$ s	$T_L/T_{L\ min}$
4.0	0.0959	1.17	85.47	291	0.29
4.5	0.0797	1.40	71.40	1061	0.07
5.0	0.0675	0.49	204.5	1321	0.16

al.[7] never considered wave energy spectral density as a measure of diffusion strength, whereas Prakash[9] does so though his work is based on Etcheto *et al.*'s theory. Moreover, as discussed in Section 1, in the case of strong diffusion, pitch angle distribution of charged particles in the equatorial magnetosphere becomes isotropic which causes wave damping which is against the very concept of Etcheto *et al.*'s theory and as such ordinarily there should exist no region of strong diffusion so far these two models are concerned. This can be checked by putting values of observed electron density n_0 at various values of L in the b_T^2 expression of Prakash[9].

In fact, in the case of strong diffusion by these models, the zero order approximation, which is the basis of Etcheto *et al.*'s theory, remains no more valid and at the same time the mean anisotropy, $A(x)$, of distribution function decreases markedly. Hence, it is quite clear that present expressions of Prakash[9] and Etcheto *et al.*[7] are not applicable to studies on strong diffusion.

The Z versus L variation in Fig. 1 and other factors clearly indicate that strong diffusion is unlikely below $L=3.8$ (59° geomagnetic latitude). It is noticeable that trends of Z versus L suggest strongest diffusion around $L=4.5$. We have confined our study up to $L=5.5$. The regions above $L=5.5$ (above 65° geomag. lat) lie in the first zone vicinity (high latitude zone of intense VLF emissions). Most of the factors (α_0, $T_{L\ min}$, B_w^{SD}) favour strong diffusion in this zone also, but to verify it the author does not possess VLF intensity data. The third zone (of intense VLF emissions, i.e. low latitude zone) has already been shown as a weak diffusion region by Prakash[9] and Prakash *et al.*[29].

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