On Jovian magnetospheric natural instability*

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In this paper an attempt has been made to study the natural instability of Jovian magnetosphere from an analysis of the electron and whistler wave interactions. It is observed that for the wave frequency of 3-300 kHz, the resonant energies fall in the range 5-50 MeV where extensive electron flux measurements have been carried out by Pioneers 10 and 11, and Voyager 1. Further, the results clearly indicate how the features in the whistler wave spectrum selectively interact with well-defined subsets of the trapped energetic electron population.

1 Introduction

Knowledge about Jupiter's magnetosphere first became available during the early age of exploration of the earth's Van Allen belts, when remote measurements of decimeter emissions from Jupiter were made\(^1\) and interpreted in terms of synchron radiation from an extremely energetic electron population trapped in a magnetic field considerably stronger than the earth's field. The dynamics of the inner magnetosphere of Jupiter is conceptually similar to that of the earth, despite the vast difference in the size and energy of the trapped particles. On the basis of radio observations, it is now well established that Jupiter has a substantial magnetosphere with a large population of energetic trapped electrons. Many workers\(^2-4\) developed, for the first time, the models for the Jovian radiation belts by assuming that electrons and protons are injected from the magnetosheath or the tail with high magnetic moments, and that they diffuse inward (conserving magnetic moment) to attain very high energies in the inner radiation belt of Jupiter. Coroniti\(^5\) analyzed theoretically the initial Pioneer data by doing a number of more detailed studies which conclusively established the importance of local loss mechanisms associated with wave-particle interactions at Jupiter. Barbosa and Coroniti\(^6\) carried out the full relativistic calculations for a given model and have compared their predictions with Pioneers 10 and 11 data. Later on, this analysis was followed by many workers\(^7-13\). Near and within the \(L_0\) plasma torus, the Voyager 1 plasma-wave instrument detected a broadband turbulence spectrum with upper frequency much less than the local electron cyclotron frequency\(^14\). These spectra are identified as whistler mode waves, and are used to study the plasma density and structure of the Jovian magnetosphere.

The Voyager 1 mission provided the first opportunity to examine directly wave-particle interaction phenomena within the magnetosphere of Jupiter and in the extensive region of disturbance upstream from the planet. The Voyager 1 plasma-wave instrument detected low-frequency radio emissions, ion acoustic waves and electron plasma oscillations for a period of three months before encountering Jupiter's bow shock. The Voyager 2 flyby of Jupiter, which occurred in July of 1979 provided the second opportunity to study plasma waves in the vicinity of Jupiter. Because of its different trajectory, the Voyager 2 mission provided new perspectives for analyzing many of the phenomena detected by Voyager 1 (Ref. 15). Some interactions in the \(L_0\) plasma torus resembled those found in the earth's plasmasphere. Near and within the \(L_0\) plasma torus, the instrument detected high-frequency electrostatic waves and strong whistler mode turbulence (chorus, hiss, and impulsive signals that appear to be associated with lightning). Electrostatic emissions related to the electron gyrofrequency harmonics and upper hybrid resonance were also detected beyond the boundary of the torus and near the magnetic equator crossings. The radiometric observations were made to infer characteristics of energetic electrons in the inner magnetosphere\(^16\), while the remote measurements did not yield any direct information about the fluxes of lower-energy electrons, distribution of ions—the mechanism responsible for charged particle acceleration and transport, or the dynamical phenomena of importance in the outer magnetos-
phere of Jupiter. Kennel developed dynamical models with whistler mode instabilities that would control the dynamics of the Jupiter's radiation belts and lead to stable the trapping limits for electrons. Thorne and Coroniti discussed a self-consistent Jupiter-radiation-belt model in which the cyclotron and lead to stable the trapping limits for electrons, control the dynamics of the Jupiter's radiation belts. They assumed that the solar wind provided a source for the radiation belts, and they applied the concept of inward radial diffusion with the conservation of magnetic moment. Making use of currently available theory of wave absorption, Ahmad et al. have discussed the absorption of very low frequency (VLF) waves in Jovian ionosphere during day and nighttime for frequencies of 2, 5 and 10 kHz. Recently, Winckler observed many effects on the natural trapped particles by echo experiments and provided more details of the behaviour of trapped electrons under conditions leading to their decay and loss. Pitch angle diffusion results in the migration of the trapped population into the loss cone and precipitation into the auroral-zone atmosphere. Thereby, decay and loss of electrons from the magnetosphere attributes to the gap in electron flux. The effect increased when the magnetosphere was more disturbed. Similarly, Namzek and Winckler give new insight into the motion of natural electrons in the outer Van Allen radiation belt. In the present paper an attempt has been made to study the natural instability of Jovian magnetosphere by analyzing the electron and whistler wave interactions.

2 Theoretical considerations

The gap in electron flux is generally attributed to an enhanced loss of electrons from the magnetosphere. Most likely, this results from pitch angle scattering of electrons into loss cone, thereby causing their precipitation into the atmosphere. Alternatively, the electrons can be removed through a resonant interaction with waves oscillating near their Larmor frequency. Such whistler mode waves are naturally generated during lightning discharges in the atmosphere, and their subsequent escape to the magnetosphere permits the resonant interaction (with the electrons) to occur. The satellite observations of electromagnetic VLF waves, deep within the earth's magnetosphere, however, have shown these sferically generated whistler to comprise only a minor portion of the total wave energy. Rather, the plasmasphere is almost continuously filled with a relatively broadband hiss. We show that this emission is probably generated by cyclotron resonance with the Jovian magnetospheric electrons themselves.

A necessary condition for this electromagnetic wave growth is that the electron distribution function be anisotropic with more energy perpendicular than parallel to the geomagnetic field. The instantaneous growth rate for this resonant interaction is proportional not only to the pitch angle growth rate, but also to the fractional number of particles near resonance. Using the fact that for a given energy electron, one may expect the frequency of this natural emission to increase significantly with decreasing L. Tverskoy suggested that the gap in electron flux was controlled by the degree to which these were absorbed in the ionosphere. As long as the wave frequency is well above the local proton gyrofrequency, wave absorption is known to decrease with decreasing frequency. At intermediate frequency it may be possible for a significant fraction of the wave energy to be reflected back into the magnetosphere without suffering appreciable absorption. There is, however, a basic error in the above argument of Tverskoy that waves are always guided by the geomagnetic field. This is more or less correct at high frequency in the absence of ducting, but becomes invalid whenever the wave frequency is below the local hybrid frequency of the propagating medium. Most of the principal features of whistler propagation can be deduced from the magnetotonic theory. The whistler energy travels with the group velocity and in a direction called the ray direction. Since the direction of the ray and that of the wave normal are not, in general, parallel, the velocity of the wave packet travelling in the ray direction will differ from the group velocity defined for the wave normal direction. Most of the whistlers observed on the ground are explained by magnetoionic guiding results from the anisotropic of the medium and by geomagnetic ducting due to the presence of field-aligned irregularities in the ionosphere. Low-frequency whistler mode waves can, in fact, propagate in a direction perpendicular to the magnetic field, enabling such waves generated near the equatorial plane to reflect and remain quasi-trapped deep within the magnetosphere. It is feasible that near-perfect magnetospheric reflection of these waves could account for the extremely low electron fluxes. However, the effective reflection coefficient might still be reduced by the resonant damping of waves as they propagate through the magnetosphere, but the observed features of the natural hiss band suggest this damping to be small.

The growth of electromagnetic radiation is generally effected by the presence of a positive slope of the electron distribution function in perpendicular velocity space. Therefore, the electron distribution function may be evolved due to diffusion process in
this velocity space at the time of growth of these radiations. This process can be responsible in driving the electrons into the velocity space to reduce the gradient \( \partial F_e / \partial v \), where \( F_e \) is the electron distribution function which may be expressed as

\[
F_e = \frac{n}{\alpha^3} \left[ \exp \left( -\frac{v_1^2}{\alpha^2} \right) \right] \left[ \exp \left( -\frac{v_1^2}{\alpha^2} \right) \right] \exp \left( -\frac{v_1^2}{\alpha_i^2} \right)
\]

Here, \( \alpha \) denotes the dispersion of momentum per unit mass, \( \alpha_1 \) and \( \alpha_i \) are, respectively, the perpendicular and parallel components of the dispersion, and \( v_1 \) and \( v_i \) stand for velocity components perpendicular and parallel to the ambient magnetic field, respectively, and \( n \) the cyclotron harmonic number. In the velocity space, the relativistic effect on the cyclotron resonance condition has been discussed in detail by Wu. The inclusion of the relativistic effect on the resonance condition qualitatively changes the physical picture. The effective ranges of \( v_1 \) and \( v_i \) in the velocity integrations are limited. The portion of the distribution function of those electrons which satisfy the resonance condition has a positive gradient, i.e., \( \partial F_e / \partial v_1 > 0 \). The amplification of both the extraordinary and ordinary modes may occur. The growth rate for the ordinary mode is smaller than that for the extraordinary mode for a fixed frequency and wave normal angle. It is very important to note that the relativistic effect on the resonance condition is not negligible even for electrons with very low frequency. Physically, it is shown that in the non-relativistic case the cyclotron motion perpendicular to the ambient magnetic field does not enter the wave-particle interaction process, whereas in the relativistic case it does. Therefore, even a small factor of relativistic correction in the resonance condition may lead to a drastically different results in the stability analysis of various processes in the plasmas. The electron will experience resonant scattering when the Doppler shifted wave frequency is equal to their cyclotron frequency. This condition can be expressed in the form

\[
\omega (1 - n \beta_i) = \Omega_\gamma \gamma
\]

where,

\[
\begin{align*}
\omega & \quad \text{Wave frequency} \\
\Omega & \quad \text{Non-relativistic gyrofrequency} \\
\gamma & \quad \text{Relativistic mass enhancement factor and is given by } \gamma = \left[ 1 - (v/c)^2 \right]^{-1/2} \\
n_i & \quad \text{Component of the wave refractive index}
\end{align*}
\]

Equation (2) can be put in terms of the wave normal angle \( \theta \), the electron pitch angle \( \phi \) and the normalized wave frequency \( \omega^* = \omega / \Omega_\gamma \) as

\[
n^2 \cos^2 \theta \cos^2 \phi (\gamma^2 - 1) = [(\omega^*)^{-1} - \gamma]^2
\]

The frequency of the observed natural hiss-band emission is found to be within the plasmasphere and is always found well below the plasma frequency \( \omega_p \) and the electron gyrofrequency \( \Omega_\gamma \). Under this condition, the whistler mode wave refractive index can be expressed as

\[
n^2 = \eta (\omega^*)^{-1} [\cos \theta - \delta \omega^*]^{-1}
\]

where,

\[
\eta = (\omega_p / \Omega_\gamma)^2
\]

and

\[
\delta = (1 + \eta^{-1}) [1 - (\omega_{\text{hy}} / \omega)^2]
\]

The parameters \( \eta \) and \( \delta \) are functions of the local cold plasma medium. Along the magnetic equator, \( \eta \) is typically between 10 and 100, although it can become unity outside the plasmapause and at higher geomagnetic latitudes. The value of \( \delta \) depends strongly on frequency. It passes through zero when \( \omega = \omega_{\text{hy}} \), the lower hybrid frequency, and for \( \omega > \omega_{\text{hy}} \), it is usually reasonable to take \( \delta = 1 \).

Combining Eqs (3) and (4) and dropping terms of order \( \omega^* \) we get the resonant wave frequency as

\[
\omega^* = \left[ \eta \cos \theta \cos^2 \phi \right]^{-1} \left[ \gamma^2 - 1 + (\delta + 2 \gamma \cos \theta) / (\eta \cos^2 \theta \cos^2 \phi) \right]^{-1}
\]

For conditions in the Jovian magnetosphere Eq. (6) has a local minimum, when \( \phi = \theta = 0 \) and is given simply by

\[
\omega_{\text{min}} = (\Omega_3 / \omega_3^2) \gamma^{-1}
\]

The energy dependence of the emitted wave frequency is contained in the terms of the scaling factor \( \varepsilon \) which is given by

\[
\varepsilon = (\gamma^2 - 1) + (\delta + 2 \gamma) / \eta
\]

This is plotted in Fig. 1 for \( \delta = 1 \) and \( \eta = 1, 10, 100 \) and \( \infty \). At higher energies the quadratic term in Eq. (8) dominates and \( \varepsilon \) becomes independent of the cold plasma parameters.

To evaluate the resonant frequency at different locations in the Jovian magnetosphere, a model for values of the total electron density \( N \) and the magnetic field strength \( B \) is to be chosen. For this purpose we used the usual dipole variation for \( \Omega_\gamma \) and adopted the reference electron density profile given by Gurnett et al. to evaluate \( \omega_p \) along the geomag-
netic equator. Although the tail field lines of the magnetosphere are found to be appreciably inflated as compared to the dipole field, indicating the effect of external current systems, but up to few $R_J$ (Jupiter radius), the night side of the Jupiter shows dipole field lines. The present results may get deviated (1-2%) due to the inclusion of non-dipolar magnetic field for $L > 5$. Kennel\textsuperscript{11} concluded that the growth rate becomes maximum at $\theta = 0$ and in most cases the major contribution to the growth rate integral stems from particles with $\phi < 60^\circ$. Thus, at any given locations, we can expect waves with frequency just above the minimum resonant frequency [Eq. (7)] to incur the most growth. The emission frequency will also increase with geomagnetic latitude. To compensate for this and to allow for the wave growth at non-zero values of $\theta$ and $\phi$, we arbitrarily assume that the typical emission on a given field line occurs at ten times the minimum resonant frequency at the equator. Thus

$$\omega_n = 10 \omega_0(L) \epsilon^{-1} \quad \ldots \quad (9)$$

where, $L$ is the dependent parameter given by $\omega_0(L) = \Omega_J^2/\omega_e^2$. The justification of using the factor 10 in Eq. (9) is that it fits well into the upper and lower boundary limits of the equatorial gyrofrequencies. The variation in this characteristic emission frequency with $L$ is plotted in Fig. 2. For each curve the energy factor $\epsilon$ has been taken from the $\eta \to \infty$ curve of Fig. 1. This provides the good representation at energies above a few 1000 keV. The behaviour of normalized wave frequency as a function of wave normal angle ($\theta$) is shown in Fig. 3. It is evident from Fig. 3 that normalized wave frequency happens to be minimum when $\phi = \theta = 0$, and increases with the increase of $\theta$, thereby anticipating the more pronounced occurrence of instability at larger angles (i.e. when $\theta = 90^\circ$) of propagation.

3 Results and discussion

The theoretical explorations of various plasma instabilities and their non-linear aspects have been carried out by many workers\textsuperscript{30,32,33} but there is still much room for further development. An excellent

**Fig. 1**—Variation of scaling factor $\epsilon$ as a function of energy for different values of $\eta$

**Fig. 2**—Quantitative plot of characteristic resonant frequency for various energy electrons as a function of Jovian $L$-values (The lower hybrid resonance frequency occurs in the shaded band.)
review of the cyclotron instability due to relativistic electrons is done by Wu. It is clear from Fig. 2, which wave frequencies are able to resonate with a given energy electron in the Jupiter's magnetosphere. Not all waves in the resonant band will grow at the same rate or to the same amplitudes. Rather, one expects preferential growth of these frequencies which interact with electrons having the most intense flux and the largest pitch angle anisotropies. The emitted waves will propagate away from the equatorial region of the growth, roughly following the magnetic lines. In order for the natural instability of the Jovian magnetosphere to develop, two conditions must apply. First, some means must be found for returning the wave energy to the generation region, and secondly, the growth incurred there must exceed all losses due to absorption and imperfect reflection. Further, application of Snell's law to waves into a region of lower refractive index. Near the equator, when $\omega > \omega_{LH}$, the waves approximately follow the field lines even when $\theta$ is large. However, at high latitudes when the wave frequency is less than $\omega_{LH}$ of the local plasma, it is possible for the wave group velocity to be oriented perpendicular to the field line. A necessary condition for the above reflection process to occur is that the waves are at some point on their path below the LH resonance frequency. To remain trapped within the Jovian magnetosphere, the waves must reflect well before this reflection. This critical frequency of the LH band is shown by the shaded area in Fig. 2. It is clear from Fig. 2 that waves generated above this frequency are lost to the ionosphere along any given L-shell. Waves of considerably lower frequency (produced by the higher energy electrons) may get reflected and thus take part in the natural instability of the Jupiter's magnetosphere. This perhaps accounts for why the center frequency of the naturally occurring hiss-band always lies between a few hundred hertz to few kilo hertz, or just below the shaded LH band.

We can also use Fig. 2 to predict the regions of electron loss. Clearly, near the perfect reflection of these waves (generated at frequencies below the LH band) would permit the natural hiss-band to grow indefinitely. By following the resonant frequency curves in Fig. 2 up to the LH band we can predict

At high frequencies above 5-20 kHz, one must rely on the reflection from the sharp density profile in the Jovian ionosphere. It seems very unlikely that this process can return a sufficient portion of the wave energy to the magnetosphere and we, thus, expect these high frequency waves to be lost by absorption in the Jovian ionosphere. This contention is substantiated by observations of VLF emissions in the magnetosphere of Jupiter. The frequency of the natural hiss-band emission is rarely above 3 kHz (Ref. 39) showing that the high frequency components of the resonant wave band do not contribute to the instability.

Lower frequency waves, however, can undergo almost perfect reflection by an alternative mechanism. Kimura and Thorne and Kennel have shown that low-frequency whistler mode wave propagating away from the equator tends to quickly increase the angle $\theta$ between their propagation vector and the ambient geomagnetic field. This results simply from an application of Snell's law to waves into a region of lower refractive index. Near the equator, when $\omega > \omega_{LH}$, the waves approximately follow the field lines even when $\theta$ is large. However, at high latitudes when the wave frequency is less than $\omega_{LH}$ of the local plasma, it is possible for the wave group velocity to be oriented perpendicular to the field line. Further, application of Snell's law in this region shows that the waves which have attained large wave normal angles are forbidden from penetrating very far past the LH resonance point. They are thus forced to reflect and return to the equatorial plane remaining more or less on the same magnetic flux tube. A necessary condition for the above reflection process to occur is that the waves are at some point on their path below the LH resonance frequency. To remain trapped within the Jovian magnetosphere, the waves must reflect well before this reflection. This critical frequency of the LH band is shown by the shaded area in Fig. 2. It is clear from Fig. 2 that waves generated above this frequency are lost to the ionosphere along any given L-shell. Waves of considerably lower frequency (produced by the higher energy electrons) may get reflected and thus take part in the natural instability of the Jupiter's magnetosphere. This perhaps accounts for why the center frequency of the naturally occurring hiss-band always lies between a few hundred hertz to few kilo hertz, or just below the shaded LH band.

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the location at which this natural instability becomes ineffective for any given energy electron. On low L-shells, the VLF waves generated by the electrons should penetrate the Jovian ionosphere and subsequently be absorbed. Several features of the electron-VLF interaction were, however, omitted from our discussion for the purpose of clarity. The analysis is certainly invalid for energies lower than about 10 keV, since the resonant frequency is then close to the electron gyrofrequency. Also, the electron energy resonant with a given wave frequency increases at higher geomagnetic latitudes.

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