Effects of ion beams on the nonlinear propagation of electron-acoustic-like wave

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In this paper a theoretical model has been developed that explains the existence of electron-acoustic-like double layers driven by an ion beam. Important aspect of the model is that a compressive type of electron-acoustic-like double layers solutions has been obtained instead of rarefactive type in the presence of an electron beam as obtained by Sutrada and Bujarbaru (Planet & Space Sci (UK), 36 (1988) 1009).

1 Introduction

Electrostatic hydrogen cyclotron (EHC) waves with ion beams and upward field-aligned current have been observed on the auroral field lines near 1RE. However, Kintner et al. concluded from their study of the correlation of ion beams and EHC waves that the source of free energy, ion beams and electron current, for the EHC waves is unclear. On the other hand, intense electrostatic waves of short wavelengths are frequently observed upstream of the earth's bow shock in presence of energetic electrons and ions. And two dominant type of waves have been used for explanations of these observations. The first type is clearly identified as electron plasma oscillations, and the second type is identified as ion acoustic waves, although the exact mechanism by which these waves are generally generated still remains to be well established. It may be mentioned that a large number of works have been done to explain theoretically the upstream electrostatic ion-acoustic-like noise. It was claimed that the ion waves could be generated by ion beam mode. However, all these theories are not suitable for bow shock conditions. It has been suggested by Marsch that this intense electrostatic waves which are frequently observed in the upstream of the earth's bow shock is associated with electron acoustic speed.

It may be mentioned that for electron acoustic waves (EAW), the frequency of the wave lies in between ion cyclotron frequency (ωi) and electron cyclotron frequency (ωe) and correspondingly their wavelength lies between a few or many Debye lengths. Typically, the phase velocities are located at and above the electron thermal speed, and, therefore, the EAW is heavily Landau-damped for a plasma in thermodynamic equilibrium. However, if there is free energy in the system under consideration in terms of nonthermal features in the distribution function or, more favourably, due to low density particle beams, then EAW can be excited against the Landau damping. It may be noted that for excitation of EAW, appropriate choices are beam density, drift and thermal speed.

Very recently Reddy et al. have studied the existence of solitons and double layers solutions associated with ion acoustic waves in a multispecies auroral beam plasmas. On the other hand, Sutrada and Bujarbaru have studied the small amplitude electron-acoustic-like double layers solutions in presence of an electron beam. They showed that electron acoustic double layers solution exists in a plasma carrying an electron beam in the direction of the magnetic field, when the beam velocity is almost of the order of the ratio of double layer velocity and the direction cosines along the magnetic field. The double layers obtained from their model is only rarefactive type. Our aim is to study the double layers solutions in terms of EAWs driven by an ion beam.

2 Time stationary analytic solution

We consider a homogeneous plasma consisting
of electrons and hot ions traversed by a cold ion beam. We assume that the EAW propagates in the x-z plane and the external magnetic field directed along the z-direction. Since both electrons and the beam ions have temperature much less than the temperature of the hot ions, the fluid equations may be used for both electrons and beam ions.

Since we assume that the EAW propagated in the x-z plane, the set of equations for the electrons are given as follows:

\[
\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x} (n_e v_x) + \frac{\partial}{\partial z} (n_e v_z) = 0 \tag{1}
\]

\[
\frac{\partial v_x}{\partial t} + \left( v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_x = -\frac{\partial \phi}{\partial x} - \frac{\theta_e}{n_e} \frac{\partial n_e}{\partial x} \tag{2}
\]

\[
\frac{\partial v_y}{\partial t} + \left( v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_y = 0 \tag{3}
\]

\[
\frac{\partial v_z}{\partial t} + \left( v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_z = -\frac{\partial \phi}{\partial z} - \frac{\theta_e}{n_e} \frac{\partial n_e}{\partial z} \tag{4}
\]

where, \( n_e \) is the density of electrons and \( \theta_e = T_e / T_i \) is the ratio of cold electron temperature and ion temperature. We assumed that beam ions are cold and moves parallel to the magnetic field \( B \) which is along the z-direction. Thus, we describe them by the usual fluid equation of motion. The z-component of continuity and the equation of motion can be written as follows:

\[
\frac{\partial n_b}{\partial t} + \frac{\partial}{\partial z} (n_b v_b) = 0 \tag{5}
\]

\[
\frac{\partial v_b}{\partial t} + \left( v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_b = -\frac{k_z}{M} \left[ \frac{\partial \phi}{\partial z} + \frac{\theta_e}{n_e} \frac{\partial n_e}{\partial z} \right] \tag{6}
\]

where,

\[ \theta_b = T_b / T_i, \text{ the temperature ratio of beam ion to the plasma ion} \]

\[ \delta = m_e / m_i, \text{ the ratio of electron and ion mass} \]

Equation (6) shows that the fluid is accelerated along \( B \) under the combined electrostatic and pressure gradient forces. For simplicity, we have considered the cases in which \( v_{b,z} \) is spatially uniform.

In Eqs (1)-(6), we have normalized the velocity, space variables, density, potential and time by the electron acoustic speed \( C_a = \sqrt{(T_i/m_i)}, \rho_c = C_a / \Omega_e, \) the equilibrium density \( n_{e0}, T_i/e \) and the inverse of the electron gyro-frequency \( \Omega_e, \) respectively.

In order to solve the Eqs (1)-(6), we now use the Galilean transformation \( \xi = k_x x + k_z z - Mt \) and \( \tau = t, \) where \( k_x \) and \( k_z \) are the direction cosines along the x- and z-directions, respectively, and \( M \) is the velocity of the moving frame.

Then Eqs (1)-(4) can be written as follows:

\[ k_x v_x + k_z v_z = M \left( 1 - \frac{1}{n_c} \right) \]

\[ \frac{M \partial v_x}{n_c \partial \xi} = k_x \frac{\partial \phi}{\partial \xi} - \frac{\theta_e}{n_e} \frac{k_x}{n_e} \frac{\partial n_e}{\partial \xi} \]

\[ \frac{M \partial v_y}{n_e \partial \xi} = -v_y \]

\[ \frac{M \partial v_z}{n_e \partial \xi} = \frac{k_z}{M} \left( \frac{\theta_e}{n_e} \frac{\partial n_e}{\partial \xi} - \frac{\theta_e}{n_e} \frac{\partial \phi}{\partial \xi} \right) \]

Integrating Eq. (10), we have

\[ v_z = -\frac{k_z}{M} \left[ \frac{\partial \phi}{\partial \xi} + \frac{\theta_e}{n_e} (1 - n_e) \right] \]

Now, in order to obtain an exact time stationary nonlinear equation relating the electron density \( n_e \) and electric potential \( \phi \), we use Eqs (7), (8), (10) and (11) in Eq. (9), and get

\[ \frac{1}{2} \frac{\partial}{\partial \psi} \left( \frac{\partial \phi}{\partial \xi} \right)^2 - \frac{1}{A} \left( 1 - n_e + \frac{k_x v_x n_e}{M} \right) = -\frac{\partial V(\phi)}{\partial \psi} \]

where, \( V(\phi) \) is the classical potential. The constant of integration of Eq. (12) is nonzero for periodic solution and zero for localized solution, where by the term 'localized' we mean that all gradients vanishes at infinity. Equation (12) is a nonlinear time stationary equation of cold electron density which depends on the electric potential \( \phi \), and it is difficult to solve analytically. To solve Eq. (12), one requires an additional relation between the cold equation fluid density and the electrostatic potential, and can be solved by assuming quasi-neutrality, or \( n_e \approx n_i + n_b \). Quasi-neutrality is valid if the Laplacian in Poisson's equation is too small. For EAW with small amplitude, the criterion is \( \lambda_{Di} << \lambda_e, \) i.e. ion Debye length is much smaller than the electron gyro-radius. With this assumption and integrating once again, we get an oscill-
tor equation \[(1/2) (\partial \phi / \partial \xi)^2 + V(\phi) = 0\] which may be solved by standard technique.

In Eq. (12), \(A\) can be defined as,
\[A = 1 + (M^2 / n_b^2 - \theta_e / n_e) \partial n_e / \partial \phi \] ...
\[(13)\]

Integrating Eqs (5) and (6) and using boundary conditions, viz. \(\phi = 0; v_b = v_0; \) \(n_b = n_{b0} / n_e = \alpha_b\) and \(n_e = 1\) at \(\xi \to \infty\) and expanding \(\phi^3\), it is possible to write.
\[n_b = \alpha_b \left[ 1 - \gamma \phi + \gamma \phi^2 - \gamma \phi^3 \right] \] ...
\[(14)\]

where, \(n_{b0}\) and \(n_{e0}\) are the equilibrium densities of beam ions and electrons, respectively.

We have neglected the higher order term of \((v_0 - M/k_z)^2\) \(\phi\), where
\[\gamma = 2 \theta_b \delta / [2 \theta_b \delta - (v_0 - M/k_z)^2] \] ...
\[(15)\]

and
\[v_0 = \text{Drift velocity of beam} \]

Since for the EAWs, the frequencies are much larger than the ion cyclotron frequencies and wavelengths are much smaller than the ion Larmor radius, the ions may be considered as unmagnetized so that their motions along the magnetic field are assumed to be important. Further, if we consider that \(0 < (\cos \theta, k_z) < (m_i / m_i)^{1/2}\) and \(C_e << v_0\), then it can be shown that the ion inertia term can be neglected in the z-component of the equation for the ions. In such situation the normalized ion density can be given by Boltzmann equation
\[n_i = (1 - \alpha_b) \exp(-\phi) \] ...
\[(16)\]

Using Eqs (14), (16) and (11) in Eq. (12) and for small amplitude limit, we find,
\[\frac{1}{2} \frac{\partial}{\partial \phi} \left( \frac{\partial \phi}{\partial \xi} \right)^2 = \frac{1}{A} (B_1 \phi - B_2 \phi^2 + B_3 \phi^3) = - \frac{\partial V(\phi)}{\partial \phi} \] ...
\[(17)\]

where,
\[B_1 = a_1 - k_z^2 (1 + a_1 \theta_e) / M^2 \] ...
\[(18)\]
\[B_2 = a_2 - k_z^2 \left[ \frac{3}{2} a_1 + (a_2 + a_1^2) \theta_e \right] / M^2 \] ...
\[(19)\]
\[B_3 = a_3 - k_z^2 \left[ \frac{4}{3} a_2 + \frac{1}{2} a_1 a_2 \right] + 2(a_3 + a_1 a_2) \theta_e \right] / M^2 \] ...
\[(20)\]

\[a_1 = 1 - \alpha_b + (\alpha_b \gamma / \theta_b) \] ...
\[(21)\]
\[a_2 = \frac{1}{2} (1 - \alpha_b) + \left( \frac{\alpha_b \gamma}{\theta_b} \right) \] ...
\[(22)\]
\[a_3 = \frac{1}{6} (1 - \alpha_b) + \left( \frac{\alpha_b \gamma}{\theta_b} \right) \] ...
\[(23)\]

Integration of Eq. (17) leads to,
\[\frac{1}{2} \left( \frac{\partial \phi}{\partial \xi} \right)^2 + V(\phi) = 0 \] ...
\[(24)\]

This is the well known 'Energy Law' where
\[- V(\phi) = \frac{1}{A} \left( B_1 \phi^2 - B_2 \phi^3 + B_3 \phi^4 \right) \] ...
\[(25)\]

In deriving Eq. (25), we have used the boundary conditions, viz.
\[V(\phi) = 0 \text{ at } \phi = 0. \]

Now, imposing the double layers boundary conditions, viz.
\[\partial V(\phi) / \partial \phi = 0 \text{ at } \phi = \psi \text{ and } V(\phi) = 0 \text{ at } \phi = \psi, \text{ respectively, in Eqs (17) and (25), we obtain,} \]
\[B_1 = B_2 \psi - B_3 \psi^2 \] ...
\[(26)\]

and
\[B_1 = \frac{2}{3} B_2 \psi - \frac{1}{2} B_3 \psi^2 \] ...
\[(27)\]

from which we find the co-efficient \(B_1\) and \(B_2\) in terms of \(B_3\) as follows.
\[B_1 = \frac{1}{2} B_3 \psi^2 \] ...
\[(28)\]
\[B_2 = \frac{3}{2} B_3 \psi \] ...
\[(29)\]

where, \(\psi\) is the amplitude of the wave.

Inserting Eqs (28) and (29) in Eq. (25), we obtain the classical potential as,
\[- V(\phi) = \frac{B_3}{4 A} \phi^2 (\phi - \psi)^2 \] ...
\[(30)\]

Using Eq. (30) in Eq. (24) and then integrating we get the double layers solutions as
\[ \phi = \frac{\psi}{2} (1 - \tanh \kappa \xi) \quad \ldots (31) \]

where

\[ x = \frac{B_3}{\sqrt{8A \psi}} \quad \ldots (32) \]

And the width of the double layers may be written in the form of

\[ d = \frac{1}{\psi} \sqrt{\frac{8A}{B_3}} \quad \ldots (33) \]

We shall now calculate the velocity \((M)\) of the double layers from Eqs (28) and (29), and to evaluate the same we assume that the beam velocity, \(v_0\), is almost of the order of \(M/k_z\), so that \((v_0 - M/k_z)^2\) can be neglected in comparison to \(2\theta_0\). Then from Eq. (15), we find \(\gamma = 1\). With this assumption the calculated value of \(M\) is given by,

\[
M = k_z \left[ \theta_b + \frac{1}{(1 - \alpha_b + \alpha_b/\theta_b)} \right]^{1/2} \times \left[ 1 + \frac{1}{6(1 - \alpha_b + \alpha_b/\theta_b)} \frac{1}{2}(1 - \alpha_b + \alpha_b/\theta_b)^2 \right. \\
- \left. \frac{3}{2} \left(1 - \alpha_b + \alpha_b/\theta_b\right) \right] \frac{1}{2}(1 - \alpha_b) \\
+ \theta_e \left(1 - \alpha_b + \alpha_b/\theta_b\right)^2 + \frac{1}{2}(1 - \alpha_b) \left(1 - \alpha_b + \alpha_b/\theta_b\right)^2 \\
+ \alpha_b/\theta_b \right] \psi \quad \ldots (34) 
\]

The linear form of the same is given by,

\[
M = k_z \left[ \theta_b + \frac{1}{(1 - \alpha_b + \alpha_b/\theta_b)} \right]^{1/2} \quad \ldots (35) 
\]

Introducing the linear velocity \(M\) in Eqs (19) and (20) we find for \(\gamma = 1\).

\[
B_2 = \frac{1 - \alpha_b + \alpha_b/\theta_b}{2} \left(1 - \frac{1 - \alpha_b + \alpha_b/\theta_b}{\theta_b} \right) \times \left[ \frac{3}{2} \left(1 - \alpha_b + \alpha_b/\theta_b\right) + \theta_e \left(1 - \alpha_b + \alpha_b/\theta_b\right)^2 \right. \\
+ \left. \frac{1}{2}(1 - \alpha_b + \alpha_b/\theta_b)^2 \right] \quad \ldots (36) 
\]

From Eq. (31) it may be concluded that for the existence of the double layers solutions, the value of \(B_3\) has to be positive. We have found that in the plasma and for \(v_0 = M/k_z\), the solution exists for different values of \(\alpha_b\), \(\theta_e\) and \(\theta_b\). From Eq. (29) it is found that since \(B_3\) is positive, \(B_2\) should be positive for positive \(\phi\) and negative for negative \(\phi\). After numerical analysis for different values of \(\alpha_b\), \(\theta_e\) and \(\theta_b\), we have seen from Eq. (36) that \(B_2\) is always positive. Therefore, we may conclude that in this case only compressive double layers exists, because it is always positive.

3 Summary

The existence of an electron-acoustic-like double layers in the presence of an ion beam propagating along the direction of the magnetic field has been discussed. In summary, we have reported analytical proofs of the existence of small amplitude electron-acoustic-like double layers solutions of the pseudo-potential system. We have numerically calculated the velocity of double layers and have studied the variations of the double layers velocity \(M\) and \((M/k_z)\) with variations of its amplitude.

In Fig. 1, we have plotted the variations of \((M/k_z)\) with the variation of amplitude \((\psi)\). It is seen that for fixed values of \(\theta_e\) and \(\theta_b\), the value of \((M/k_z)\) increases with the increase of amplitude \((\psi)\) for a particular value of beam density, \(\alpha_b\), but decreases with increase of \(\alpha_b\) for a particular value of \(\psi\).

In Fig. 2, it is seen that for fixed values of \(\theta_e\) and \(\theta_b\), and for \(\psi < 0.1\), the values of \((M/k_z)\) increase with the increase of \(\theta_b\) and as \(\theta_b\) increases further, \((M/k_z)\) increases slowly. But as \(\psi\) exceeds the value 0.1, \((M/k_z)\) varies in the opposite sense.

The variations of the half-width of the double layers \((d)\) with its amplitude \(\psi\) for fixed values of \(\theta_e\) and \(\theta_b\), and for a particular value of \(\alpha_b\) have been plotted in Fig. 3. It is observed that in that
Fig. 1—Variation of \( \langle M/k_z \rangle \) with \( \psi \) for different values of beam density \( \alpha_b \) [Curves (a), (b) and (c) correspond to \( \alpha_b = 0.06, 0.08 \) and 0.1, respectively.]

Fig. 2—Variation of \( \langle M/k_z \rangle \) with beam temperature \( \theta_b \) for different values of \( \psi \) [Curves (a), (b), (c), (d) and (e) correspond to \( \psi = 0.01, 0.05, 0.1, 0.15 \) and 0.2, respectively.]

Fig. 3—Variation of double layer half-width \( d \) with its amplitude \( \psi \) for different values of beam density \( \alpha_b \) [Curves (a), (b) and (c) correspond to \( \alpha_b = 0.001, 0.01 \) and 0.1, respectively.]

Fig. 4—Variation of double layer half-width \( d \) with beam density \( \alpha_b \) for different values of \( \psi \) [Curves (a), (b), (c) and (d) correspond to \( \psi = 0.03, 0.05, 0.1 \) and 0.2, respectively.]
case, the value of $d$ decreases with the increase of $\psi$ for a particular value of $\alpha_b$ (Fig. 3).

In Fig. 4, are shown the variations of the half-width of the double layers ($d$) with beam density $\alpha_b$ (for different values of $\psi$). It is found that the values of $d$ decreases with the increase of $\alpha_b$ for a particular value of $\psi$.

It may be noted that we have used quasi-neutrality condition in our analysis which is valid when the ion Debye length is smaller than the electron gyro-radius. Therefore, the double layers which is discussed in the present paper do not fit into the original definition of a double layer which is a localized dipole sheet of a space charge surrounded by quasi-neutral plasma. However, the electrostatic shocks with small amplitude and shorter scale length may be understood as oblique ion/electron acoustic double layers\textsuperscript{16}.

It is found that in case of hot ion plasma ($T_i >> T_e$), the ion Landau damping is severe, but it can be reduced by driving a current through the plasma. Therefore, it can become undamped in a $T_i >> T_e$ plasma by adjusting current. It has been also shown that electrostatic shock solution can exist in a $T_i >> T_e$ plasma under sheet conditions and that the resulting shock potential jump is sufficient for energizing auroral primary electrons\textsuperscript{17}.

In conclusion, we can say that electron-acoustic-like double layers solution exists in a plasma carrying an ion beam in the direction of the magnetic field, when the beam velocity is of the order of $(M/k_e)$. The theoretical results may be applicable to the regions of space plasmas, where this type of plasma is observed to exist.

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References