Theoretical limits of the application of an image restoration technique in astronomy

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The technique of image restoration by using inverse method has been applied to restore astronomical image; also, the Wiener filter has been used to minimize the effect of noise. An attempt has been made to find out how noisy the astronomical data has to be, before the attempted image restoration technique becomes useless. It is found that reasonable restored astronomical images could be produced when S/N ratio is $\gtrsim 1$, and this technique gives unacceptable results if S/N ratio is $< 1$.

1 Introduction

The astronomical images are generally degraded by atmospheric turbulence, and this turbulence, is the main problem in optical astronomy and observational work. The degradation in observed images will be caused not only by the atmosphere of the earth, but also by the effect of the optical instruments. The total degradation of the telescope images by optical inhomogeneities in the earth's atmosphere is generally called "seeing" (Ref. 1). Image restoration means the removal or reduction of degradation which appeared while the image was being obtained. Degradation includes the blurring by the effect of the optical system and image motion as well as noise in the image. In astronomical work this means an estimate of the ideal images which are observed if there were no degradation. The general aim of all methods of image restoration is to bring the degraded image towards what they should be, if they have been observed and recorded without the degradation effects. The present work is an application of image restoration technique in astronomy using inverse method and Wiener filter. Also the effect of standard deviation of smearing function (Gaussian function) on the restored images has been studied. For real astronomical data, we have worked on NGC 6814 galaxy, and the dimensions of the image used are $64 \times 64$ points.

2 Image restoration technique

Many researchers have investigated theoretically and experimentally different methods and techniques to improve the degraded images. The problem of image restoration has not yet been solved satisfactorily, and no unique solution has been found so far. The presence of noise made the problem worse and, in practice, a perfect restoration is impossible to obtain. No amount of image restoration can increase or add more information content to the image. Apriori information of the degraded image such as the nature of the noise, way of recording the image and the purpose of the recorded image are very important in deciding which method of restoring the image should be used.

In astronomical images the noise which can be defined as an additive contamination of an image will be one of the following three types:

(i) Noise in the astronomical object light

(ii) Noise in the sky background

(iii) Noise in the equipment which it used in imaging.

The visual effect of noise will appear in the contrast, appearance and resolution of the image.

Restoration techniques are based on the Fourier transform, which has recently become easy and available to use due to the introduction of the Fast Fourier Transform (FFT) by Cooly and Tucky in 1965.

Algebraic details of the optimum restoration in presence of noise have been described by Brault and White. Here, we present only essential details; so in the presence of noise the observed image will be given by:

$$O(x) = A(x) * T(x) + N(x) \quad \ldots (1)$$
where,

$O(x)$ Observed image

$A(x)$ Smearing function

$T(x)$ Original undegraded image

$N(x)$ Noise term

$\star$ Convolution symbol

If we then take Fourier transform of this equation we get,

$$O(k) = A(k) T(k) + N(k) \quad \ldots (2)$$

To restore this image, we divide Eq. (2) by the Fourier transform, $A(k)$, of the smeared function (These are according to Brault and White$^4$) and get,

$$R(k) = \frac{O(k)}{A(k)} = \frac{T(k) + N(k)}{A(k)}, \text{ where } A(k) \neq 0 \quad \ldots (3)$$

By the first term of Eq. 3 (in the absence of the noise), a perfect restoration is obtained, but the second term (presence of the noise) will produce an additive error. The value of the second term will increase if $A(k)$ becomes small, and at high frequency, the amplitude of the noise will be amplified to an intolerable level.

The signal-to-noise ratio (S/N), which can be defined as the ratio of the amplitude of wanted signal to the amplitude of the noise at any point in the transmission system, is the best way to characterize the noise and its level in observed images. If the smearing function has no zero value and the S/N ratio is high, the restoration by the method of inverse filtering works well. But, if the S/N ratio is lowered by adding more noise, the performance of the restoration by this method becomes poor$^5$.

Generally, we can say that the method of image restoration by inverse filter produces reasonable results with a very high S/N ratio and proper value of blur.

The inverse method of restoration is very sensitive to noise and not enough to produce good results in the presence of noise. This sensitivity to noise can be avoided by another restoring technique such as the Wiener filter.

3 Wiener deconvolution filter

Wiener deconvolution filter is an optimal filter for recovering an unknown signal from noise. It is designed to minimize the mean squared error between the original image and the estimated one in the presence of noise$^6$ as follows:

$$E_{\text{min}}^2 = |T(x) - O(x)|^2 \quad \ldots (4)$$

where,

$E$ Estimate of the original image

$T(x)$ Original image

$O(x)$ Observed (noisy) image

The Wiener filter transform function is given by,

$$H(k) = \frac{P_T(k)}{P_T(k) + P_N(k)} = \frac{|T(k)|^2}{|T(k)|^2 + |N(k)|^2} \quad \ldots (5)$$

where,

$P_T(k)$ Power spectra of the original image

$P_N(k)$ Power spectra of the noise

$P_A(k)$ Power spectra of the smearing function

The use of this filter is sometimes difficult due to the unknown power spectra of the original image [$P_T(k)$]. The total power spectra of the degraded noisy image is the sum of the signal and noise power spectra. Figure 1 shows the total power spectra of the degraded noisy image, and the way of estimating the power spectra of the signal and noise.

Fig. 1—Total power spectra of the degraded noisy image, and the way of estimating the power spectra of the signal and noise.
spectrum of the degraded noisy image, [part $P_{\text{signal}}$ represents the signal power spectra and part $P_{\text{noise}}$ represents the noise power spectra]. To produce the Wiener filter, we estimated the value of the power spectra of the signal and noise by taking the mean value of noisy part and subtracting it from the total power spectra of the degraded noisy image. The result of that subtraction is the power spectra of the signal. We can write the Wiener filter according to our way of work as:

$$H(k) = \frac{P_{\text{tot}}(k) - P_{\text{noise}}(k)}{P_{\text{tot}}(k)} \quad \ldots (6)$$

where,

- $P_{\text{tot}}(k)$ Total power spectra of the blurred noisy image
- $P_{\text{noise}}(k)$ Noise power spectra.

Fig. 2—Image of real astronomical data ($64 \times 64$)
Fig. 4—Same as Fig. 3, but for $k=15$, $P_{\text{noise}}=10$ (a); and $k=15$, $P_{\text{noise}}=20$ (b).

Fig. 5—Same as Fig. 3, but for $k=10$, $P_{\text{noise}}=10$ (a); and $k=10$, $P_{\text{noise}}=20$ (b).
Fig. 6 - Same as Fig. 3, but for $k = 1$. $P_{\text{max}} = 20$.

Fig. 7 - Same as Fig. 3, but for $k = 0.5$. $P_{\text{max}} = 20$. (b)
If the noise is very small or zero \( P_{\text{noise}}(k) = 0 \), the Wiener filter will be reduced to the normal inverse filter; that means the Wiener filter has inverse behaviour in low spatial noise frequency regions.

The value of Wiener filter in this work is changeable. So, when we change the value of the noise power spectra, the power spectra of the signal will change, and that change will produce a new value of Wiener filter.

The way of estimating the value of the noise and signal power spectra is illustrated in Fig. 1, where all the points greater than the point (a) represent the noise power spectra and all the points less than (a) represents the signal power spectra, and it is the same for the points (b) and (c).

4 Theoretical limits

To find out the theoretical limits of the application of image restoration technique, the effects of standard deviation of the smearing function and the Wiener filter on the restored images have been studied.

For real astronomical data we worked on a CCD image of NGC 6814 galaxy. After we identified the data, a part of this data \( (64 \times 64 \text{ points}) \) has been fed to the computer (Fig. 2). We have found that the inverse filter restoration technique produces acceptable results if the value of the standard deviation of the point spread function is more than one, but its performance becomes unacceptable if the value of the standard deviation is \( \leq 1 \).

The effect of standard deviation of Gaussian point spread function and Wiener filter (by changing noise power spectra, \( P_{\text{noise}} \)) on the restored real astronomical image \( (64 \times 64 \text{ CCD image of NGC 6814}) \) is shown in Figs 3-7. The level of signal and noise in the observed images is controlled by the value of \( S/N \), for which we have used here signal/\( \sqrt{\text{signal}} \). Restorations have been compared for two different values of noise power \( (P_{\text{noise}} = 10, 20) \) for a set of standard deviation of point spread function \( (k) \) ranging over values from 100 to 0.5.

For images having standard deviation \( \geq 1 \), the restored image looks better when the power spectra of the noise used in Wiener filter is greater than or equal to the power spectra of the signal. That is because we almost subtracted all the noise components of frequency. But, if the noise power spectra is less than the signal power, the amount of the noise component of frequency subtracted is small, and the Wiener filter reduces to an inverse filter if the noise power is very small or zero.

For astronomical data, the technique of image restoration is good for the images if the standard deviation of point spread function is greater than one, and the signal is equal to or greater than noise \( (S/N \text{ increase}) \); but it is useless if the noise is greater than the signal \( (S/N \text{ decrease}) \).

5 Conclusions

In this theoretical work of image restoration technique it has been found that reasonable restored images can be produced when the signal in the image is \( \geq 1 \), i.e. \( S/N \geq 1 \), but this technique produces hopeless results if the level of the noise is greater than the level of the signal \( (S/N \text{ ratio was less than one}) \). Also the proper estimate of the noise and signal power spectrum makes the Wiener filter more useful.

Deconvolution filtering usually increases with frequency, thus it restored and enhanced a high frequency spectrum (noise) more than low frequency (signal), and for this reason, whatever the filter used in deconvolution method of restoration, its performance was still poor for noises images. It is unwise to try for restoring a noisy image of an unknown object by deconvolution methods. This method can, however, be quite useful in cleaning and deblurring the images.

References
1 Young A T, Astrophys J (USA), 189 (1974) 587.