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Whistler wave amplification by compressed dipolar field

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The impact of solar wind distorted dipolar field on magnetospheric parameters and quiet time parallel whistler mode propagation has been studied. It is shown that in the compressed dipolar field the resonant velocity of energetic electrons as well as their integral flux increase above their values in a dipole field. Because of this, the temporal wave growth and power gain of whistler mode waves are increased significantly. The increments in temporal wave growth and power gain are directly proportional to the solar wind intensity, and the effects are more pronounced at higher L shells than at lower L shells.

1 Introduction

It was probably Mead[1] who computed for the first time the deformation of the geomagnetic field due to the impact of solar wind on the magnetosphere. Williams and Mead[2] measured trapped energetic electron fluxes at 1100 km above the earth's surface and studied the deformation of the dipolar geomagnetic field. This solar wind interaction with the centred dipolar geomagnetic field creates different current systems which distort the earth's magnetic field. Choe and Beard[3] and Beard[4] showed that these current systems in the magnetopause, plasmasphere and neutral sheet cumulatively affect the geomagnetic field. Richmond[5] developed a general method of computing the geomagnetic field variations due to three-dimensional magnetospheric current systems. Richmond's model can be adopted to study distortions in the earth's field due to field-aligned currents. Mead and Cahill[6] and Sugiura *et al.*[7] developed models to compute geomagnetic field distortions based on data recorded aboard the Explorer 12 and OGO 3/5 satellites. Mead and Fairfield[8] and other workers developed their own models to compute the effect of induced current systems on the distortion of the earth's magnetic field. Their studies were based on experimental data of the geomagnetic field taken aboard IMP satellites. It is observed that solar wind variability results in a corresponding variability in the geomagnetic field distortion[9].

Since all the models discussed above are unable to compute rapid variations in the earth's magnetic field, the axisymmetric Mead field approximation for the distorted geomagnetic field given by Mead[1] has been adopted by us in this study. The gyrofrequency (f_H) of the electrons in and near the equatorial region of Mead's distorted geomagnetic field is a function of the 'stand off' distance of the magnetopause from the point dipole. An intense solar wind will compress the magnetosphere more, thus giving a smaller 'stand off' distance b. The value of b can be a minimum of 5 R_0, where R_0 is the earth radius (6370 km), whereas it is normally $\sim 10\ R_0$. Mead[1] showed that the strength of the perturbed field depends directly on the solar wind intensity, i.e.

Distortion extent $\propto$ Solar wind intensity

The gyrofrequency of electrons in and near the equatorial region of Mead's distorted field (f_{Hd}) and undistorted field (f_{H0}) at a certain L value is given by

$$f_{Hd} = f_{H0} \times \left[1 + \frac{B_1 R_0^3 L^3}{B_0 b^3} + \frac{9}{2}\left(1 - \frac{B_1 R_0^3 L^3}{4 B_0 b^3}\right) \times \left(\frac{Z}{L R_0}\right)^2 \right]$$

Here, suffix 'd' stands for the distorted value and '0' for the undistorted value, $B_0 = 0.31$ G, $B_1 = 0.25$ G (for details, *see* Mead[1]), L is the McIlwain parameter, b the "stand off" distance measured in earth radii from the point dipole to the magnetopause, and Z the length of the interaction region measured from the equatorial plane.

Recently Singh[10] studied the effect of the distorted geomagnetic field on the Storey angle (ψ_S), which plays an important role in the study of various VLF events such as inverted V-structures, saucers, etc. (Refs 11-13). He observed that the distorted geomagnetic field affects the Storey angle (ψ_S) up to $L \gg 3$ and thus the range of frequencies over which VLF signals are observed changes significantly. Singh and Prasad[14,15] studied the effect of geomagnetic field distortions on wave particle interactions and electron precipitation. They found that the precipitated electron flux increases with a decrease in b, suggesting that an intense solar wind will cause an increased flux of precipitated electrons.

Rycroft[16-18] studied in great detail the growth rate of ELF/VLF emissions considering transverse resonance between ELF/VLF waves and energetic electrons in the magnetosphere and explained the observation of discrete emissions. Cornilleau-Wehrlin *et al.*[19] and Solomon *et al.*[20-21] studied the growth rates of ELF/VLF emissions. Their study was based on simultaneous measurements of whistler mode wave spectra (200-3000 Hz) and of electrons in the energy range 15-300 keV obtained aboard the satellites GEOS 1/2.

In this paper the effect of solar wind distorted geomagnetic field on VLF wave amplification has been presented.

2 Method of calculation and ionospheric model

To study the interaction between the distorted geomagnetic field and the anisotropic distribution of energetic electrons we consider L values of 4.0 and 5.0. The temporal growth rate γ is computed from the following expression[17,21,22]

$$\gamma = 2\pi^2 (A - A_c) \cdot \eta f_H (1-x)^2 \quad \ldots (1)$$

where $A [= T_\perp / T_\parallel - 1$, $T_{\parallel(\perp)}$ being the parallel (perpendicular) energy of the energetic electrons] is the temperature anisotropy of the energetic electrons, $\eta \left(= \frac{J_\parallel}{V_\parallel N_0}, J_\parallel \text{ being the electron flux, } V_\parallel \text{ the parallel resonant velocity of energetic electrons, and } N_0 \text{ the cold plasma density} \right)$ the fractional concentration of energetic electrons, and A_c the critical anisotropy below which no amplification takes place. A_c is given by the following expression (normalized frequency $x = f/f_H$)

$$A_c = \frac{f}{f_H - f} = \frac{x}{1-x} \quad \ldots (2)$$

The gain in wave intensity due to a cyclotron resonance interaction with energetic electrons in the equatorial region is given by[23,24]

$$G = \frac{2\gamma Z}{V_g} \quad \ldots (3)$$

where V_g is the group velocity of whistler mode waves along the geomagnetic field line and can be expressed as

$$V_g = 2c \frac{f_H}{f_p} \left(\frac{f}{f_H} \right)^{1/2} (1-x)^{3/2} \quad \ldots (4)$$

where c is the velocity of light and f_p is the electron plasma frequency. The resonant velocity of the energetic electrons can be expressed as

$$V_\parallel = \frac{c\,(f_H - f)^{3/2}}{f_p\, f^{1/2}} \quad \ldots (5)$$

The equatorial plane cold plasma densities (N_0) for the considered L values are taken from Singh[25] and correspond to the diffusive equilibrium model[26]. Their values are 400 el. cm^{-3} at $L=4$ and 182 el. cm^{-3} at $L=5$. The Ariel 3/4 satellites recorded VLF emissions at 3.2, 9.6, 16.8 and 22.3 kHz (Refs 27-28). Since most of the emissions are recorded in 3-5 kHz range (Ref. 29), we consider 3.2 kHz as the interacting wave frequency.

3 Results and discussion

Choe and Beard[3] showed that the solar wind plasma compressing the magnetosphere can cause a distortion of up to 60% in the dipolar field. Here, we will investigate the effect of the compressed dipolar field, having a distortion of 60%, 40% and 20%, on whistler mode waves. We write the relation for the undistorted geomagnetic field (f_{H0}) and the distorted geomagnetic field (f_{Hd}) as

$$f_{Hd} = M \; f_{H0}$$

where $M = 1.6$ for 60% distortion

$= 1.4$ for 40% distortion

$= 1.2$ for 20% distortion

First of all we will study how the compressed geomagnetic field affects the parallel resonant velocity ($V_\parallel$), the interaction length (Z), the whistler wave group velocity (V_g), the fractional concentration of hot plasma (η), and the anisotropy ($A - A_c$). We will study only the quiet time solar

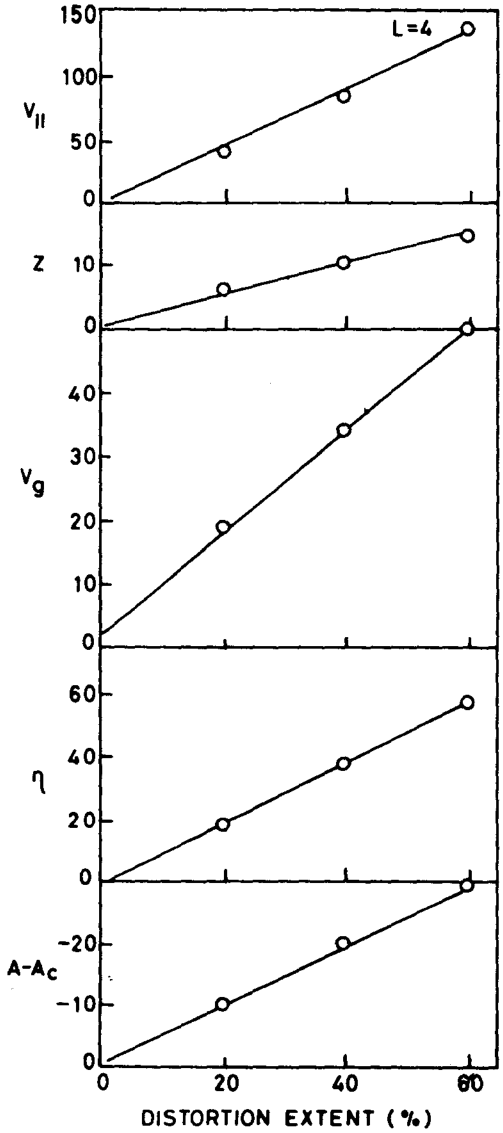

Fig. 1—Variations of per cent changes in $V_{\parallel}$, Z, V_g, η and $(A-A_c)$ with distortion extents at $L=4$.

wind effects and thus do not consider any variations in the cold plasma density (N_0).

Resonant velocity ($V_{\parallel}$)

Eq. (5) shows that $V_{\parallel} \propto (f_H - f)^{3/2}$ and so the resonant velocity will increase under the compressed dipolar field. The plasma frequency of the electrons (f_p) remains unchanged since we are considering only a quiet-time effect and no storm-time effects[30-32]. The variation (in %) of the change in the parallel resonant velocity ($V_{\parallel}$) with distortion extent at $L=4$ is shown in Fig. 1.

Interaction length (Z)

We know that the interaction length $Z \propto (V_{\parallel}/f_H)^{1/3}$ (Refs 33 and 34) and that (approximately) $V_{\parallel} \propto f_H^{3/2}$, thus the parameter Z also increases due to the enhanced solar wind. Variation of Z with distortion extent is also depicted in Fig. 1.

Group velocity (V_g)

Since N_0 (hence plasma frequency also) does not change during quiet time solar wind impact, V_g will increase with gyrofrequency (f_H). The variation of V_g with distortion extent, too, is depicted in Fig. 1.

Fractional concentration (η)

It is well known that the source of magnetospheric energetic electrons (and protons) is the solar wind plasma. Etcheto *et al.*[35] showed that energetic electron injection rate increases with K_p level. Thus it can be said that the solar wind distortion of the magnetosphere during quiet to storm times will increase the trapped electron flux ($J_{\parallel}$) largely and so η. The direct measurement of $J_{\parallel}$ with varying intensity of the solar wind has not been reported so far. Figure 13 of Solomon *et al.*[21] depicts the variation of η with reduced frequency x. It is clear from the figure that, for a fixed frequency of VLF hiss, a doubling of the electron gyrofrequency will ultimately double the η value. Thus a distortion of 60% in dipolar geomagnetic field will increase the fractional concentration of energetic electrons by 60%.

Pitch angle anisotropy ($A-A_c$)

There is no reported relation between $(A-A_c)$ and distorted geomagnetic field values. The study of electron diffusion by VLF waves[35,36] reveals that solar wind enhancement will increase the rate of diffusion and, when the diffusion increases, the anisotropy starts to decrease[21,36,37]. Figure 13 of Solomon *et al.*[21] is the experimental proof of the $(A-A_c)$ decrease as f_H increases (or x decreases). Figure 15 of Solomon *et al.*[21] shows that for $f/f_H=0.4$, $(A-A_c)$ is 0.4, and for $f/f_H=0.2$, $(A-A_c)$ is 0.2. Considering a fixed frequency f, we see that for a 100% increase in f_H, we get a 50% decrease in $(A-A_c)$. The $(A-A_c)$ variation (%) is also depicted in Fig. 1.

The variation of per cent changes in the values of $V_{\parallel}$, Z, V_g, η and $(A-A_c)$ with distortion extent at $L=4$, depicted in Fig. 1, shows increase in the values of all the parameters under the compressed geomagnetic field, except in $(A-A_c)$. It is clear from the figure that the increments are almost directly proportional to the extents of distortion.

The computations of wave growth rate (γ) under a distorted as well as an undistorted geomag-

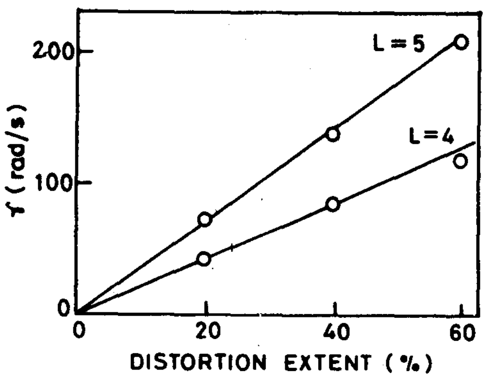

Fig. 2—Variation of per cent increment in temporal growth rate (γ) with distortion extents at $L=4$ and $L=5$.

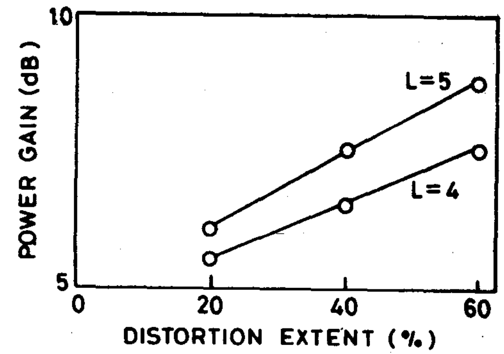

Fig. 3—Variation of per cent increment in power gain with distortion extents at $L=4$ and $L=5$.

netic field show that the wave growth rate increases significantly with increasing solar wind pressure. A 20% distortion at $L=5$ in the dipolar field produces a wave growth increment of 210%, i.e. the wave growth rate under an enhanced solar wind is 3.1 times the wave growth rate under an undistorted dipolar field. The variation of per cent change in wave growth rate with distortion extent is depicted in Fig. 2.

Figure 2 shows that the wave growth increments are larger at $L=5$ than at $L=4$ for the given distortion extent due to the fact that $L=5$ is farther from the earth. Thus at higher L shells we will get intense whistler waves in comparison to those produced at lower L shells.

The solar wind impact on the magnetosphere also affects the total power gain of whistler mode waves. A 20% distortion will produce a whistler wave having a power gain larger by 6.04 dB than the power gain of such waves generated for an undistorted field at $L=5$. The variation of increased power gain with distortion extent is shown in Fig. 3, which shows that power gain changes are larger for large L shells and greater distortion extents.

Singh and Prasad[14] showed that for smaller "stand off" distances b, the precipitated flux is higher, i.e. for a more intense solar wind we get an increased electron gyrofrequency [*see* their Eq. (1) and Fig. 1] and precipitated electron flux. Thus, due to an increasing solar wind pressure the integral flux and fractional concentration should also increase. Due to the large precipitated flux, a large fraction of energetic electrons will release their energies to the wave before being dumped into the lower ionosphere, thereby increasing the wave growth and power gain of the waves. Thus our conclusion is consistent with the work of Singh and Prasad[14].

The impact of the solar wind depends upon the angle between the geomagnetic equator and the solar wind direction. Mead[1] showed that compression of field lines takes place on both the dayside and nightside, though it is greater on the dayside in comparison to that at dusk or on the nightside. Figure 10 of Mead[1] shows that differences in distortions for dusk and noon are equal to the differences in distortions for night and dusk. Therefore our 60%, 40%, 20% distortion extents represent noon, dusk and nightside dipolar compressions. It is, therefore, clear that the solar wind will not only affect daytime whistler events but also nighttime whistler events.

4 Conclusions

The effect of the compressed dipolar geomagnetic field on whistler wave amplification has been studied. It is found that with such a distorted field, the resonant velocity of electrons increases as do the integral flux and fractional concentration of hot energetic electrons along with the group velocity of waves and the length of wave-particle interaction (WPI) region (Z). The wave growth rates are found to increase significantly with solar wind intensity (distortion extent). The compressed field produces whistler waves having a power gain larger by at least 5 dB.

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