A comparative study of energy and pitch angle diffusion during cyclotron resonance

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Adopting linear and non-linear theories of wave-particle interaction, the relative importance of pitch angle diffusion over particle energy diffusion is examined for different ELF/VLF waves at different equatorial and off-equatorial locations for $L$ values lying between 2 and 8. The result shows that the pitch angle diffusion is important in the plasmasphere where $\sigma \leq 0.5$, and the energy diffusion is important outside the plasmasphere where $\sigma = 0.6 - 0.8$ (here $\sigma = f/f_{H}$, $f$ being the wave frequency and $f_{H}$, the electron gyrofrequency). As such, the result indicates the dominance of wave-growth over particle precipitation outside the plasmasphere and is useful for interpretation of high latitude and polar emissions.

1 Introduction

There are a number of processes in the magnetosphere to cause scattering of energetic electrons trapped in the Van Allen radiation belts. Dungey$^1$ suggested that electromagnetic disturbances can produce significant diffusion in $L$-space, energy and pitch angle ($\alpha$). The relative importance of any type of diffusion depends on the type of disturbance, nature of the magnetic storm and $K_{s}$ index as well as energy and location of the particles under consideration$^2$. Cyclotron interaction between whistler mode (ELF/VLF) waves and energetic electrons is one of the main processes to drag these trapped electrons into the loss cone to be lost in the lower ionosphere. In this process, the broad band (or incoherent) noise is amplified and energy of the electrons is reduced. If the electrons, somehow, gain energy, the whistler waves are not amplified but damped and thus no precipitation of energetic electrons takes place$^3-5$. The VLF hiss or plasmaspheric hiss are the kinds of incoherent noise. The interaction between incoherent waves and trapped electrons in terms of pitch angle diffusion has been studied by many workers$^6-7$.

The pitch angle diffusion of energetic electrons by coherent electromagnetic whistler mode waves, too, has been studied extensively$^8,9$. A lot of work in this field is done by Stanford workers$^{10}$. The examples of highly coherent signals are natural whistlers, triggered VLF emissions, large scale power grid radiations and signals that are injected into the magnetosphere by VLF ground transmitters$^{10,11}$. Whereas Ashour-Abdalla$^2$ studied variation of trapped particle flux during such interaction. Inan$^{10}$ computed precipitated flux of energetic electrons by a moderate monochromatic VLF signal.

Most of the work done so far concentrated on either measuring the precipitated flux of energetic electrons ($\phi_{p}$) or wave intensity ($B_{z}^2$) of amplified waves with the help of pitch angle diffusion. This has been done because energy diffusion was found to be very small as compared to pitch angle diffusion. Inan et al.$^{12}$ have shown that during non-linear interaction of coherent whistler waves and energetic electrons the pitch angle diffusion can be as high as 90\%, whereas energy diffusion was $\approx 2\%$. Singh$^{13}$ has recently found energy diffusion to be as high as 50\%.

In this paper, non-linear cyclotron resonance between coherent whistler mode signal$^{14} (f < f_{H})$, where $f$ is the wave frequency and $f_{H}$ the electron gyrofrequency) and radiation belt electrons have been studied. The location for interaction is taken both at equator (the most probable wave amplification region) and away from the equator (the less probable region). An attempt has also been made to see whether the pitch angle diffusion is always
greater than energy diffusion of the particle or not.

2 Method of calculation

2.1 Equatorial resonance

Studying non-linear cyclotron resonance between coherent VLF signal and radiation belt electrons, Inan\textsuperscript{10} computed normalized pitch angle changes \((d\alpha /\alpha)\) and normalized energy changes \((dK/K)\). The ratio of \((d\alpha /\alpha)\) and \((dK/K)\) is presented as follows\textsuperscript{10}.

\[
\eta = \frac{d\alpha}{\alpha} \left/ \frac{dK}{K} \right. = \left[ \frac{V_t}{V_p} \cos^2 \alpha \right] / \alpha \sin 2\alpha \quad \ldots (1)
\]

Here, \(K\) is the kinetic energy of the particle, \(V_t\) the parallel resonant velocity and \(V_p\) the phase velocity. But, we know that \(V_t/V_p = (f_0/f)/(\text{Ref. 3})\). Hence Eq. (1) can be expressed as

\[
\eta = \frac{d\alpha}{\alpha} \left/ \frac{dK}{K} \right. = \left[ \frac{f_0-f}{f} + \cos^2 \alpha \right] / (\alpha \sin 2\alpha)
\]

\ldots (2)

2.2 Off-equatorial resonance

Pitch angle diffusion coefficient \((D_{\alpha\alpha})\) and energy diffusion coefficient \((D_{\nu\nu})\) of the particle are given by

\[
D_{\alpha\alpha} = \langle \Delta \alpha^2 \rangle / J_b \quad \ldots (3)
\]

\[
D_{\nu\nu} = \langle \Delta \nu^2 \rangle / J_b \quad \ldots (4)
\]

where \(J_b\) is the bounce period of electron. For almost linear interaction between coherent whistler signal and energetic electron, Ashour-Abdalla\textsuperscript{2} derived expressions for \(D_{\alpha\alpha}\) and \(D_{\nu\nu}\) for resonance away from the equator as follows.

\[
D_{\alpha\alpha} = \frac{1}{2J_b \sec^4 \alpha} \left[ \frac{1}{V_t^2} \left[ \frac{V_t^3}{V_i^3} (1 - \sigma) \right] \right]^2 \cdot M.N \quad \ldots (5)
\]

and

\[
D_{\nu\nu} = \frac{V_t^2 \sigma^2}{2J_b V^2} \cdot M.N \quad \ldots (6)
\]

where,

\[
\begin{align*}
M &= \left( 1 - \frac{k V_t}{\sigma} \right)^2 \\
N &= \pi \left/ \left[ \frac{dB}{dS} \right] \left[ - \frac{k \mu}{V_i} + \frac{1}{V_i} \right] - f(S_i) \right.
\end{align*}
\]

Here, \(k\) represents the wave number.

All the parameters expressing \(D_{\alpha\alpha}\), \(D_{\nu\nu}\), \(M\), \(N\) are in normalized form\textsuperscript{2}. With the help of Eqs (3)-(6), we can write

\[
\frac{(d\alpha^2)}{\alpha^2} \left/ \frac{(d\nu^2)}{\nu^2} \right. = \left[ \frac{V_t}{V_i} \right] \left[ \frac{V_t^3}{V_i^3} (1 - \sigma) \right] \times \left[ \frac{1}{V_t} - \frac{V_t^3}{V_i^3} \right]^2 \quad \ldots (7)
\]

We can write Eq. (7) as

\[
\eta = \frac{d\alpha}{\alpha} \left/ \frac{d\nu}{\nu} \right. = \left[ \frac{V_t}{V_i} \right] \left[ \frac{V_t^3}{V_i^3} (1 - \sigma) \right]
\]

\ldots (8)

Converting Eq. (8) in initial units from normalized units by writing \(V = V \cdot k / \Omega\) and \(\sigma = \omega / \Omega\) (Ref. 2) we have

\[
\eta = \frac{V_t^2 \Omega}{a V_\perp \omega \Omega} \left[ \frac{\Omega}{\omega} - \frac{V_t^3 k}{V_i^3} \left( \frac{1 - \omega}{\Omega} \right) \right]
\]

\ldots (9)

But, \(V_t = V \cos \alpha\), \(V_\perp = V \sin \alpha\) and \(k V_t = \omega - \Omega\) are cyclotron resonance conditions. Thus,

\[
\eta = \frac{(\omega - \Omega)}{a \omega V_\perp} \left[ \frac{\Omega}{\omega} - \left( \tan^3 \alpha \right) \frac{\omega - \Omega}{\Omega} \times \Omega - \omega \right]
\]

\ldots (10)

Since, \(K = \frac{1}{2} m V^2\), where \(m\) is particle mass,

\[
\frac{dK}{K} = 2 \cdot \frac{dV}{V}. \text{Therefore,}
\]

\[
\eta = \frac{d\alpha}{\alpha} \left/ \frac{dK}{K} \right. = \frac{2 \cot \alpha}{\omega} \frac{\Omega}{\omega} \left[ 1 - \left( \tan^3 \alpha \right) \left( \frac{\Omega - \omega}{\Omega} \right)^3 \right]
\]

\ldots (11)

It is worth noticing that, though, both of the theories of Inan\textsuperscript{10} (equatorial resonance) and Ashour-
Abdalla\(^2\) (off-equatorial resonance) include plasma frequency as effective parameter, our Eqs (2) and (11) are independent of plasma frequency \(f_p\).

3 Results and discussion

3.1 Equatorial resonance

The computations are done for \(L=2\) to \(L=8\). The interacting wave frequency lies in ELF/VLF ranges. To avoid unnecessary details we do the computations in terms of normalized frequency \(\sigma\). Figure 1 shows \(\eta\) variation with pitch angle \((\alpha)\) for equatorial resonance at normalized frequencies \((\sigma)\) 0.2, 0.5 and 0.8. Since at \(\alpha=90^\circ\), \(\alpha \sin 2\alpha\) will be zero [Eq. (2)], the \(\eta - \alpha\) curve at this pitch angle will show asymptotic behaviour. Because of this, \(\eta - \alpha\) variations are not shown at \(\alpha=90^\circ\).

Figure 1 clearly shows that \(\eta\) is \(> 1\) for normalized frequency \(\sigma\leq 0.5\), but at \(\sigma=0.6-0.8\), \(\eta\) will have values \(=1\) indicating that only at \(\sigma > 0.6\), energy diffusion will be either comparative to or more than pitch angle diffusion. Such a thing occurs only for \(\alpha=50-70^\circ\). A Maxwellian pitch angle distribution have greater energy at these pitch angles than at \(\alpha < 50^\circ\). Thus, it is clear that energy diffusion is significant at \(\sigma \geq 0.6\) and it is not always negligible in comparison to pitch angle diffusion.

3.2 Off-equatorial resonance

It has been seen that pitch angle diffusion is not always larger than energy diffusion for whistler mode ELF/VLF waves resonating with energetic electrons in cyclotron mode at the equator. Now for the study of these diffusions for resonances away from the equator, one may ask, where from off-equatorial resonance starts? There is no sharp boundary between these two resonances.\(^{15-17}\) Carlson \textit{et al}.\(^{15}\) have considered equatorial resonance between \(\pm 1^\circ\) geomagn. latitudes around the equator. Since the wavegrowth vanishes\(^{17}\) after geomagnetic latitude \(\lambda = 30^\circ\), we can say that off-equatorial resonance occurs at \(\lambda = 1-30^\circ\). Because many workers may not think our point adequate we, in this case, too, do our calculations in terms of normalized frequency. Table 1 shows \(\eta\) values for \(\sigma = 0.5, 0.8\) at different \(\alpha\) values. It can be shown that \(\eta = 1\) for \(\sigma > 0.5\). It is seen from Table 1 that for \(\sigma = 0.8\), \(\eta\) is less than 1 for \(\alpha \geq 65^\circ\).

Table 1 shows \(\eta\) values for only few pitch angles. If a pitch angle is less than minimum pitch angle shown in Table 1, \(\eta\) will increase. But, what about larger pitch angles which are larger than maximum pitch angle shown in the Table 1? The Eq. (11) shows that there is a limiting pitch angle \((\alpha_L)\) for which \(\eta\) will have zero value. The expression for limiting pitch angle is given by.

\[
\alpha_L = \tan^{-1} \left( \frac{f_H}{f_H - f} \right).
\]

If \(\alpha > \alpha_L\), \(\eta\) will become negative and in this case we will not get amplification of the whistler mode wave. It means that there is a range \(\alpha_0 - \alpha_L\) of

<table>
<thead>
<tr>
<th>(\sigma)</th>
<th>(\eta) values at pitch angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.36</td>
</tr>
<tr>
<td>0.8</td>
<td>[&gt; 1]</td>
</tr>
</tbody>
</table>

*No cyclotron resonance is possible.
pitch angles only for which cyclotron resonance will occur. Here, $\alpha_0$ is the equatorial loss cone pitch angle.

Figure 2 shows variation of $\alpha_L$ with McIlwain parameter ($L$). The lower panel of Fig. 2 shows equatorial loss cone angle $\alpha_0$ at different $L$ shells. Figure 2 clearly reveals that

(i) Interacting pitch angle range increases with the increase in $L$, and

(ii) Interacting pitch angle range increases with the increase in frequency ($f$).

Recently after extending the non-linear theory of VLF-ELF wave generation given by Villalon et al. Singh has shown similar result.

The pitch angle vs. electron energy curve ($\alpha - E$ distribution) for resonant energies applicable for ELF/VLF whistler mode cyclotron waves is Maxwellian type, i.e. at $\alpha = 0^\circ$, the number of electrons as well as their energy is little and at $\alpha = 90^\circ$, the number density and electron energy are highest; hence large part of total energy lies between $\alpha = 50^\circ$ and $90^\circ$. Since, for $\sigma > 0.6$ equatorial and off-equatorial resonances have $\eta < 1$, it can be said that large part of energy is being lost by electrons to the wave at such pitch angles, though they may or may not lie in the loss cone to be lost in the atmosphere. Such loss of energy is not negligible but countable.

We have shown that energy scattering dominates the pitch angle scattering for interacting frequency range $\sigma \geq 0.6$. Energy scattering is directly related with wavegrowth and pitch angle scattering is associated with particle precipitation. It suggests that even for small pitch angle diffusion, large electron energy changes will be required, i.e. though we may get good wavegrowth of waves, we may not observe simultaneous particle precipitation. This is the case for large number of recorded natural whistlers.

Enhanced wavegrowth is responsible for many whistler mode wave phenomena such as frequency broadening, beat excitation of side bands, triggering and new wave generation. Helliwell et al. have discussed, in detail, the process of whistler-mode side-band generation in the magnetosphere on the ground of enhanced wavegrowth. Helliwell et al. gave a model for magnetospheric hiss and adopting the enhanced wavegrowth theory explained the generation of band limited VLF noise. The side-band frequencies in this case have intensity increase of about 10 dB or more.

Nunn and Sazhin have also presented a model for the generation mechanism of hiss triggered chorus. Band-limited VLF hiss emissions in the off-equatorial region of the magnetosphere can deform the electron distribution to give enhanced whistler wavegrowth.

Laaspere and Hoffman have discussed correlation between low energy electrons and auroral hiss. These hiss events were recorded aboard 0G04 spacecraft which was launched on 28 July 1967 into a nearly polar ($86^\circ$) orbit with an apogee of 900 km and perigee of 400 km. Figures 3 and 4 in Ref. 25 clearly show that intensity is larger (frequency broadening is seen alongwith) at around 8 kHz which corresponds to $\sigma \geq 0.6$. Maeeda and Lin, in the same manner, too explained the frequency band broadening of magnetospheric VLF emissions near the equator. These events were recorded aboard Explorer-45 satellite outside the midnight sector of the plasmasphere. The wavegrowth enhancements were observed at $\sigma \geq 0.5$ (or at $\sigma$ greater than half gyrofrequency). Ungstrup reported auroral VLF hiss, which has also larger wavegrowths at these frequencies. These hiss events were recorded at F13, C31/1 and C31/2 satellites of Norway (first one) and Sweden during 26 June 1966-6 Dec. 1967.

Recently, Singh has shown that precipitation increases with frequency and $L$ value. But lesser...
values of $\eta$ at $f \geq 0.6 f_h$ show that one should get less precipitation at higher frequencies. Actually, when a wave interacts with energetic electron distribution in the magnetosphere, then:

(i) Either it causes triggering which gives rise to side band generation or generation of new VLF waves of lesser frequencies etc. alongwith particle precipitation such that precipitation increases with frequency and McIlwain parameter. Such a phenomenon occurs throughout the plasmasphere ($L=3-8$). It has already been discussed in earlier two paras.

(ii) No triggering at all takes place producing wavegrowth and lesser precipitation or no precipitation.

Whistlers with wavegrowth and no precipitation are generally observed. Helliwell\textsuperscript{14} has reported VLF emissions up to normalized frequency $\sigma=0.8$. A large number of emissions were not accompanied by simultaneous particle precipitation.

Let us study now VLF emissions having wavegrowth but lesser particle diffusion. We will also seek the region in which our results are totally applicable. It is well known that a whistler mode wave has reduced normalized frequency $\sigma$, as it moves away from the equator, because of increasing geomagnetic induction ($B_0$). Wave amplification may take place up to and not beyond $\lambda=30^\circ$ (Ref. 17). Thus, for a whistler frequency having $\eta<1$, it is necessary that $\sigma$ should never be below 0.6 in the propagation region of $\lambda=0^\circ$ (equator) to $\lambda=30^\circ$. This is possible at $L \geq 7$, which corresponds to magnetopause location, i.e. outside the plasmasphere.

Sazhin et al.\textsuperscript{28} have reported the observation of very intense VLF waves at normalized frequency $\sigma \geq 0.5$. These VLF events were recorded aboard AMPTE-UKS satellite on 2 and 9 Oct. 1984 at or around the magnetopause ($L \geq 7$). They\textsuperscript{28} tried to explain the intensification on the basis of wave trapping in the magnetopause region, but they were of the view that some other process may also be there to cause intensification. The intensification of these high latitude VLF emissions may be explained very well by the present model. But as per the present model, we should have weak diffusion at these frequencies/location. Adopting the theory proposed by Sonnerup\textsuperscript{29} for the formation of magnetopause boundary layer, Sazhin et al.\textsuperscript{28} found the computed diffusion to be weak. Tsurutani and Thorne\textsuperscript{30} have also shown that contribution of whistler mode ELF-VLF waves is too small towards primary mechanisms of diffusion processes in the vicinity of magnetopause. Such waves due to trapping and reflection could only be detected aboard satellites.

Much more explanation is beyond the scope of the paper, and is also impossible for us due to lack of required data.

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