Generation of perturbation electric fields suitable for triggering of equatorial spread-F by gravity waves in the E-region and zonal winds in the F-region

S. Prakash
Physical Research Laboratory, Ahmedabad 380 009
Received 1 March 1996

A new mechanism for the generation of perturbation electric fields which can produce irregularities in electron density in the F-region, which in turn can act as seed irregularities suitable for the growth of equatorial spread-F is described. The mechanism involves (i) gravity wave winds in the E-region producing electron density irregularities which in turn make the flux tube integrated Pedersen and Hall conductivities non-uniform in this region, (ii) eastward winds in the base of F-region and in the intermediate region which give rise to downward electric fields through dynamo action, (iii) these downward electric fields transport the plasma from 130 km region into the lower E-region and thus enhance the Hall conductivity in the E-region, (iv) the eastward electric fields transport the ionization upwards, thereby decreasing the conductivity in the region between 130 km and the base of the F-region, and (v) the zonal currents, driven by the electrojet fields and the F-region dynamo fields, while flowing in the non-uniformly conducting E-region give rise to the required perturbation electric fields. Using realistic values of various parameters, the amplitudes of perturbations in the vertical plasma drift velocities was found to be as large as 6.5 ms\(^{-1}\). This amplitude is much larger than 1.3 ms\(^{-1}\) required to produce 5% perturbation in electron density near the base of the F-region normally used in the simulation of the equatorial spread-F.

1 Introduction

The equatorial spread-F (ESF), which is a late evening phenomenon, has been extensively studied using a large number of space- and ground-based techniques. While many of the features of ESF are now well understood, there are a few which need further investigations. One of the outstanding problems is the triggering of ESF. Even though the Rayleigh-Taylor instability (reviews by Basu and Kelley\(^1\); Ossakow\(^2\)) has been found to be quite suitable to explain the growth of ESF irregularities, there are some features of ESF which indicate that the Rayleigh-Taylor instability (RTI) does not grow from the background ion density variations produced through thermal fluctuations but instead from the initial perturbations produced by some other agency. Forward scatter measurements\(^3\) in the African sector show regular wave-like region of enhanced plasma density. Rottger\(^3\) argued that the spatial resonant gravity wave mechanism is responsible for the organized structures in ESF. Klostermeyer\(^4\) concluded that with the spatial resonance effect, including the nonlinear processes, the gravity waves (GW) could organize the F-region plasma into large scale horizontal structures. On some nights the ESF irregularities seem to grow when the F-region plasma is either at its peak or is coming down while the growth rate of RTI would be largest when the plasma is moving up, i.e. the primary electric field is eastward. The Jicamarca VHF backscatter radar data for March 21, 1979 of Kelley et al.\(^5\) and for October 6, 1984 of Hysell et al.\(^6\) provide some very good examples. This indicates that the growth rate of RTI does not seem to be large enough to explain the growth of ESF from the ion density variations due to thermal fluctuations in a period of few tens of minutes. With their VHF backscatter radar studies, Hysell et al.\(^6\) demonstrated the existence of the layering of the plasma which he believed was due to a GW effect.

Kelley et al.\(^5\) concluded that while a pure GW theory explaining ESF is not tenable, it can induce finite amplitude modulation of the bottom-side F-layer even if the spatial resonance is only partially attained. It is now generally believed that the initial trigger is due to GW in F-region. The following observations indicate that GW producing the required perturbation electric fields (PEF) may not be exclusively located in F-region.

(a) The radar maps over Jicamarca, such as
given by Kelley et al. and Hysell et al., show that on many strong ESF days the plumes are uniformly spaced and the spacing between them varies from 150 km to less than 50 km. If these ESF plumes are initiated by GW, then the wavelength of GW should be less than 150 km. Hines showed that at 225 km altitude, GW have a lower wavelength cut-off of 100 km due to dissipation by molecular viscosity. At higher altitudes, such as at 400 km where ESF is initiated, the lower wavelength cut-off would be much larger than 100 km. Therefore, GW with zonal wavelengths in the range of 50 to 150 km, even if present, will be highly attenuated.

(b) The early Jicamarca studies of Farley et al. showed that the onset of ESF usually began when the F-region was above certain minimum altitude \((h_{\text{min}})\) and the irregularities occurred at \(h_{\text{min}}\) and then moved upwards. Sometimes \(h_{\text{min}}\) was as low as 240-250 km although usually it ranged between 300 and 350 km. If PEF were due to GW in F-region, then due to loading from the lower region, the amplitudes of PEF would be largest in the higher density regions. Hence the triggering of ESF should take place in the high density region or simultaneously throughout the base of F-region but this is not what is observed.

(c) In the F-region, the Brunt-Vaisala period \((T_{N})\) is about 14.7 min, and hence the maximum possible phase velocity of GW with zonal wavelength of 50 to 150 km would be 57 to 170 ms\(^{-1}\). As the zonal wind velocity in the F-region ranges between 100 and 200 ms\(^{-1}\) from the studies of Meriweather et al., Sipper et al., and Wharton et al., the phase velocity of GW would be smaller or somewhat equal to the zonal wind velocity. The GW propagating from the lower region are, therefore, not likely to penetrate into the F-region.

(d) Prakash and Pandey showed that GW which have wavefronts parallel to the geomagnetic field \((B)\) can generate perturbing electric fields through dynamo action which can be transmitted from one region to another. In the F-region this condition can be satisfied only if GW propagate in a very narrow angle around the zonal direction. This puts a severe restriction on the probability of the availability of GW for producing PEF. This also rules out the possibility of the GW of auroral origin producing a suitable trigger for ESF.

In continuation with our earlier studies, Prakash examined the suitability of the gravity wave wind (GWW) in the E-region and the zonal winds in F-region to provide suitable PEF through electron density irregularities they could generate in the E-region. Dagg pointed out the importance of small scale electric fields to explain the ESF irregularities. Using horizontal winds derived from a stream function, Farley concluded that at lower latitudes some fluctuations in electron density could be produced but these would probably be too weak to be significant. Figure 1 gives a block diagram of the mechanism presented in this paper for the generation of the required PEF with horizontal wavelengths of 50 to 150 km. In the E-region, GWW gives rise to electron density irregularities and thus make the conductivity in the E-region non-uniform. The electric fields generated in and below the F-region by the zonal winds are transmitted to the E-region and give rise to electric currents there and also bring down the ionization into the 110 km region and thus increase the Hall conductivity of the E-region. These currents in the non-uniformly conducting E-region would give rise to PEF with scale sizes related to GW which produced the electron density irregularities.

This paper is organized in the following manner. In Section 2, we derive an expression relating PEF to perturbations in flux tube integrated (FTI) Pedersen and Hall conductivities. In Section 3, we present the data from which the parameters of GW relevant for this study are derived. Section 4 gives the results of numerical simulation of the electron density and electron density irregularities in E-region generated by GWW in this region. We also calculate FTI conductivities and their amplitudes of variations. In Section 5, we derive an expression relating electron density irregularities to perturbations in FTI conductivities. In Section 6, numerical simulation of FTI Pedersen conductivities above 130 km altitude is presented. In Section 7, we derive the threshold values of PEF required for producing 5% perturbations in electron density in the F-region during half the wave period. In Section 8, we combine the expressions derived in Sections 2 and 5 and derive full relationship between PEF with the electron density irregularities and discuss the nature of each parameter appearing in the expression. The amplitude of PEF for the realistic values of various parameters responsible for its generation is also given.

2 Theory: Derivation of the expression for perturbation electric field

In this section we write down the current equations for the E-, M- and F-regions and derive the expression for PEF which can give rise to seed irregularities in the F-region. Here we use both the Cartesian and the cylindrical coordinate systems, represent-
ed respectively by \((X, Y, Z)\) and \((\rho, \theta, Y)\), where \(X\) is towards north and \(Z\) is vertically upwards, \(Y\) is towards the magnetic west and is common to both the systems. \(\theta\) is the angle of inclination of the radius vector \(\rho\) in the meridional plane from the vertical. The meridional plane is represented by \(Z, X\) and by \(\rho, \theta\) in the two coordinate systems.

For convenience, we divide the region encompassed by the geomagnetic field lines into three domains: (i) from 90 to 130 km, where the Hall conductivity is large and will be referred to as E-region and denoted by the suffix E, (ii) from 130 km to the region below the base of F-region and will be referred to as the intermediate region (middle region) and denoted by the suffix M, and (iii) from the base of F-region and above and this will be referred to as the F-region and will be denoted by the suffix F. The current equations for the M- and F-regions are the same and hence the equations for the M-region are not written down separately and can be recovered from those of F-region, whenever necessary. We write the current equations separately for regions (i) and (iii) and then add them to represent the total current over a given field line.

\[
\begin{align*}
\mathbf{j}_E &= \sigma_{1E}(E_y + Bw_{pE}) - \sigma_{2E}(E_\rho - Bw_{pE}) \quad \cdots (1a) \\
\mathbf{j}_\rho &= \sigma_{1E}(E_\rho - Bw_{pE}) + \sigma_{2E}(E_y + Bw_{pE}) \quad \cdots (1b) \\
\mathbf{j}_\theta &= \sigma_{0E}E_{\theta E} \quad \cdots (1c) \\
\mathbf{j}_y &= \sigma_{1F}(E_y + Bw_{yF}) - n_e e (gB\Omega_i) - n_e e (gB\Omega_e) \quad \cdots (2a) \\
\mathbf{j}_\rho &= \sigma_{1F}(E_\rho - Bw_{yF}) \quad \cdots (2b) \\
\mathbf{j}_\theta &= \sigma_{0F}E_{\theta F} \quad \cdots (2c)
\end{align*}
\]

The parameters \(j_x, j_\rho, j_\theta\) represent current densities in zonal, radial, and parallel to \(B\) directions respectively. \(w_x, w_\rho, w_{y}\) are large scale winds, \(\sigma_{1E}, \sigma_{2E}, \sigma_{0E}\) represent Pedersen, Hall and parallel conductivities per electron. The last term in the expression for \(j_y\) represents ion drift currents, where \(n_e\) is the electron density, \(e\) is the electronic charge, \(g\) is the acceleration due to gravity, \(\Omega_i\) and \(\Omega_e\) are respectively the ion and electron gyro-frequencies. The electron drift current has been neglected as it is much less than the ion drift current. Haerendel\textsuperscript{18} pointed out that a proper treatment of ESF bubbles requires the use of flux tube integrated variables. The three current densities are, therefore, integrated over the flux tubes.
in their respective regions and added to give total currents represented by \( J_y \), \( J_p \), and \( J_\theta \). Three different types of terms appear in the integrals.

(a) There are six terms containing \( E_y \) and \( E_p \) and as these fields can be assumed to be constant for \( \lambda_y \) and \( \lambda_p > \text{few tens of km} \), they can be taken out of the integrals. Therefore, the integrand of these terms can be expressed as product of the electric fields and FTI conductivities represented as \( \Sigma_{1E}, \Sigma_{2E}, \Sigma_{1F} \) over the appropriate regions.

(b) In the ion drift current, the drift velocity of ions can be assumed to be constant and hence the electron density can be easily integrated. The integrated electron density in the intermediate region and the F-region has been represented by \( N_{eM} \) and \( N_{ef} \) respectively.

(c) There are six different terms containing large scale winds and FTI conductivities. In case the winds do not remain constant in a given region, they cannot be taken out of the integral and, therefore, the integrands of these terms are represented as a product of FTI conductivities and weighted means of the winds. In the F-region the weighted means are taken over \( \sigma_{1F} \) and \( \sigma_{2F} \). Although, the weighted mean can be taken over \( \sigma_{1E} \) and \( \sigma_{2E} \) separately, it was considered unnecessary as the details of the winds in the E-region are not known and further they are believed to be much less than the winds in the F-region. In the E-region, as \( \sigma_{2E} \) is larger than \( \sigma_{1E} \), the weighted means were taken over \( \sigma_{2E} \) and represented as \( W_{yE}, W_{pE} \) and these were used for \( \sigma_{1E} \) term also.

The combined integrated values of the three currents \( J_y, J_p, J_\theta \) are given by the following equations.

\[
J_y = \Sigma_{1E}(E_y + B W_{OE}) - \Sigma_{2E}(E_p - B W_{OE}) + \Sigma_{1F}(E_y + B W_{OF}) - N_{ef} e g B \Theta_1 \quad \ldots (3a)
\]

\[
J_p = \Sigma_{1E}(E_p - B W_{OE}) + \Sigma_{2E}(E_y + B W_{OE}) + \Sigma_{1F}(E_p - B W_{OF}) \quad \ldots (3b)
\]

\[
J_\theta = \int \sigma_{1E} E_{\theta E} \rho d\theta + \int \sigma_{0F} E_{\theta F} \rho d\theta \quad \ldots (3c)
\]

The combined current for all the three regions can be written as

\[
J_y = \Sigma_{1E}(E_y + B W_{OE}) - \Sigma_{2E}(E_p - B W_{OE}) + \Sigma_{1M}(E_y + B W_{OM}) + \Sigma_{1F}(E_y + B W_{OF}) - (N_{eM} + N_{ef}) e g B \Theta_1 \quad \ldots (4a)
\]

\[
J_p = \Sigma_{1E}(E_p - B W_{OE}) + \Sigma_{2E}(E_y + B W_{OE}) + \Sigma_{1M}(E_p - B W_{OM}) + \Sigma_{1F}(E_p - B W_{OF}) \quad \ldots (4b)
\]

\[
J_\theta = \int \sigma_{1E} E_{\theta E} \rho d\theta + \int \sigma_{0M} E_{\theta M} \rho d\theta \quad \ldots (4c)
\]

As these currents have been integrated over \( \theta \), they are not functions of \( \theta \) but of \( \Theta \) and \( \rho \) only. It may be noted that the field line integrated current \( J_p \) represents the meridional currents. In the equatorial region, zonal and meridional currents form a part of the same current loop which is driven by the large scale dynamo fields outside the equatorial region. During evening hours, when the electric fields reverse from east to west direction, the zonal currents converge towards the reversal region and hence during this period the meridional currents are positive. The reverse is true during the morning reversal periods. Normally, the evening electric field reversal takes place over a period of more than one to two hours which corresponds to a distance of 1500 to 3000 km or more in the zonal direction. This distance is much larger than the zonal wavelength of GW used here. The current equations can, therefore, be linearized for perturbations induced by GW and the linearized equations can be used to carry out perturbation analysis. In this study the perturbations are produced by GW and we introduce perturbations of the form

\[
\Delta q = \delta q \exp i(K_y \Theta + K_p \rho - \omega t) \quad \ldots (5)
\]

where \( q \) represents \( \phi, \Sigma_{1E}, \Sigma_{2E}, J_y, J_p, J_\theta \) where \( \phi \) represents the electrical potential. \( \nabla \cdot J \) in the cylindrical coordinates is given by the following equation.

\[
\nabla \cdot J = \partial(\Delta J_y)/\partial y + 1/\rho \partial(\rho \Delta J_p)/\partial \rho + \partial(\Delta J_\theta)/\rho \partial \theta \quad \ldots (6)
\]

As the radius of the field line \( \rho_p \) is much larger than the wavelength of the perturbing waves under consideration, Eq. (6) can be simplified as

\[
\nabla \cdot J = \partial(\Delta J_y)/\partial y + \partial(\Delta J_p)/\partial \rho + \partial(\Delta J_\theta)/\rho \partial \theta \quad \ldots (7)
\]

For charge neutrality, we substitute the value of \( J \) in the above equation from Eqs (4a), (4b) and (4c) and equate it to zero and we get

\[
(\Sigma_{1E} + \Sigma_{1M} + \Sigma_{1F})[K_y^2 + K_p^2] \delta \phi = -i[K_y(E_y + B W_{OE}) + K_p(E_p - B W_{OE})] \delta \Sigma_{1E}
\]
\[-i\left(-K_y(E_y-BW_{yE})+K_y(E_y+BW_{yE})\right)\delta\Sigma_{2E}\]  

Denoting
\[K_y/K_y=\tan \varphi\]  
and,
\[(E_y+BW_{yE})(E_y-BW_{yE})=R\]  
and representing
\[\Sigma_{1E}+\Sigma_{1M}=\Sigma_{1L}\]  

where suffix L represents values in the lower region which includes, the E-region and the M-region. As \(\delta \phi = i \delta E_y/K_y\), we get from Eq. (8)
\[\delta E_y=\left[-(R+\tan \varphi)\cos^2 \varphi(\delta\Sigma_{1E}/\delta\Sigma_{2E})\right.\]
\[+\left.(1-R \tan \varphi) \cos^2 \varphi(E_y-BW_{yE})\right]\]
\[\times(\delta\Sigma_{2E}/\delta\Sigma_{2E})\left(\Sigma_{2E}/(\Sigma_{1L}+\Sigma_{1E})\right)\]  

The expression for \(\delta E_\rho\) is given by
\[\delta E_\rho=\delta E_yK_y/K_y\]  
Using Eq. (9), the above equation can be written as
\[\delta E_\rho=\delta E_y \tan \varphi\]  

As can be seen from the above equation, PEF can be produced, both through the perturbations in FTI Pedersen and Hall conductivities in the E-region. It will be shown later that the perturbations in FTI Hall conductivity are relatively more effective in the generation of PEF.

### 3 Gravity waves in the E-region

We discuss here the expression and parameters of GW which are required for the present study. As pointed out earlier in Section 1, the VHF backscatter radar maps over Jicamarca show, on some days, a number of plumes with equal spacing which is in the range of 50 to 150 km. If PEF seeding these plumes are produced by GWW, then the horizontal spacing of these plumes would correspond to the zonal wavelength of GW. If PEF is produced through electron density irregularities in the E-region, as is hypothesized in this paper, the electron density profile over a low latitude non-equatorial station such as at SHAR can be used to estimate the vertical wavelength of GW. The in situ measurements of electron density over SHAR, given in Fig. 2, show that in the E-region during late evening hours, the electron density irregularities are very prominent and the electron density in them may vary by as much as

---

![Fig. 2](https://via.placeholder.com/150)  

**Fig. 2**—Electron density versus altitude for six rocket launches conducted from SHAR (13°42'N, 80°E, dip 14°N), India. For clarity the scale for each of these electron density profiles is shifted by one decade.
a factor of ten. The vertical scale size of irregularities determined through scaling with eyes was about 24 km for flights SHAR 560.16 and 560.26. If these irregularities are produced through GWW, they would indicate the presence of GW with large amplitudes and with wavelengths right up to 24 km.

From the vapour release experiments, Kochanski\textsuperscript{19} found that the dominant modes of GW have vertical wavelength of 12 km and 16 km at 90 and 120 km altitude respectively. In this altitude range, the vertical wavelength of GW could be as high as 24 km and the amplitude as much as 80 ms\textsuperscript{-1}. The scale sizes derived from the electron density profiles over SHAR are in fair agreement with those derived from the above vapour release experiments. Following Hines\textsuperscript{8}, the GWW in the wavelength range up to a few hundred km can be represented as a plane wave by the equation

\[ W = W_0 \exp\{Z/2H + i(K_x X + K_y Y + K_z Z - \omega t)\} \]  \hspace{1cm} \ldots (15)

where \( H \) is the scale height of the neutral density and

\[ K_x W_x + K_y W_y + K_z W_z = 0 \]  \hspace{1cm} \ldots (16)

The period \( T_w \) of GW is given by

\[ T_w = T_g \left[ 1 + \frac{K_z^2}{(K_x^2 + K_y^2)} \right]^{0.5} \]  \hspace{1cm} \ldots (17)

where \( T_g \) is the Brunt-Vaisala period which is about 4.9 min in E-region. A field line with an apex of 350 km will have a dip angle of 20\degree in E-region. As the condition \( K_z = 0 \) has a very special significance in the generation of electric field and perturbation in \( \Sigma_{2E} \), we calculate the period of GW when this condition is satisfied. The condition \( K_z = 0 \) implies

\[ K_x/K_y = \tan I \]  \hspace{1cm} \ldots (18)

Therefore Eq. (17) reduces to

\[ T_w = T_g \left[ 1 + 1/(\tan^2 I + K_x^2/K_y^2) \right]^{0.5} \]  \hspace{1cm} \ldots (19)

It can be seen from the above equation that for non-zero values of \( K_y \), \( T_w < T_g \) cosec \( I \). For \( I = 20\degree \), \( T_w < 2.92 \) \( T_g \) or 14.3 min. In the present study, the maximum value of \( K_x/K_y \) is encountered for \( \lambda_x = 24 \) km and \( \lambda_y = 50 \) km corresponding to \( K_x/K_y = 0.48 \), and in this case \( T_w = 1.88 \) \( T_g \). This would correspond to a period of 9.2 min. It can, therefore, be seen that GW which have wavefronts parallel to the geomagnetic field lines have relatively smaller periods.

4 Numerical simulation of electron density and electron density irregularities and conductivities in the E-region

The time dependent continuity equation was solved numerically for the simulation of electron density irregularities by Prakash \textit{et al.}\textsuperscript{21} for the altitude range of 90 to 200 km. These were used for the calculations of the FTI Pedersen and Hall conductivities and perturbations in these conductivities for a low latitude station. For these simulations, the ion production rates were taken from Strobel \textit{et al.}\textsuperscript{22} and the recombination coefficient from Hanson \textit{et al.}\textsuperscript{23}, and the time dependent continuity equation was solved for ions in a moving frame following the motion of plasma in \( Y \) and \( Z \) directions. For the simulations of electron density irregularities, GW with wavelengths, wind amplitudes and periods commensurate with the observations were used. The GW equations were taken from Hines\textsuperscript{8}, their range of parameters from Kochanski\textsuperscript{19} and the convergence rate due to GWW through the wind shear mechanism\textsuperscript{24} from Prakash and Pandey\textsuperscript{25}. For the present study, we shall use the following results of this simulation: (i) the electron density profiles and the Pedersen and Hall conductivities per electron, (ii) the FTI Pedersen and Hall conductivities, (iii) perturbations in the FTI Pedersen and Hall conductivities induced by the electron density irregularities, and (iv) relationship between the amplitude of GWW and the perturbations in the FTI Pedersen and Hall conductivities.

We first discuss the density profile in the absence of GW, i.e. the amplitude of GWW is made zero. Based on VHF radar observations over Jimacara of Woodman\textsuperscript{26,27}, the vertical and the eastward drifts were assumed to be \( \pm 30 \) ms\textsuperscript{-1} and 100 ms\textsuperscript{-1} respectively. We assume the geomagnetic field to be 0.5 G and therefore the corresponding electric fields would be \( E_y = \pm 1.5 \) mV\textsuperscript{-1} and \( E_x = -5.0 \) mV\textsuperscript{-1}. The numerical simulation studies show that when \( E_y = 1.5 \) mV\textsuperscript{-1} and \( E_x = -5.0 \) mV\textsuperscript{-1}, the maximum value of electron density is \( 4.9 \times 10^3 \) cm\textsuperscript{-3} occurring at an altitude of 108 km, while when \( E_y = -1.5 \) mV\textsuperscript{-1} and \( E_x = -5.0 \) mV\textsuperscript{-1}, the maximum value of electron density is \( 3.9 \times 10^3 \) cm\textsuperscript{-3} occurring at an altitude of 106.5 km. It may be noted that in both the cases the electron density profile has a valley at altitudes of 128 to 129 km.

We now consider the shape of the electron density profile in the above two cases where \( E_y = \pm 1.5 \) mV\textsuperscript{-1}. In the first case, while the maximum value of electron density occurred at an altitude of 108 km., it decreased to half of its
maximum value at 100 and 115.5 km altitudes. We refer these three altitudes as \( Z_0, h_1 \) and \( h_2 \). In the second case, \( Z_0, h_1 \) and \( h_2 \) were respectively at 106.5, 98 and 113 km. In the E-region, the Hall conductivity per electron, \( \sigma_{2E} \), remains constant within 10% of the maximum value in the altitude range of 90 to 114 km and above 117 km it decreases with altitude with a scale height of 7 km. The contribution of the electron density at higher altitudes to the Hall conductivity would, therefore, be negligible. In the first case, the Hall conductivity integrated over the altitude range of 97 to 119 km is 86% of the conductivity integrated over the altitude range of 90 to 200 km. Similarly in the second case, the Hall conductivity integrated over the altitude range of 95.5 to 117.5 km is 88% of the conductivity integrated over the altitude range of 90 to 200 km. It can, therefore, be concluded that the Hall conductivity is confined to a very limited region and this property of the Hall conductivity can be made use of in the integration of the expression given in Eq. (26). For this expression to be easily integrable, the electron density profile should be represented either (i) as a sine or cosine function of \( Z \) or (ii) as a step function, where the electron density is constant within an altitude range and negligible outside or (iii) as a combination of both. It can be seen from the above simulation that the electron density in the E-region can be well represented by the function

\[
n_{e} = n_{e0} \cos[\pi (Z - Z_0)/\Delta h] \quad \ldots (20)
\]

where

\[
\Delta h = 1.5 (h_1 - h_2) \quad \ldots (21)
\]

and \( n_{e0} \) is the maximum value of electron density in the E-region. In the above two cases, \( \Delta h \) is \( \approx 22 \) km.

The Pedersen conductivity per electron has a maximum value at 122.5 km and a minimum value at 96.5 km and the ratio of conductivity at these two altitudes is 15. The value of Pedersen conductivity at 169 km is the same as at 96.5 km. In the altitude range of 100 \((h_1)\) and 115 \((h_2)\) km, the Pedersen conductivity varies by a factor of 8.4. Therefore, the Pedersen conductivity term in Eq. (26) cannot be used to determine the perturbations in the Pedersen conductivity. From the numerical simulations, it is found that \( \delta \Sigma_{2E} < 0.18 \delta \Sigma_{2E} \) and it is shown in Section 8 that the contribution of the Pedersen conductivity term to \( \delta E_3 \) can be negligible.

We now discuss numerical simulation of the irregularities in the electron density and the irregularities in FTI conductivities. Simulation studies have been carried out for GW with horizontal wavelengths ranging from 50 to 150 km and vertical wavelengths from 12 to 24 km. \( E_y \) was varied from \(-1.5 \) mV m\(^{-1}\) to \(+1.5 \) mV m\(^{-1}\) and \( E_y \) from \(-10 \) mV m\(^{-1}\) to \(+5 \) mV m\(^{-1}\). The amplitude of GWW was varied from 0 to 40 ms\(^{-1}\). The zero wind case represents the density profile in the absence of GW.

As the radial electric field gives rise to the vertical transport of the plasma, the Hall conductivity in the E-region increases with the decrease of \( E_y \). It is found that the Hall conductivity increases by 18% when \( E_y \) is varied from 0 to \(-5.0 \) mV m\(^{-1}\). The values of \( \delta \Sigma_{2E}/\Sigma_{2E} \) are calculated for \( E_y = -5 \) mV m\(^{-1}\), \( E_y = 0 \), amplitude of GWW of 30 ms\(^{-1}\), \( \lambda_y = 100 \) km, and vertical wavelength ranging from 16 to 24 km. The values of \( \delta \Sigma_{2E}/\Sigma_{2E} \) are found to be 4.4%, 4.4% and 6.5% respectively for \( \lambda_y \) of 16, 20 and 24 km. The average value of \( \delta \Sigma_{2E}/\Sigma_{2E} \) for the above wavelength range is 5%.

The values of \( \delta \Sigma_{2E}/\Sigma_{2E} \) are calculated for different amplitudes of GWW when \( K_y = -2\pi/100 \) km and \( K_y = -2\pi/20 \) km. The values of \( \delta \Sigma_{2E}/\Sigma_{2E} \) are respectively 2.7%, 5.2% and 6.5% for GWW amplitude of 10, 20 and 30 ms\(^{-1}\) respectively. It can therefore be seen that \( \delta \Sigma_{2E}/\Sigma_{2E} \) is somewhat linearly related to GWW when they are small.

5 Expression relating parameters of electron density irregularities and perturbations in flux tube integrated conductivities

In this section we derive a relationship between the electron density irregularities and the perturbations in the FTI Pedersen and Hall conductivities under simplifying assumptions. For simplicity we assume that the electron density irregularities can be represented as monochromatic waves with a constant fractional amplitude. We also assume that the background electron density can be represented as a cosine function of the altitude. This approximation is quite valid as shown in Section 4. It may be noted that this approximation does not take into account the nonlinearity in the electron density irregularities and hence can be used mainly to explain the physical process which gives rise to perturbations in the FTI Hall conductivity. It may not give the numerical values of perturbations correctly in all the cases.

The electron density irregularities with vertical scale sizes up to 24 km have been observed in the E-region over SHAR during early evening by Gupta\(^{28}\) and late evening hours by Prakash et al.\(^{25}\)
and Sinha and Prakash\textsuperscript{30} with amplitude ranging from 0.4 to 0.8 times the background electron density. The horizontal scale sizes of electron density irregularities are difficult to determine from the rocket data. The fractional amplitude of irregularities observed over SHAR was found to be largest during the late evening hours. If the electron density irregularities are produced by GW and they can be assumed to be monochromatic, then the fractional amplitude of the electron density irregularities can be represented by
\begin{equation}
\Delta b = \delta b \sin(K_x X + K_y Y + K_z Z - \omega t) \quad \ldots (22)
\end{equation}

The total electron density can therefore be represented as
\begin{equation}
n_e = n_0 + \delta b n_0 \sin(K_x X + K_y Y + K_z Z - \omega t) \quad \ldots (23)
\end{equation}
where $\delta b$ is the fractional amplitude of the electron density irregularities and $n_0$ is the background electron density. Let $(X, Y, Z_0)$ be the coordinates of a mid-point in the E-layer through which a given geomagnetic field line passes. While $X$ and $Y$ will vary from one field line to another, $Z_0$ can be assumed to be a constant, as it is a mid-point of the E-region which can be assumed to be horizontally stratified. As the extent of the E-region is much less than the radius of curvature of the field line, the geomagnetic field line in the above region can be represented by a straight line given by
\begin{equation}
z = -x \tan l \quad \ldots (24)
\end{equation}
where $z$ and $x$ are the coordinates of the field line with the point $X, Y, Z_0$ as the origin. If the thickness of the E-layer is $\Delta h$, then from Eq. (23), the conductivities $\Sigma_{2E}$ and $\Delta \Sigma_{2E}$ integrated over the field line are given by
\begin{equation}
\Sigma_{2E} = \int_{-\Delta h/2}^{\Delta h/2} \sigma_{20} n_e \cos \omega t \, dz \quad \ldots (25)
\end{equation}
\begin{equation}
\Delta \Sigma_{2E} = \int_{-\Delta h/2}^{\Delta h/2} \sigma_{20} n_e \delta b \cos \omega t \, dz \quad \ldots (26)
\end{equation}
where $\sigma_{20}$ represents the maximum Hall conductivity per electron in the E-region.

In 100 to 120 km region, the Hall conductivity per electron remains fairly constant. The expression given in Eq. (26) becomes easily integrable when the electron density as a function of $Z$ can be represented either (a) as a sine or cosine function of $Z$ or (b) as a step function, where the electron density is constant within an altitude range and negligible outside it or (c) as a combination of both. As per the discussion in Section 4, we use Eq. (20) to represent the electron density in the Hall conductivity region, and as
\begin{equation}
z = Z - Z_0 \quad \ldots (27)
\end{equation}
Eq. (20) can be written as
\begin{equation}
n_e = n_0 \cos[\pi(Z - Z_0)/\Delta h] = n_0 \cos(\pi z/\Delta h) \quad \ldots (28)
\end{equation}
Substituting the value of $n_e$ from Eq. (28) and $x$ from Eq. (24), Eq. (26) can therefore be written as
\begin{equation}
\Delta \Sigma_{2E} = \frac{\Delta h^2}{\Delta h} \int_{-\Delta h/2}^{\Delta h/2} \sigma_{20} \delta b \cos \omega t \, dz \quad \ldots (29)
\end{equation}
where
\begin{equation}
\Psi = (K_z - K_z \cot l) \Delta h/2 \quad \ldots (30)
\end{equation}
From Eq. (29), the amplitude $\delta \Sigma_{2E}$ is therefore given by
\begin{equation}
\delta \Sigma_{2E} = \delta b \pi n_0 \sigma_{20} \Delta h \cos \Psi / \pi ^2 \quad \ldots (31)
\end{equation}
Substituting the value of $n_e$ from Eq. (20) in Eq. (25), we get
\begin{equation}
\Sigma_{2E} = \frac{\Delta h^2}{\Delta h} \int_{-\Delta h/2}^{\Delta h/2} \sigma_{20} \cos(\pi z/\Delta h) \cos \omega t \, dz \quad \ldots (32)
\end{equation}
Integrating RHS of the above equation, we get
\begin{equation}
\Sigma_{2E} = 2 n_0 \sigma_{20} \cos \Psi / \pi \quad \ldots (33)
\end{equation}
From Eqs (31) and (33), we get
\begin{equation}
\frac{\delta \Sigma_{2E}}{\Sigma_{2E}} = \frac{\delta b \cos \Psi / (1 - 4 \Psi ^2 / \pi ^2)}{\delta b \Gamma} = \delta b \Gamma \quad \ldots (34)
\end{equation}
where
\begin{equation}
\Gamma = \cos \Psi / (1 - 4 \Psi ^2 / \pi ^2) \quad \ldots (35)
\end{equation}
As $Z_0$ is a constant, $K_z Z_0$ in Eq. (29) can be represented by a constant $C$. This is equivalent to
$K_z = 0$. The contribution of $K_z$ is to introduce a phase shift $C$. Thus the perturbations in the FTI Hall conductivities over different field lines are a function of $X$ and $Y$ but not of $Z_0$ and hence $\Delta \Sigma_{2E}$ can be written as

$$\Delta \Sigma_{2E} = \delta \Sigma_{2E} \sin (K_x X + K_y Y + C - \omega t) \quad \ldots (36)$$

From the field line geometry, $K_x$ and $X$ can be respectively represented by $K_p$ and $p$ in the expression for $\Delta \Sigma_{2E}$ through the following equations.

$$K_x = K_p \csc l \quad \ldots (37)$$

and

$$\rho = X \sin l \quad \ldots (38)$$

Therefore

$$K_x X = K_p \rho \quad \ldots (39)$$

Eq. (36) can therefore be written as

$$\Delta \Sigma_{2E} = \delta \Sigma_{2E} \sin (K_p \rho + K_y Y + C - \omega t) \quad \ldots (40)$$

From Eqs (9) and (37), we get

$$K_x = K_p \tan \varphi \sin l \quad \ldots (41)$$

hence

$$K_x / K_y = \tan \varphi \sin l = \tan \alpha \quad \ldots (42)$$

where $\alpha$ is the angle of inclination of the wave vector of GW in the horizontal plane from the $Y$ axis. From Eqs (17) and (41), the period of gravity waves would be given by

$$T_w = T_b \left[1 + K_x^2 / K_y^2 \left(1 + \tan^2 \varphi \sin^2 l\right)\right]^{0.5} \quad \ldots (43)$$

We now study the nature of $\Psi$ and $\Gamma$ which determines the value of $\delta \Sigma_{2E}$. The function $\Gamma$ will be maximum when $\Psi = 0$ and its maximum value is one. $\Gamma$ is symmetrical around $\Psi = 0$, and its value decreases with increase of $\Psi$ and changes sign when $\Psi = (2n + 1) \pi / 2$, where $n$ is an integer. For example, $\Gamma$ has values of 1.0, 0.73, 0.33, 0.0, -0.07 and 0 respectively for $\Psi$ of 0, 0.5 $\pi$, $\pi$, 1.5 $\pi$, 2 $\pi$ and 2.5 $\pi$. From Eq. (30), $\Psi$ will be zero when $K_x = K_x \cot l$. This condition is the same as Eq. (18) implying that the wavefronts of GW should be parallel to the geomagnetic field line for $\Psi$ to be zero. As $l$ is positive in the northern hemisphere and negative in the southern hemisphere, the above condition is satisfied when $K_x / K_y$ is positive in the northern hemisphere and negative in the southern hemisphere. As $K_y$ is negative for most of the GW, it implies that the GW satisfying the requirement of $K_y = 0$ would have a direction of propagation inclined towards the equator from the zonal direction, both in the northern and the southern hemispheres.

6 Flux tube integrated Pedersen conductivities

The estimation of $\delta E_y$ from Eq. (12) requires the values of FTI Pedersen conductivities $\Sigma_{1E}$, $\Sigma_{1M}$ and $\Sigma_{1F}$ for the appropriate ionospheric conditions. The ESF is normally triggered during the late evening hours and by that time the plasma would have drifted up to acquire certain minimum altitude. Rastogi and Woodman pointed out that an upward drift of the ionospheric plasma for half an hour or so is a requirement for the occurrence of the nighttime ESF. Due to lack of systematic measurements of electron density in the E-region and the M-region during late evening hours before ESF is triggered, it is necessary to make full use of the scanty information available in the literature. Andersen and Mendillo calculated FTI Pedersen conductivity using their model calculations of the electron density. For these calculations, they had set a lower limit to the electron density as $10^4$ cm$^{-3}$ in the altitude range of 125 km to the bottomside of the F-region which was later decreased by a factor of two. As will be seen later that even this reduced value is larger than what is observed through the in situ measurements as well as from the model calculations, at least in the 125 to 200 km region. Hanson et al. calculated the electron density during late evening hours setting the photo ionization source as zero. The values of electron density from these calculations after sunset are as follows. In case the ionospheric plasma drifts upwards with a velocity of 25 ms$^{-1}$, the electron density one hour after sunset is $3 \times 10^3$ cm$^{-3}$ below the base of the F-region. Above this altitude the electron density increases with altitude somewhat exponentially attaining a value of $10^5$ and $10^6$ cm$^{-3}$ at 350 and 450 km respectively. They observed that the vertical diffusion of plasma does not change the electron density by more than 20% even up to a dip angle of 28°. This angle corresponds to a field line apogee of 559 km. The ion density observed with the retarding potential analyzer in the ESF bubbles ranges from $10^3$ to $10^4$ cm$^{-3}$ or more. As the bubbles might be in different phases of development, these values represent upper limits to the electron density present below the base of the F-region at the time and place when the ESF was started.

A number of high altitude rockets were launched from the American zone at times when
the E-region was already triggered, and the data for the E-region were available only on a few of them. The values of electron density from the up leg of the flight from Punta Lobos at 2100 hrs from Morse et al. were $5 \times 10^3$, $3 \times 10^3$, and $1 \times 10^4$ cm$^{-3}$ respectively in the 100 km region, in the valley region (around 130 km), and in the 200 to 240 km region. The values of electron density from the down leg of the flight from Natal (Brazil) at 2122 hrs from Kelley et al. were $3 \times 10^4$ and $3 \times 10^3$ cm$^{-3}$ respectively in the regions around 100, and 130 to 250 km. These values are much larger than those observed over SHAR and also those given by our model calculations in the E-region. However, these large values of electron density in the E-region do not adversely affect the estimation of the amplitude of PEF, which from Eq. (47) is proportional to $\Sigma_{2E}/(\Sigma_{1E} + \Sigma_{1M} + \Sigma_{1F})$, which is dependent on the ratio of electron density in the E-region and in the intermediate region and not on their absolute values. It may be noted that in this case the value of $\Sigma_{1F}$ may have to be increased proportionately to get same values of $\delta E_y$. The values of electron density from the down leg of the above flight from Punta Lobos and from the down leg of Plumex 1 launched from Kwajalein Islands at 0031 hrs (Ref. 38) were similar to those observed over SHAR. The values of electron density from Plumex 1 were $5 \times 10^3$, $1.5 \times 10^3$, and $3 \times 10^3$ cm$^{-3}$ respectively around 100, 150 and 200 km. From Plumex 2, launched at 2157 hrs (Ref. 39), the values of electron density were $1.8 \times 10^3$, $2.5 \times 10^3$, $3.0 \times 10^3$ and $1 \times 10^3$ cm$^{-3}$ respectively at 200, 250, 300 and 340 km.

Six RH-560 rockets (with apogee of about 325 km) were launched from SHAR, two during evening hours, one just before and three after the ESF was observed with an ionosonde. The in situ measurements of electron density, carried out during late evening hours over SHAR, are given in Fig. 2. The electron density profiles are plotted according to the local time of the launch. The flights SHAR ASV 30 and SHAR ASV 26 were conducted during the evening twilight periods when the F-region plasma was drifting upwards and the ESF was still observed. During these flights the electron density had not yet acquired the nighttime values. The flight SHAR 448 was launched when the F-region plasma had already stopped drifting upwards and was 5 min before the ESF was observed at 400 km. Before this launch, the F-region plasma had drifted upwards with a velocity of 25 ms$^{-1}$ for more than half an hour. The flights SHAR 560.16, 560.25 and 560.26 were during the periods when the F-region plasma was drifting downwards and the ESF had already developed. During the flight SHAR 560.25, $hF$ was 350 km at 2040 hrs and the plasma was drifting down with a velocity of 25 ms$^{-1}$. During the flight SHAR 560.16 $hF$ was 380 km at 2000 hrs and the plasma was drifting down with a velocity of 40 ms$^{-1}$ for 45 min and thereafter with a velocity of 15 ms$^{-1}$ for rest of the time. During the flight SHAR 560.26, $hF$ was 345 km at 2140 hrs and the plasma was drifting down with a velocity of 35 ms$^{-1}$.

During the period, when the plasma was drifting downwards, the lower region was receiving ionization from the higher altitudes. Therefore, the electron density profiles from the flights SHAR 560.16, 560.25 and 560.26 would at least in the lower region represent electron densities larger than those observed during the periods when the ionospheric plasma was drifting upwards. The examination of these profiles show that there is usually a valley in the electron density in the 130 km region and the electron density in the 150 km region was between $2 \times 10^3$ and $3 \times 10^3$ cm$^{-3}$. The value of electron density in the 200 km region was between $3 \times 10^3$ and $4 \times 10^3$ cm$^{-3}$. The values of electron density on flight SHAR 448 were $6.3 \times 10^3$, $1.5 \times 10^3$, and $3 \times 10^3$ cm$^{-3}$ respectively at 125, 150 and 200 to 285 km altitude (H S S Sinha, 1995, private communication). As pointed out earlier, this flight over SHAR was conducted when the F-region plasma had already drifted upwards for more than half an hour. From the simulation studies of Prakash et al. for $E_z = -5.0$ mV m$^{-1}$ and $E_v = 1.5$ mV m$^{-1}$, the values of electron density were $4.9 \times 10^3$, $1.1 \times 10^3$, $1.6 \times 10^3$ and $2 \times 10^3$ cm$^{-3}$ respectively at 108, 125, 150 and 200 km altitude, while for $E_z = -1.5$ mV m$^{-1}$, the values of electron density were $3.9 \times 10^3$, $6.0 \times 10^2$, $1.0 \times 10^3$ and $2 \times 10^3$ cm$^{-3}$ respectively at 106.5, 125, 150 and 200 km altitude. These values are lower than all the above in situ measurements.

Keeping in mind that data from the flights in the American zone and from SHAR as well as the results from the model calculations, we have used values of electron density as $1.5 \times 10^3$ and $3 \times 10^3$ cm$^{-3}$ respectively in the 150 km region and the region around 200 km. The values of electron density below 150 km were taken from the model calculations discussed in Section 4.

The FTI Pedersen conductivities $\Sigma_{1E}$, $\Sigma_{1M}$ and $\Sigma_{1F}$ and the zonal winds averaged over FTI integrated conductivities were calculated in the altitude range of 150 km to the apex of various field lines using the collision frequencies from MSIS 89 model both for the solar maximum ($F_{10.7} = 150$)
and minimum conditions \((F_{10.7} = 75)\). Two representative electron density distributions A and B given below were used for these calculations. As the collision frequencies are larger during the solar maximum than during solar minimum, \(\Sigma_{1M}\) will be larger during solar maximum than during solar minimum. Therefore, the requirements on conductivity for the triggering of ESF are more difficult to satisfy during the solar maximum, and hence only the values for solar maximum are given in these figures. Based on the rocket experiments and the model calculations, we assume two electron density profiles. In Case A, the electron density is \(1.5 \times 10^3\), \(3 \times 10^3\) and \(5 \times 10^3\) \(\text{cm}^{-3}\) respectively at \(150\), \(200\) and \(300\) km and thereafter increasing exponentially with a scale height of 10 km. In Case B, the electron density is \(1.5 \times 10^3\), \(3 \times 10^3\) and \(10^4\) \(\text{cm}^{-3}\) respectively at \(150\), \(200\) and \(300\) km and thereafter increasing exponentially with a scale height of 10 km. It may be noted that between these specified altitude ranges, the electron density was assumed to increase exponentially.

As the zonal winds play an important role in the generation of PEF, we adopt as an example the values of winds as used by Andersen et al.\(^{33}\), which were mainly based on the measurements of Meriwether et al.\(^{10}\), Sipler et al.\(^{11}\) and Wharton et al.\(^{12}\)

\[
w_{\text{alt. } \geq 300\text{ km}} = 200[\cos{(21 - L T)(15°/h)}] \text{ms}^{-1}
\]

\[
w_{(200 \leq \text{alt. } < 300)} = w_{300}(\text{alt. } - 200)/100 \text{ ms}^{-1}
\]

\[
w_{(140 \leq \text{alt. } < 200)} = -50 \text{ ms}^{-1}
\]

\[
w_{(\text{alt. } < 140)} = 0 \quad \ldots (44)
\]

The values of \(\Sigma_{2E}, \Sigma_{1L}, \Sigma_{1F}\) and \(W_{YT}\) and factor \(G_2\), to be defined later, were calculated using the above parameters and are given in Figs 3 and 4 respectively for electron density models A and B for the apex altitudes of 200 to 400 km. A sharp change in \(\Sigma_{1L}\) is seen in these figures just above 300 km altitude. This is due to the fact that the apogee region of the field line above 300 km does not contribute to \(\Sigma_{1L}\). Above 300 km, \(\Sigma_{1F}\) starts increasing. From Figs 3 and 4, the FTI conductivity at the base of F-region is about 0.7 mho m\(^{-3}\) which is nearly one third of the minimum value of the conductivity from Andersen and Mendillo\(^{32}\). This is what would be expected as the electron density in the 150 km region from the present model is about \(1.5 \times 10^3\) \(\text{cm}^{-3}\) which is nearly one seventh of what has been assumed by the above authors. The conductivities given in Figs 3 and 4 are appropriate only for the lower part of the base of the F-region as in this altitude region one can neglect the diffusion of the plasma along the field lines.

7 Requirement on the amplitude of perturbation electric field

We now calculate minimum amplitude of PEF which is required to provide seed irregularities in the base of the F-region for the triggering of ESF. The simulation studies of ESF carried out by Zal-
222

As $\Sigma_{1E} W_{pE} + \Sigma_{1M} W_{pM}$, can be represented by $\Sigma_{1L} W_{pL}$, the above equation reduces to

$$
E_p = \left[ J_p - \Sigma_{2E} (E_y + B W_{pE}) + \Sigma_{1L} B W_{pL} + \Sigma_{1F} B W_{pF} \right] / [\Sigma_{1L} + \Sigma_{1F}] \quad \ldots (49)
$$

As discussed earlier, the meridional current $J_p$ would be positive during the evening reversal, but its value is not known and can be determined only through model calculations. For the present calculations, we assume $J_p$ to be small or zero. For $E_y = 0$ and neglecting $W_{pE}$ and $W_{pF}$ as discussed earlier, we get

$$
E_p / B = [\Sigma_{1L} W_{pL} + \Sigma_{1F} W_{pF}] / [\Sigma_{1L} + \Sigma_{1F}] = W_{yt} \quad \ldots (50)
$$

Hence $W_{yt}$ represents the zonal wind averaged over the conductivities in all the three regions. Substituting the values of $E_p / B$ from Eq. (50) in Eq. (47), and rearranging the terms, we get

$$
\delta E_y / B = (- \cos \varphi \sin \varphi \delta \Sigma_{1E} / \delta \Sigma_{2E} + \cos^2 \varphi) \times \left( \delta \Sigma_{2E} / \Sigma_{2E} \right) [\Sigma_{2E} / (\Sigma_{1L} + \Sigma_{1F})] \\
\times \left( \Sigma_{1L} W_{pL} + \Sigma_{1F} W_{pF} \right) / (\Sigma_{1L} + \Sigma_{1F}) \quad \ldots (51)
$$

Splitting the RHS of Eq. (51) into $G_1$ and $G_2$, we get

$$
G_1 = (- \cos \varphi \sin \varphi \delta \Sigma_{1E} / \delta \Sigma_{2E} + \cos^2 \varphi) (\delta \Sigma_{2E} / \Sigma_{2E}) \quad \ldots (52)
$$

$$
G_2 = \left[ (\Sigma_{1L} W_{pL} + \Sigma_{1F} W_{pF}) / (\Sigma_{1L} + \Sigma_{1F}) \right] \quad \ldots (53)
$$

Eq. (51) can be written as

$$
\delta E_y / B = G_1 G_2 \quad \ldots (54)
$$

8.1 Estimation of $\delta E_y / B$

We now estimate the value of $\delta E_y / B$ using realistic values of various parameters. There are two terms on the right hand side of Eq. (52) and they will be referred to as $F_1$ and $F_2$ and we discuss them one by one. The term $F_1$ is maximum when $\varphi_m = -0.5 \tan^{-1} (\delta \Sigma_{1E} / \delta \Sigma_{2E})$. As $(\delta \Sigma_{1E} / \delta \Sigma_{2E}) < 0.18$ and hence $\varphi_m = -5.0^\circ$. The first sub-term of $F_1$ corresponds to the contribution of the perturbations in the Pedersen conductivity $\delta \Sigma_{1E}$, while the second sub-term of $F_1$ corresponds to the contribution of the perturbations in the Hall conductivity $\delta \Sigma_{2E}$. Therefore, it can be concluded that the contribution to $\delta E_y$ by the perturbation in
Pedersen conductivity is much less than that due to the perturbation in the Hall conductivity. The second term \( \delta \Sigma_{2E}/\Sigma_{2E} \) consists of two sub-terms \( \delta b \) and \( \Gamma \). From the numerical simulation given in Section 4, \( \delta \Sigma_{2E}/\Sigma_{2E} \) was found to be somewhat linearly related to the amplitude of GWW, therefore \( \delta b \) is expected to be proportional to the zonal component \( \delta W_x \) of GWW. As \( \delta W_x = \delta W_H \cos \alpha \), therefore for a given value of \( \delta W_H \) the amplitude \( \delta b \) can be assumed to be proportional to \( \delta W_H \cos \alpha \). \( \delta b \) can therefore be represented as \( \delta b_0 \cos \alpha \), where \( \delta b_0 \) is the amplitude of the electron density irregularities when \( \alpha = 0 \) for a GWW with an amplitude of \( \delta W_H \). The angle \( \alpha \) is related to \( \varphi \) through Eq. (42). \( \Gamma \) is related to \( \varphi \) through Eqs (30), (35) and (41). As \( I \) is positive in the northern hemisphere, from Eqs (30) and (35) \( \Gamma \) can be maximum when \( K_x \) has the same sign as \( K_y \). As \( K_x \) is normally negative for GW, hence \( K_y \) should also be negative. This would make \( K_x \) also negative through Eq. (37). Thus from Eq. (9), \( \varphi \) will be positive when \( K_y \) is negative and vice versa. Figure 5 gives a plot of \( G_1/\delta b_0 \) and \( G_1/\delta b_0 \) when \( K_y = 2\pi/100 \text{ km}^{-1} \) for two values of \( K_z \) which are \(-2\pi/20 \text{ km}^{-1}\) and \(-2\pi/24 \text{ km}^{-1}\). These two cases will be respectively referred to as Case 3 and Case 4. From Fig. 6 when \( K_y = -2\pi/24 \text{ km}^{-1} \), \( G_{1m} = 0.42 \) and its value is maximum at \( \varphi = 7.5^\circ \). The range of angles for which the value of \( G_1/\delta b_0 \) is more than half of its maximum value is given by \(-35^\circ \) to \( 46^\circ \) (corresponding to \( \alpha \) of \( -13^\circ \) to \( 20^\circ \)). It may be concluded that GW propagating in the near zonal direction would be most effective in the generation of PEF. From Figs 5 and 6, the values for \( G_{1m}/\delta b_0 \) for \( K_y = -2\pi/20 \text{ km}^{-1} \) are nearly 2/3 of the value for \( K_y = -2\pi/24 \text{ km}^{-1} \), however, the range of angles \( \varphi \) for these values of \( K_y \) remains nearly the same. It can also be seen that the maximum value of \( G_{1m}/\delta b_0 \) does not vary significantly when \( K_y \) is changed from a positive value to a negative value while it changes appreciably with \( K_x \). It may be noted that the values of the first term in Eq. (52) are respectively 0.96 and 1.0 for the values of \( \varphi = -17.5 \) and 7.5 for which \( G_1/\delta b_0 \) is maximum. Also it can be seen from Figs 5 and 6 that the values of the parameter \( G_{1w}/\delta b_0 \) are within 10 to 15% of the value at \( \varphi = 0 \), hence for the rough estimation of the first term, it is sufficient to assume a value corresponding to \( \varphi = 0 \). Eq. (51) can therefore be written as

\[
\delta E_y/B = (\delta \Sigma_{2E}/\Sigma_{2E})G_2 \quad \ldots (55)
\]

There are two terms \( F_1 \) and \( F_2 \) on RHS of Eq. (53). The term \( F_2 \) gives the values of zonal winds averaged over the FTI Pedersen conductivities. To study the nature of the \( F_2 \) term, we take into account the winds prevailing in these regions. We
use the wind model adopted by Andersen and Mendillo\(^\text{12}\) and given in Section 6. If the base of the F-region is above 300 km, the average wind \(W_{\text{F}}\) would be same as the ambient wind \(W_{\text{F}}\). The first subterm of \(\Sigma_{2m}\) depends on \(\Sigma_{1n}, \Sigma_{1e}, W_{\text{d}}, W_{\text{F}}\) and \(W_{\text{d}}\) which depends on the distribution of the electron density and winds over a given field line. The product of \(F_{2}\) and \(F_{1}\) represented by \(G_{2}\) in Eq. (53) would maximize when

\[
\Sigma_{1F} = \Sigma_{1L}/(1 - 2 W_{\text{F}}/W_{\text{d}}) 
\]  

It can be seen from this equation that \(G_{2}\) can have a maxima only if \(W_{\text{d}} < 0.5 W_{\text{F}}\) and the maximum value of \(G_{2m}\) would be given by

\[
G_{2m} = (\Sigma_{2E}/4 \Sigma_{1L})(W_{\text{F}}^{2}/(W_{\text{F}} - W_{\text{d}})) 
\]  

From Eq. (56), \(\Sigma_{1F}\), at which \(G_{2}\) maximizes, decreases with the increase of \(W_{\text{d}}\) and according to Eq. (57), \(G_{2m}\) will increase with the increase of \(W_{\text{d}}\). This shows that the meridional winds below the base of the F-region would not only bring down the altitude at which PEF is maximum but also increase its value.

From Eq. (57), \(G_{2m}\) is proportional to \(\Sigma_{2E}/\Sigma_{1L}\). As \(\Sigma_{1E}\) is proportional to \(\Sigma_{2E}\) and hence using Eq. (11), \(\Sigma_{2E}/\Sigma_{1L}\) can be written as

\[
\Sigma_{2E}/\Sigma_{1L} = 1/(C_{1} + \Sigma_{1M}/\Sigma_{2E}) 
\]  

where \(C_{1}\) is the ratio of \(\Sigma_{1E}/\Sigma_{2E}\). From numerical simulation in Section 4, \(C_{1}\) is 0.17, 0.20 and 0.24 respectively for \(E_{v} = -1.5 \text{ mV m}^{-1}\), 0 and +1.5 mV m\(^{-1}\) and when \(E_{v} = -5 \text{ mV m}^{-1}\). From Eq. (58), \(\Sigma_{2E}/\Sigma_{1L}\) will be large when \(\Sigma_{1M}/\Sigma_{2E}\) is small.

Figures 3 and 4 give the values of \(\Sigma_{1L}\), \(\Sigma_{1F}\), \(\Sigma_{2E}\), \(W_{\text{d}}\), \(G_{2}\) for different altitudes for the assumed electron density profile Case A and Case B specified in Section 6. For these calculations the winds used are given by Eq. (44). In case \(W_{\text{d}} = 0\), then \(G_{2}\) would be maximum when \(\Sigma_{1F} = \Sigma_{1L}\) and its value would be \(\Sigma_{2E} W_{\text{d}}/4\Sigma_{1L}\). In Fig. 3, the base of the F-region starts at 300 km, \(G_{2}\) is maximum at 335 km altitude where the plasma frequency is 3.65 MHz. The maximum value of \(G_{2}\) is 130. In Fig. 4, \(G_{1}\) is maximum at 325 km altitude where the plasma frequency is 3.13 MHz. The maximum value of \(G_{2}\) is 124. Thus an increase in electron density between 200 km and the base of the F-region by a factor of two brings down the altitude as well as the electron density at which \(G_{2}\) is maximum. The altitude where \(G_{2m}\) occurs is, however, 10 km higher in Case A than in Case B. This can be explained through the increase in \(W_{\text{d}}\) in Case B over Case A.

The values of \(E_{v}/B\) were calculated using the average value of \(\delta \Sigma_{2E}/\Sigma_{2E}\) for the vertical wavelengths of 16 to 24 km and \(G_{2m}\) from Figs 3 and 4. While the average value of \(\delta \Sigma_{2E}/\Sigma_{2E}\) from Section 4 is 5\%, the values of \(G_{2m}\) from Figs 3 and 4 are respectively 130 and 124 ms\(^{-1}\). Therefore the values of \(\delta E_{v}/B\) are 6.5 and 6.2 ms\(^{-1}\) respectively for the Case A and Case B.

For our calculations we have used zonal winds of 200 ms\(^{-1}\) and \(\lambda_{z}\) of 16 to 24 km. We have used GWW amplitude of 30 ms\(^{-1}\) instead of 80 ms\(^{-1}\) observed with the vapour release experiments of Kochanski\(^\text{20}\). \(\delta \Sigma_{2E}/\Sigma_{2E}\) was calculated for \(K_{x} = 0\) while it would be larger when the wavefront of GW makes smaller angle with the geomagnetic field line. Thus a larger value of \(\delta \Sigma_{2E}/\Sigma_{2E}\) can be expected. Therefore the values of \(\delta E_{v}/B\) can be larger than the present estimates. The present estimates of \(\delta E_{v}/B\) give values as high as 6.5 ms\(^{-1}\) which are much larger than required for the production of seed irregularities with 5\% perturbations in the electron density as used by Zalesak et al.\(^\text{40}\) for their simulation studies. It may be noted that \(\delta E_{v}/B\) does not depend on the scale height of the electron density in the base of the F-region used for the model calculations. Sekar et al.\(^\text{41}\) pointed out that under certain conditions, the seed irregularities even as small as 0.5\% may be sufficient for the growth of ESF and this would bring down the requirements on \(\delta E_{v}/B\) by a factor of ten. The values of \(\delta E_{v}/B\) would be smaller or larger than the ones estimated above depending on the values of the zonal winds in the F-region, vertical wavelength and wind amplitude of GW, FTI conductivities and angle between the wavefront of GW and geomagnetic field. In addition, the minimum required amplitude of PEF would be determined by the growth rate of RTI. The values of \(\delta E_{v}/B\) estimated here have sufficiently large margins and hence the requirement on \(\delta E_{v}/B\) may be satisfied even if the zonal winds or some other parameters are somewhat smaller in value than used here. Using HF radar operating at 5.5 MHz at Trivandrum (8.6\(^\circ\)N, 77\(^\circ\)E, magnetic dip 0.5\(^\circ\)S), Nair et al.\(^\text{42}\) found that the dominant vertical velocity fluctuations have periods of the order of few tens of minutes and amplitudes greater than about 5 ms\(^{-1}\). Subbarao and Krishna Murthy\(^\text{43}\) with their HF radar operating at 2.2 MHz at the same station found that the fluctuations in the 4 to 30 min period have amplitudes of about 1.0 ms\(^{-1}\). The PEF estimated from the present model are up to about 6.5 ms\(^{-1}\) which are within the range of above observations.

Maruyama and Matuura\(^\text{44}\) defined the seasonal...
longitudinal condition when ESF is not likely to occur. Essentially their condition for ESF activity was that the direction of the wind during night be mainly perpendicular to the geomagnetic field line. Mendillo et al.\textsuperscript{45} concluded that the major difference between the nights when ESF did or did not occur was due to the absence or presence of equatorward winds. Tsnuda\textsuperscript{46} described the seasonal condition when ESF is likely to occur.

They gave stress on the special relationship between the solar terminator and the local magnetic field declination which probably results in a smaller meridional winds. Bittencourt and Sahai\textsuperscript{47} showed that transsequatorial winds can produce a large difference in the height between the conjugate points. While meridional winds towards the equator would transport the plasma downwards, they would transport the plasma downwards while moving away from it towards the poles. As the ion neutral collision frequency is larger in the lower region, this will result in a net increase of the FTI Pedersen conductivity $\Sigma_{1M}$ below the base of the F-region. When the meridional winds are small or absent, the conductivity in the intermediate region will be minimum and thus from Eq. (57) PEF will be larger, Zalesak et al.\textsuperscript{40} gave a growth rate of RTI which is proportional to the ratio of the FTI Pedersen conductivity in the F-region and the total conductivity over a given field line, which according to our nomenclature would be given by $\Sigma_{1F} / (\Sigma_{1E} + \Sigma_{1M} + \Sigma_{1F})$. Thus a smaller value of $\Sigma_{1M}$ would not only make the growth rate of RTI larger but also give a larger PEF for production of seed irregularities.

8.2 Estimation of $\delta E_p/B$

While the PEF $\delta E_p$ can initiate the growth of irregularities in the region of upward gradients, $\delta E_p$ can initiate growth of irregularities in the region of horizontal gradients in the steep vertical walls of the ESF plumes.

From Eq. (14), $\delta E_p = \delta E \tan \varphi$, therefore using Eq. (51) we get

$$\frac{\delta E_p}{B} = (\cos \varphi \sin \varphi \frac{\delta \Sigma_{1E}}{\delta \Sigma_{2E}} + \cos^2 \varphi \tan \varphi) \times \frac{\delta b \Gamma (\Sigma_{1L} W_{sl} + \Sigma_{1P} W_{sp}) (\Sigma_{1L} + \Sigma_{1P})}{(\Sigma_{2E} / \Sigma_{1L} + \Sigma_{1P})}$$

For $\delta \Sigma_{1E}/\delta \Sigma_{2E} = 0.18$, the first term will be maximum when $\varphi = -35^\circ$. We denote the product of the first two terms on the right hand side of Eq. (59) as $G_3$. From Fig. 5 when $K_z = -2\pi/24$ km$^{-1}$, then $G_{3m} = 0.36$ and its value is maximum at $\varphi = -57^\circ$. The range of angles for which the value of $G_3/b_0$ is more than half of its maximum value is given by $-78^\circ$ to $-22^\circ$ (corresponding to $\alpha$ of $-58^\circ$ and $-8^\circ$). From Fig. 6 when $K_z = -2\pi/24$ km$^{-1}$, then $G_{3m} = 0.23$ and its value is maximum at $\varphi = 45^\circ$. The range of angles for which the value of $G_3/b_0$ is more than half of its maximum value is given by $15^\circ$ to $67^\circ$ (corresponding to $\alpha$ of $5^\circ$ and $39^\circ$). From Figs 5 and 6, the values for $G_{3m}/\delta b_0$ for $K_z = -2\pi/20$ km$^{-1}$ are nearly 3/4 of the value for $K_z = -2\pi/24$ km$^{-1}$, however, the range of angles $\varphi$ remain nearly the same for $K_z = -2\pi/24$ km$^{-1}$. It may be concluded that GW propagating at around $45^\circ$ from the zonal direction towards the equator would be most effective in the generation of PEF in the radial direction. As the angle $\varphi$ corresponding to these gravity waves is large, their period would be small as can be noted from Eq. (43).

As the last two terms in the expression for $\delta E_p$ and $\delta E_y$ are same, hence their ratio would be same as the ratio of $G_3/b_0$ and $G_3/b_0$. It can therefore be seen from Figs 5 and 6 that the ratios $\delta E_p$ and $\delta E_y$ are 0.80, 1.00, 0.55 and 0.65 respectively for Cases 1, 2, 3 and 4, implying that the values of $\delta E_p$ and $\delta E_y$ can be equally large subject to the presence of the appropriate GW.

9 Summary and conclusions

In this paper we have examined the possibility of electron density irregularities in the E-region giving rise to perturbation electric fields, which when transmitted to the F-region, can produce seed irregularities in the base of the F-region large enough to trigger ESF. The mechanism of generation of these perturbation electric fields is summarized in the block diagram given in Fig. 1. In this mechanism, the zonal winds in the intermediate region and in the F-region give rise to the radial electric fields $E_r$. These fields together with zonal fields drive currents in the E-region where the FTI Pedersen and Hall conductivities are perturbed by the electron density irregularities produced by gravity wave wind. This results in the perturbations of the zonal and the radial electric fields which in turn produce seed irregularities in the base of the F-region. These seed irregularities may give rise to the growth of ESF if the condition for the growth of RTI are favourable. A summary of the salient features of the physical mechanism through which these irregularities are produced is as follows.

(i) The electron density during the nighttime in the E-region is maintained not only by the nighttime sources of ionization but also by the downward transport of ionization by the radially inward electric fields generated by the zonal winds in the
F-region. For example, when $E_y$ is zero, the integrated Hall conductivity in the E-region will increase by 18% when $E_p$ is changed from zero to $-5.0 \text{ mV m}^{-1}$ (refer Section 4).

(ii) From the studies of Kochanski\textsuperscript{20}, the gravity waves in the E-region can have vertical wavelengths as large as 24 km and the wind amplitude as 80 ms$^{-1}$. Electron density irregularities have been observed over SHAR (13°42′N, 80°E, dip 14°N), India, with amplitudes as large as 40 to 80% over the background and vertical wavelength over 24 km and these can be accounted for through numerical simulation by Prakash et al\textsuperscript{21} using convergence rate from Prakash and Pandey\textsuperscript{22}.

(iii) The electron density irregularities give rise to perturbations in the FTI conductivities. An expression relating the perturbations in the FTI conductivities with other parameters is given by Eq. (31). The perturbation in the Hall conductivity was found to be more than five times larger than in the Pedersen conductivity. As given in Section 4, the average value of $\Delta \Sigma_2 \Sigma_2 \Sigma_2$ for $\lambda_z$ varying from 16 to 24 km is about 5%.

(iv) The zonal winds in the F-region give rise to the radial electric fields\textsuperscript{48}. A comprehensive expression relating the radial electric fields with the zonal winds, FTI conductivities, meridional currents and other parameters of the E-region has been derived and is given by Eq. (48). Zonal winds up to 200 ms$^{-1}$ have been observed\textsuperscript{10-12}.

(v) The zonal and radial electric fields give rise to currents in the E-region while flowing through the non-uniformly conducting E layer, and these would give rise to perturbations in the zonal and radial electric fields. The electric fields produced through the perturbations in the Hall conductivity are more than five times larger than those produced through the perturbations in the Pedersen conductivity. Expressions for the perturbation electric field in the zonal and radial directions are respectively given by Eqs (51) and (59).

(vi) The perturbation electric field in the zonal direction is the product of $G_1$ and $G_2$, where $G_1$ and $G_2$ are respectively given by Eqs (52) and (53). $G_1$ is mainly dependent on the E-region parameters and is a function of $\varphi$. As discussed in Section 8, the gravity waves propagating in the near zonal direction slightly inclined towards the equator are most suitable for the production of the perturbation electric field in the zonal direction. This rules out the possibility of their arrival from the polar region.

(vii) For the calculation of the perturbation electric field in the radial direction, $G_1$ has to be replaced by $G_1 \tan \varphi$. As discussed in Section 8, the gravity waves propagating around the angle of 45° towards the equator are most suitable for the generation of perturbation electric field in the radial direction.

(viii) $G_2$ mainly depends on the zonal winds and the conductivities in the base of the F-region and the lower region. It maximizes for a value of $\Sigma_{1F}$ given by Eq. (56) and its maximum value $G_{2m}$ is given by Eq. (57). It can be seen that $G_{2m}$ is proportional to the ratio $\Sigma_{2F}/(\Sigma_{1F} + \Sigma_{1L})$ and it would be large if $\Sigma_{1M}/\Sigma_{2E}$ is small. For the calculation of conductivities, the electron density below 130 km was taken from our numerical calculation and above 130 km using data from various rocket experiments and from model studies of Hanson et al\textsuperscript{24}.

(ix) For $\Sigma_{1M}$ to be low, the electron density in the 130 km to the base of the F-region should be low particularly in the 130 to 200 km region as the collision frequency in this region is much larger than above 200 km. The ESF during late evening and night hours is triggered when the ionospheric plasma has continued to drift upwards for more than half an hour or so after the evening twilight or during night. The upward drift above 130 km and the downward drift below 130 km reduces the electron density in the intermediate region resulting in a small value of $\Sigma_{1M}$.

(x) Strong meridional winds in the F-region can bring down the plasma into the intermediate region and thereby increase the value of $\Sigma_{1M}$ and thus decrease the value of perturbation electric field. The values of $\Sigma_{1M}$ given in Figs 3 and 4 are for the case when the meridional winds are absent in the F-region.

(xi) The maximum values of the perturbation electric field for two different electron density profiles, Case A and Case B, are calculated using Eq. (51) for $q=0$, zonal winds of 200 ms$^{-1}$, $\delta \Sigma_{2E}/\Sigma_{2E} = 5\%$ and $G_{2m}$ from Figs 3 and 4. The perturbation velocities are found to be respectively 6.5 and 6.2 ms$^{-1}$ for Case A and Case B. These values are larger than 1.3 ms$^{-1}$ required for producing the seed irregularities with 5% amplitude. As the values of the parameters which go into the calculations of the perturbation electric field are known only for some isolated epochs, attempts have been made here to calculate the typical values of perturbation electric field. The maximum value of $\delta \Sigma_{2E}/\Sigma_{2E}$ could not be determined correctly as the simulation studied could be carried out only when $K_e=0$ when $\delta \Sigma_{2E}/\Sigma_{2E}$ is not expected to acquire a maximum value. For the par-
parameters from Figs 3 and 4, the perturbation velocity maximizes towards the lower end of the base of the F-region where the plasma frequency is about 3.65 and 3.13 MHz for two different electron density profiles. The critical frequencies at which ESF is normally observed with ionosonde in the initial stages of development can even be smaller than these values. The variability of zonal winds in and below the base of the F-region is an important factor in determining the region where the perturbing electric field become maximum and the conditions for triggering equatorial spread-F are created.

Acknowledgements

The author is thankful to Prof. P K Kaw, Prof. B H Subbaraya and Dr H S S Sinha for many useful discussions. This research was supported by Council of Scientific and Industrial Research, New Delhi, and Physical Research Laboratory, Ahmedabad, India.

References

1 Basu S & Kelley M, Radio Sci (USA), 14 (1979) 471.
4 Klostermeyer J, J Geophys Res (USA), 83 (1978) 3753.
20 Kochanski A, J Geophys Res (USA), 69 (1964) 3651.
27 Woodman R F, Space Res (UK), 12 (1972) 969.