Association of fire-hose instability and geomagnetic activity in the plasma sheet

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During maximum solar activity years (1978-79) and at low geomagnetic activity, the temperature of the plasma sheet (PS) in the tail region of earth's magnetosphere is found to be decreasing. The heating of the plasma sheet ions can occur both adiabatically and non-adiabatically. The strong ion heating associated with 'substorm-onset occurs in a non-adiabatic fashion with a strong increase of specific entropy, and this heating is found to be localized since both low and high entropy plasmas are present in the plasma sheet. During high activity periods, plasma sheet was found to be surviving against particle loss and heat transport. The chaotic behaviour of the plasma sheet during the selected substorm events is also discussed.

1 Introduction
The near-earth plasma sheet between about 10 \( R_E \) and 20 \( R_E \) is a region well covered by satellite observations. During last five years, numerous statistical studies using plasma measurements by satellites have contributed a significant progress in our understanding of the earth's plasma sheet and have resolved many of the questions. The tail data analysis suggests that the plasma sheet ions are undergoing adiabatic heating in the plasma sheet boundary layer (PSBL) and in the distant tail. The temperature of the ULF Alfvén wave propagating through the resonance layer of the plasma sheet was found to be a function of plasma \( \beta \)-parameter and geomagnetic activity (\( AE \) ) index. Heating events in the PSBL and in the central plasma sheet (CPS) was found to occur at the onset of expansive phase activity. The heating of the plasma sheet during the period of high activity is a feature which is readily apparent in the statistical studies done by Goertz and Smith. The validity of the reconnection theory during the thinning and thickening processes of the plasma sheet was tested recently by Bindu et al.

To study the heating mechanism in the plasma sheet of geomagnetotail, 22 substorm events during maximum solar activity years (1978-79) were selected and the variations of polytropic index \( \gamma \) (ratio of specific entropies) with \( AE \) index and sunspot number \( \chi \) were found out. The association of fire-hose instability in the plasma sheet with field-line-curvature radius \( R_c \) and chaos parameter \( K \) were verified and also the variation of the growth rate of this instability in adiabatic and non-adiabatic media were found out.

2 Materials and methods
The equation for the scalar thermal pressure \( P \) in space plasmas can be written as

\[
P = \alpha N_i \gamma \frac{k_B T_i}{N_i^{\gamma/3}} \quad \cdots (1)
\]

where, \( \alpha \) is a constant which depends on the specific entropy of the plasma and \( \gamma \) is the polytropic index. The specific entropy function, \( \alpha \), is defined as

\[
\alpha = k_B T_i / N_i^{\gamma/3} \quad \cdots (2)
\]

where \( k_B \) is the Boltzmann's constant and \( T_i \) and \( N_i \) are the ion temperature and ion number density, respectively. Since plasma \( \beta \)-parameter is the ratio of plasma pressure \( (n k_B T) \) to magnetic pressure \( (B_0^2 / 8 \pi) \), it directly depends on the temperature \( T \) of the plasma sheet. So the thermal plasma pressure \( P \) can be written in terms of plasma \( \beta \)-parameter as

\[
P = \beta B_0^2 / 8 \pi \quad \cdots (3)
\]

So, from Eq. (1) we can understand that the temperature of the plasma sheet and the polytropic
index are related to each other. This gives the relation between heat flow in the plasma sheet and polytropic index \( \gamma \).

The polytropic index depends on the specific entropy of the plasma. In the plasma sheet, the electron pressure \( P_e \) is much smaller than the ion pressure \( P_i \) (for example, on 20 June 1979 at 2021 hrs UT, \( P_i \) was 13.48 keV/cm\(^3\) and \( P_e \) was 0.25 keV/cm\(^3\)). The number density \( N_i \) of electrons is also much lower than that of ions \( N_i \) (on 20 June 1979 at 2021 hrs UT, \( N_i \) was 0.52/cm\(^3\) and \( N_e \) was 0.01/cm\(^3\)). Therefore, we have just taken ion thermal pressure and number density and their values are given in Table 1 for the 22 substorm events in the maximum solar activity year derived from GEOS-2 observations. The \( \gamma \) values for the selected events are computed using Eq. (1) and the heating mechanism in the plasma sheet is analysed. The polytropic index can attain values ranging from \( \gamma = 0 \) (isobaric behaviour of the plasma), over \( \gamma = 1 \) (isothermal) and \( \gamma = (f+2)/f \) (adiabatic with \( f \) denoting the degrees of freedom) to \( \gamma = \infty \) (isometric, constant density). Since in the plasma sheet ions are fairly isotropic, we can set \( f = 3 \) and thus get \( \gamma = 5/3 \) in the adiabatic case.

In this paper, we have computed the polytropic index \( \gamma \) for individual substorm events and identified the events as adiabatic and isothermal with their \( \gamma \) values. The values of \( \gamma \) obtained agree well with the results of Baumjohann et al. Therefore, it can be established that the polytropic indices for individual events will correlate with the values obtained by regression analysis.

### 3 Fire-hose instability and geomagnetic activity

Let us discuss whether the fire-hose instability in the plasma sheet can be associated with substorm activity. During the growth phase of a substorm the plasma sheet becomes thinner, indicating that the curvature radius of field lines decreases. Therefore, the pressure gradient must increase further and, consequently, the plasma pressure is localized in a certain range of radial distance. The thinning of the plasma sheet corresponds to the increase of plasma pressure and also to the increase of magnetic field strength. The field line curvature also changes drastically during the course of a substorm which, in turn, changes the current system in the plasma sheet. These peculiar features during the course of substorms will trigger the fire-hose instability in the plasma sheet.

Consider the plasma sheet particles gyrating about field lines with their perpendicular velocities \( V_{\perp} \), while they move along them with parallel velocities \( V_{\parallel} \). As a result of the curvature of field lines, centrifugal drifts arise, resulting in currents around the field lines. This centrifugal current density \( I_c \) is directly proportional to particle density, \( V_{\parallel} \) and particle mass and inversely proportional to the radius of curvature of the field lines and their intensities, i.e. \( I_c = P_i/(B_0 R_c) \). Since there are more positive particles moving in a region between field lines, a geometrical current \( I_g \) opposing \( I_c \) results. This \( I_g \) is directly proportional to the magnetic moment and particle density and inversely proportional to curvature radius of field lines, i.e. \( I_g = P_i/B_0 R_c \).

These currents in the plasma sheet must be compared with the current \( I_m \) which is necessary to maintain the deformation of uniform magnetic field. This current \( I_m = (B/\mu_0 R_c) \) and it points in the same direction as of \( I_c \). If the plasma sheet currents are smaller than the current required to deform the field lines, the perturbation swings back and waves are excited. If \( |I_c - I_g| > I_m \), the perturbation grows and instability sets in the plasma sheet. To analyse this behaviour, \( I_c, I_g \) and \( I_m \) are written in terms of chaos parameter \( \bar{K} \) (which
is defined as $K = R_p/\rho_{\text{max}}$, where $R_p$ is the curvature radius of field lines and $\rho_{\text{max}}$ is the maximum Larmor radius of ions as

$$I_e = P_{\parallel}/(B_0 K^2 \rho_{\text{max}}) \quad \ldots (4)$$

$$I_g = P_{\perp}/(B_0 K^2 \rho_{\text{max}}) \quad \ldots (5)$$

$$I_m = B/(\mu_0 K^2 \rho_{\text{max}}) \quad \ldots (6)$$

where, $P_{\parallel}$ and $P_{\perp}$ are the parallel and perpendicular components of ion pressure, $\rho_{\text{max}}$ is the maximum ion Larmor radius and $B$ the field intensity in the plasma sheet at substorm onsets. The firehose instability occurs when $(P_{\parallel} - P_{\perp}) > (B^2/\mu_0)$ and has the growth rate

$$\omega_g = [\gamma(P_{\parallel} - P_{\perp} - B^2/\mu_0)^{-1/2} k \quad \ldots (7)$$

where, $k$ is the wave number.

4 Results and discussion

Figure 1 shows the graphical representation of the variation of polytropic index $\gamma$ with $AE$ index for $\alpha = 0.5$, 1 and 1.5. For $AE < 300$ nT, $\gamma$ is less than unity. This indicates that the quiet plasma sheet undergoes cooling. The more possible cooling mechanisms are particle loss in the earth’s ionosphere or heat transport away from the PSBL by electromagnetic or electrostatic waves. On the other hand, during the more disturbed times ($AE > 400$ N T and $\alpha = 0.5$ and 1), $\gamma > 1.6$, indicating that in the active plasma sheet, heating mechanisms are stronger than dissipation. The specific entropy function $\alpha$, an important factor, suggests the plasma sheet dynamics during magnetospheric substorms. For purely adiabatic behaviour of the plasma sheet, $\alpha$ has values 0.5 and 1.5 (Ref. 10). For $\alpha > 0.5$, $\gamma$ is $< 1.6$. The behaviour of the plasma sheet on some selected events and their corresponding $\gamma$ values and $AE$ indices are shown in Table 2.

Figure 2 shows the nature of $\gamma$ as a function of sunspot number, $\chi$, for different values of $\alpha$. When $\chi$ exceeds 200, $\gamma$ values remain constant for different values of $\alpha$. The behaviour of the plasma sheet is purely adiabatic for $\chi > 150$, but for $90 < \chi < 150$, $\gamma$ is very close to the adiabatic value of 5/3. When $\chi < 90$, $\gamma$ is less than unity and is an example of the isothermal behaviour of the plasma sheet at low activity times.

Table 2—Behaviour of plasma sheet during some selected events and their corresponding $\gamma$ values and $AE$ indices

<table>
<thead>
<tr>
<th>Occurrence of substorm events</th>
<th>Time (hrs UT)</th>
<th>$AE$ (nT)</th>
<th>$\gamma$</th>
<th>Behaviour of plasma sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 Nov. 1978</td>
<td>2010</td>
<td>199</td>
<td>0.9</td>
<td>Isothermal</td>
</tr>
<tr>
<td>21 Nov. 1978</td>
<td>2100</td>
<td>202</td>
<td>0.9</td>
<td>Isothermal</td>
</tr>
<tr>
<td>26 Feb. 1979</td>
<td>2026</td>
<td>431</td>
<td>1.65</td>
<td>Adiabatic</td>
</tr>
<tr>
<td>25 Mar. 1979</td>
<td>1956</td>
<td>439</td>
<td>1.64</td>
<td>Adiabatic</td>
</tr>
<tr>
<td>23 May 1979</td>
<td>2036</td>
<td>267</td>
<td>0.67</td>
<td>Adiabatic</td>
</tr>
<tr>
<td>27 May 1979</td>
<td>1832</td>
<td>426</td>
<td>1.64</td>
<td>Adiabatic</td>
</tr>
</tbody>
</table>

Figure 3 [(a) and (b)] shows the nature of $\gamma$ as functions of field line curvature $R_p$ and of the total ion pressure $P_i$. Figure 3(a) shows that there is a continuous decrease of $\gamma$ with increase of $R_p$, i.e. there is a change from adiabatic behaviour to non-adiabatic behaviour of the plasma sheet. Since chaos parameter $K$ also increases with $R_p$, there is a decrease of $\gamma$ with $K$. When the plasma sheet becomes more chaotic, the thermodynamics of the plasma sheet changes appreciably. From Fig. 3(b), it is clear that there is a continuous increase of $\gamma$ with the total ionic pressure. According to Eq. (1), there should be an increase of $\gamma$ with $P_i$. Baumjohann et al. showed that there is a linear relation.
between the logarithmic values of ion pressure and density and, in fact, a regression analysis yields a correlation coefficient $r = 0.74$. The regression coefficient, i.e. the magnitude of the polytropic index, is $\gamma = 1.69$, which is very close to 5/3. At different regions of the plasma sheet, the behaviour of the particles is different. So the correlation between ion pressure and ion density will be different. Therefore, the polytropic index of the individual events changes appreciably. During more disturbed times ($AE > 300$), the correlation coefficients between $P_i$ and $\gamma$ have values slightly less than those in the quiet times. Since $\gamma$ is a function of specific entropy, it cannot be expected to remain constant in space or time. If we assume that $\alpha$ varies independently and randomly, the slope of the lines in Fig. 3(b) gives the value of $\gamma$, while the variation in $\alpha$ determines the correlation coefficient. For a large value of specific entropy and $\alpha$, the correlation coefficient is very small.

The variation of the field line curvature $R_c$ and chaos parameter $K$ with $AE$ index during the selected 22 events are shown in Fig. 4. From Fig. 4 we can see that $R_c$ and $K$ are maximum for strong storm ($AE = 348$) and minimum for weak storm ($AE = 120$). As $AE$ increases, $R_c$ and $K$ also increase gradually and reach a maximum for maximum $AE$. Since $K$ depends on the field intensity in the plasma sheet and mass of the moving particle, for a constant mass of the particle $R_c$ and $B$ are large during strong storms; so there is an increase of $K$ with $AE$ index. Since $R_c$ and $K$ are playing important roles in fire-hose instability, a clear picture of the instability in the plasma sheet associated with geomagnetic activity can be obtained.

The variation of the growth rate of fire-hose instability with wave number $k$ for adiabatic and non-adiabatic behaviours of the plasma sheet can be seen in Fig. 5. For adiabatic case, the instability curve is not so sharp as that for the non-adiabatic case. From this we can infer that a non-adiabatic medium is more favourable for the growth of fire-hose instability.
References