A comprehensive study of 2x2 planar phased array of rectangular microstrip antenna on Ni-Co based ferrite substrate at 10 GHz is presented. The far-zone field expressions have been derived using vector wave function technique and pattern multiplication approach. The pattern characteristics and other important antenna parameters like half power beam width (HPBW), direction of maximum radiation, total shift of major and minor lobe, side lobe level (SLL), radiation conductance, directive gain and impedance bandwidth are estimated for two values of progressive phase excitation difference, i.e. \( \beta_x = \beta_y = \pi/2 \) and \( 2\pi/3 \). The results of ferrite based array geometry are compared with those of dielectric based (PTFE quartz reinforced) array antenna. The results are quite interesting and the antenna geometry is suitable to be employed as a scanned array for radar applications.

1 Introduction
In recent years, ferrite substrates have been the subject of much interest for microstrip antennas and arrays. The high dielectric constant of the ferrite reduces the antenna dimensions and when biased with a DC magnetic field, the antenna exhibits a number of novel properties\(^1\)-\(^5\). These include frequency tuning agility, the generation of circular polarization, reduction of surface waves and radar cross-section control. Ferrite materials are also used to generate beam scanning antennas. In the present paper a 2x2 planar phased array of rectangular microstrip antenna on Ni-Co based ferrite substrate for two different values of progressive phase excitation difference between the elements at 10 GHz has been investigated. The array factor of the geometry and the far-zone field expressions are obtained using the pattern multiplication approach and vector wave function technique. The field patterns and important radiation parameters of the array geometry have been computed and plotted. These results of ferrite based array geometry have also been supported with those of same array configuration designed on PTFE quartz reinforced dielectric for the same input parameters.

2 Theory
The geometry and co-ordinate system of a 2x2 planar phased array of rectangular microstrip antenna is shown in Fig. 1. It consists of four identical elements printed on Ni-Co ferrite substrate (Ni\(_{0.62}\) Co\(_{0.02}\) Fe\(_{1.948}\) O\(_4\)) of thickness \( h \) and having \( \varepsilon_{\text{eff}} = 6.86 \) and \( \mu_{\text{eff}} = 20.7 \), where \( \varepsilon_{\text{eff}} \) and \( \mu_{\text{eff}} \) are the effective permittivity and permeability, respectively. The length of each element is \( L \) and width is \( W \). The array elements which are positioned along x-axis are separated by a distance \( d_x \) and those along y-direction are separated by a distance \( d_y \). Each element can be excited by a microstrip transmission line connected to the edge or by a co-axial line from the back at the plane \( \phi = 0 \). The total fields of the present array geometry can be expressed by the fields of the single element positioned at the origin multiplied by a factor which is referred as the array factor. Since the entire array is taken as uniform, the normalized form of array factor (AF) for this geometry is obtained using the procedure given by Balanis\(^6\) and Bahl and Bhartia\(^7\) which is as follows:

\[
AF = 0.25 \frac{\sin \{Kd_x \sin \theta \cos \phi + \beta_x\}}{\sin \{0.5(Kd_x \sin \theta \cos \phi + \beta_x)\}} \times \frac{\sin \{Kd_y \sin \theta \sin \phi + \beta_y\}}{\sin \{0.5(Kd_y \sin \theta \sin \phi + \beta_y)\}}
\]

where

\( K = \) Phase propagation constant for EM wave
\( \beta_x, \beta_y = \) Progressive phase excitation difference along x- and y-directions, respectively
\( d_x, d_y = \) Separation between the elements along x- and y-directions, respectively

The analysis of single element rectangular microstrip antenna has been reported in the literature. Here, we have developed a theory of 2x2 element planar phased array antenna designed on ferrite substrate considering variation in progressive phase excitation difference among the antenna elements, which can be effective in the system by using phase shifters. Neglecting coupling between the elements, the far-zone field expressions for array geometry are obtained as follows:

\[ E_{\theta r} = 0 \]  
\[ E_{\phi r} = -j2V_o W K e^{-jKr} \frac{\sin((Kh/2)\sin \theta \cos \phi)}{((Kh/2)\sin \theta \cos \phi)} \times \frac{\sin((KW/2)\cos \theta)}{((KW/2)\cos \theta)} \sin\theta \]

where,

\[ E_{\theta r} \text{ and } E_{\phi r} = \text{Components of total electric field vector for EM wave} \]

\[ V_o = \text{Edge voltage at } \phi = 0 \]
\[ h = \text{Thickness of substrate} \]
\[ W = \text{Width of patch} \]

Here, the expression for \( E_{\phi r} \) given in Eq. (3) involves additional terms containing \( \beta_x, \beta_y, d_x, d_y \). These factors are derived by considering the appropriate geometrical configuration of the array geometry. For the present calculation we need the value of \( L \) and \( W \) of rectangular patch, which have been determined using the following equations.

Fig. 1—Geometry and co-ordinate system of 2x2 element rectangular microstrip planar phased array
\[ L = \frac{c}{2 f_r \sqrt{\varepsilon_{\text{eff}} \mu_{\text{eff}}}} \] \hspace{1cm} \ldots (4)

\[ W = \frac{\lambda_0}{2} \left( \frac{\varepsilon_{\text{eff}} \mu_{\text{eff}} + 1}{2} \right)^{-1/2} \] \hspace{1cm} \ldots (5)

where, \( c \) is the velocity of EM wave.

**2.1 Field patterns**

The total field pattern \( R(\theta, \phi) \) is generally obtained from the relation

\[ R(\theta, \phi) = |E_{\theta}|^2 + |E_{\phi}|^2 \] \hspace{1cm} \ldots (6)

The values of \( R(\theta, \phi) \) are computed for a case taking \( f_r = 10 \text{ GHz} \), \( \varepsilon_{\text{eff}} = 6.86 \), \( \mu_{\text{eff}} = 20.7 \), \( L = 0.13 \text{ cm} \), \( W = 0.18 \text{ cm} \), \( h = 0.16 \text{ cm} \), \( K = 24.94 \text{ cm}^{-1} \), \( d_x = d_y = 1.5 \text{ cm} \). We have also calculated the total field pattern for this array geometry designed on dielectric substrate (PTFE quartz reinforced) taking parameters \( \varepsilon_r = 2.47 \), \( L = 0.95 \text{ cm} \), \( W = 1.14 \text{ cm} \), \( h = 0.16 \text{ cm} \), \( K = 3.28 \text{ cm}^{-1} \) and \( d_x = d_y = 1.5 \text{ cm} \).

For both the cases the results are plotted in \( \phi = 0 \) plane for two values of progressive phase excitation difference, i.e. \( \beta_x = \beta_y = \pi/2 \) and \( 2\pi/3 \), as shown in Figs 2-5.

It is observed that the patterns of array geometry are directive in nature containing secondary beams oriented in different positions in case of ferrite substrate. It is also found from Figs 2-5 that the position of the main beam and the secondary beams are shifted considerably on changing the progressive phase excitation difference, i.e. \( \beta_x = \beta_y = \pi/2 \) and \( 2\pi/3 \). We have measured different pattern characteristics of array geometry for \( \beta_x = \beta_y = \pi/2 \) and \( 2\pi/3 \) in both the cases of substrates and are given in Table 1.

![Fig. 2.—Variation of \( R(\theta, \phi) \) of array geometry for \( \phi = 0 \) plane and \( \beta_x = \beta_y = \pi/2 \) and \( 2\pi/3 \) for Ni-Co ferrite substrate](image)

**Fig. 2**—Variation of \( R(\theta, \phi) \) of array geometry for \( \phi = 0 \) plane and \( \beta_x = \beta_y = \pi/2 \) and \( 2\pi/3 \) for Ni-Co ferrite substrate
2.2 Radiation conductance

The expression for radiation conductance of the array geometry may be expressed as

\[ G = \frac{2P_r}{V_0^2} \]  \hspace{1cm} \ldots (7)

where,

\[ P_r = \frac{1}{2Z_0} \int \int \{ |E_{\theta r}|^2 + |E_{\phi l}|^2 \} r^2 \sin \theta \, d\theta \, d\phi \]  \hspace{1cm} \ldots (8)

\[ Z_0 = \text{Free space impedance} = 120 \pi \]

The field pattern, \( R(\theta, \phi) = |E_{\theta r}|^2 + |E_{\phi l}|^2 \)

\[ = \frac{A^2 \sin^2 \left\{ (Kh/2) \sin \theta \cos \phi \right\} \sin^2 \left\{ (KW/2) \cos \theta \right\}}{\{(Kh/2) \sin \theta \cos \phi(KW/2) \cos \theta\}^2} \]

where,

\[ A = -j2\pi \frac{r \sin \theta \cos \phi + \beta_x}{4\pi r} \]

2.3 Directive gain

The directive gain of an antenna in a given direction is defined as the ratio of the radiation intensity \( U \) in that direction to the average radiated power \( P_r \) (Ref. 10). It is expressed as

\[ D_g = \frac{4\pi U}{P_r} \]  \hspace{1cm} \ldots (10)

Fig. 3—Variation of \( R(\theta, \phi) \) of array geometry for \( \phi = 0 \) plane and \( \beta_x = \beta_y = \pi/2 \) and \( 2\pi/3 \) for dielectric substrate
Therefore, where, \( Q_T \) is the total quality factor given as

\[
D_g = \frac{4\pi M_e}{I} \quad \ldots (11)
\]

where,

\[
I = \int_0^{2\pi} \int_0^{\pi} M_e \sin \theta \, d\theta \, d\phi \quad \ldots (12)
\]

\[
M_e = R(\theta, \phi) = \sqrt{\left| E_{\theta} \right|^2 + \left| E_{\phi} \right|^2}
\]

which is evaluated from Eq. (9).

2.4 Impedance bandwidth

Impedance bandwidth for an antenna can be given as

\[
BW = \frac{f_r}{Q_T} \quad \ldots (13)
\]

where, \( Q_R, Q_C \) and \( Q_D \) are the radiation, conductor and dielectric (substrate) loss quality factors, respectively, and defined as follows:

\[
Q_R = \frac{c \sqrt{E_{\text{eff}}}}{4 f_r h} \quad \ldots (15)
\]

\[
Q_C = 2\pi h \sqrt{5.7 f_r} \quad \ldots (16)
\]

\[
Q_D = \frac{1}{\tan \delta} \quad \ldots (17)
\]

![Fig. 4—Comparison of \( R(\theta, \phi) \) for the array geometry designed on Ni-Co ferrite and quartz reinforced for \( \phi = 0 \) plane and \( \beta_x = \beta_y = \pi/2 \).](image-url)
where, $\tan \delta$ is the loss tangent of substrate materials and can be determined using microwave technique for dielectric measurement.

The values of radiation conductance, directive gain and impedance bandwidth have been calculated for the array geometry using above expressions for two different values of progressive phase excitation, i.e. $\beta_x = \beta_y = \pi/2$ and $2\pi/3$ by giving the same input parameters for both the substrates. The integral involved in Eq. (8) has been solved using numerical method\textsuperscript{11}. The calculated values are given in Table 2.
Table 2—Calculated values of antenna parameters of array geometry

<table>
<thead>
<tr>
<th>Antenna Parameters</th>
<th>Ferrite (Ni_{0.06}Co_{0.02}Fe_{1.94}O_4)</th>
<th>Dielectric (PTFE quartz reinforced)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation conductance (G)(mho)</td>
<td>β=π/2 1.026x10^{-3}</td>
<td>β=2π/3 1.196x10^{-3}</td>
</tr>
<tr>
<td>Directive gain(D_g)(dB)</td>
<td>5.49</td>
<td>4.29</td>
</tr>
<tr>
<td>Impedance bandwidth (BW)(%)</td>
<td>3.2</td>
<td>3.73</td>
</tr>
</tbody>
</table>

It is observed from Table 2 that there is a significant change in the values of radiation conductance, directive gain and impedance bandwidth on the variation of progressive phase excitation difference among the elements of the array geometry.

3 Discussion and conclusions

It is found that there is a significant change in the radiation characteristics of the antenna under investigation due to (i) variation of progressive phase excitation difference among the elements and (ii) designing on ferrite substrate.

Figures 2 and 3 represent the field patterns in \( \phi = 0 \) plane of the array geometry designed on ferrite and dielectric substrate, respectively, for two values of progressive phase excitation, i.e. \( \beta_x = \beta_y = \pi/2 \) and \( 2\pi/3 \).

Some salient features of this array geometry are summarized as follows:

(i) On changing the value of \( \beta \), the position of principal maxima and secondary maxima are shifted by a maximum value of 24° and 25°, respectively. However, in case of dielectric based array geometry, there is a total shift of 15° only for principal maxima, while secondary maxima is found absent. Thus, it is evident that ferrites are more suitable substrate material for designing a scanned array.

(ii) The patterns are having relatively narrow beam with a HPBW 10° for \( \beta = \pi/2 \) as well as low value of SLL (−9.6 dB) for \( \beta = 2\pi/3 \) for ferrite based array geometry. To have low value of SLL of array is an essential requirement and considerable importance in many applications.

(iii) A maximum value of 6.67 dB of directive gain and impedance bandwidth 3.73% are obtained for the present array geometry. These results are in close agreement with the recent experimental values reported by Staraj et al.\(^1\) and Yang\(^2\).

(iv) The size of antenna is considerably reduced when designed on ferrite substrate. This considerable reduction in size of array geometry has potential application in miniaturization of antenna system for satellite and cellular communication.

The overall results of HPBW, SLL, \( G \), \( D_g \) and \( BW \) show that the array geometry provides improved radiation performance which may be utilized to form scanned arrays.

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References


