Theoretical model on mass of soil particles passing through two-layered nonwoven geotextile

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A two-layered nonwoven geotextile, associated with one layer of fine fibres having a thickness selected to provide optimal filtration characteristics and another layer of coarse fibres to provide the required mechanical properties, has been developed to study its theoretical filtration performance. Based on the theoretical filtration of single layer nonwoven geotextile, a mathematical model on mass of soil passing through two-layered nonwoven geotextile is established. The model is validated by slurry test using a self-designed apparatus. The experimental results obtained for five different specimens are compared with the theoretical solution, and it is found that the present model predicts to an accuracy of about 93.2% for needle-punched two-layered nonwoven geotextiles.

Keywords: Filtration performance, Pore size, Soil retention capacity, Two-layered nonwoven geotextiles

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1 Introduction

Nonwoven geotextiles are widely used in drainage systems, and a great deal of new products is developed to improve the soil retention capacity. Although mechanical properties are proverbially important, the selection of geotextile for a given soil and prevalent hydraulic gradients is based on its filtration behaviour. Filtration performance is related to the capacity to retain soil particles over time without clogging, while permitting water to percolate through the fabric. Soil retention capacity is related to the type of nonwoven geotextile (fabric structure), type of soil (size of particles and granulometric curve distribution), and mechanical and hydraulic in situ conditions. Water percolation capacity is also related to the type of nonwoven geotextile and soil as well as mechanical and hydraulic in situ conditions. For the given soil and flow conditions, the filtration performance mainly depends on fabric structure, such as fabric thickness, porosity, pore sizes and their distribution curve. Lombard et al. have studied the relationship between pore sizes and structure of heat-bonded geotextiles and found that the pore sizes and their distribution curve of every elementary plane in nonwoven geotextile are theoretically invariable only if fibres in the elementary plane have the same diameter. However, for nonwovens with different thickness, the pore sizes distribution curves are different even if they are made up of fibres with the same diameter. In other words, thickness is an essential parameter for the filtration performance of nonwoven geotextile. Giroud et al. found that the pore sizes of nonwoven geotextile made from the same material decrease with increasing thickness. A relatively thin nonwoven geotextile is desirable in many practical cases. However, a thin nonwoven geotextile may not have the required mechanical properties to withstand mechanical damage and to resist deformations that could alter its pore sizes. So, the development of a two-layered nonwoven geotextile is needed for practical uses, which consists of a layer constructed with fine fibres to percolate water and retain soil, and another layer constructed with coarse fibres to provide enough mechanical strength.

A two-layered nonwoven geotextile has already been developed by the author and the effects of manufacturing parameters on its properties studied. A theoretical model on mass of soil particles passing through the two-layered nonwoven geotextiles is established in this paper.
1.1 Research Background

Studies have been done on the filtration performance of nonwoven geotextile, including experimental tests and theoretical analyses. Lombard et al.\(^9\) studied the theoretical pore size distribution of geotextile based on Poisson Polyhedron theory and got the following expression:

\[
F_{g(r)} = 1 - \left[ (\sigma r + 1)^2 \exp(-2\sigma r) \right]^N \quad \ldots(1)
\]

where \(F_{g(r)}\) is the theoretical pore sizes distribution of geotextile; \(r\), the equivalent radius of pores in geotextile; \(\sigma = \frac{8 \mu_g}{\pi T_g d_f \rho_f}\), the total fibre length per unit area of an elementary plane, usually called specific length; \(T_g\), the thickness of geotextile; \(N = T_g/2d_f\), the number of all elementary plane in geotextile; \(\rho_f\), the fibre density; \(d_f\), the fibre diameter; and \(\mu_g\), the mass per unit area of geotextile.

Based on Eq. (1) and random probability theory, Liu\(^11\) discussed theoretical filtration performance of single-layered nonwoven geotextile and obtained two related expressions. One is the probability \(P_{g_p(x)}\) of particles with radius \(x\) passing through nonwoven geotextile, that is

\[
P_{g_p(x)} = \varepsilon \left[ 1 - F_{g(x)} \right]^N = \varepsilon \left[ (1 + \sigma x)^2 \exp(-2\sigma x) \right]^N \quad \ldots(2)
\]

where \(\varepsilon = 1 - \left( \mu_g / T_g \rho_f \right)\) is the porosity of nonwoven geotextile.

The other is the mass of soil particles passing through unit area of nonwoven geotextile, that is

\[
U_p = \sum_{I=1}^{I} U_I = \sum_{I=1}^{I} \frac{U_I}{M_I} \int_0^{O_{100}} (M - IP_{g_p(x)})^{-1} \cdot P_{g_p(x)} \phi_x dx \quad \ldots(3)
\]

where \(U_p\) is the mass of soil particles passing through unit area of nonwoven geotextile; \(U_I\), the mass of soil particles on per unit area of geotextile at the beginning of filtering; \(M\), the number of soil layer; \(I\), the number of soil layers of every “instant filtration”, according to the presume in Liu’s study \(I = 1\); \(r_{M}\), the theoretical maximal pore radius of nonwoven; and \(\phi_x\), the proportion of particles with radius \(x\) at the starting of filtering.

The relationship between filtration performance and structure parameters of two-layered nonwoven geotextile is theoretically discussed based on the above theoretical expressions.

2 Materials and Methods

2.1 Theoretical Model

Presume that the two-layered nonwoven geotextile consists of Layer A and Layer B, whose characteristics are shown in Table 1.

The pore size and its distribution in each layer of two-layered nonwoven geotextile are different since the fibres of each layer have different diameter. Figure 1 shows the cumulative probability of pore size distribution of Layer A and Layer B, in which \(O_{A1}\) and \(O_{B1}\) are their pore size cumulative distribution respectively (\(O_{A1} > O_{B1}\) as shown in Table 1) and \(O_{100}\) indicates 100% of pores smaller than or equal to \(O_{100}\), i.e. the size of the largest pore in geotextile. Hence, a particle with radius \(x\) can surely pass through the whole two-layered nonwoven geotextile only if it can pass through Layer B, in other words whether the particle can pass through two-layered nonwoven geotextile depends on \(O_{B1}\).

### Table 1 — Characteristics of Layers A and B

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mass per unit area of nonwoven kg/m²</th>
<th>Thickness m</th>
<th>Fibre diameter (d_f) m</th>
<th>Fibre density (\rho_f) kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer A</td>
<td>(\mu_{g1})</td>
<td>(T_{g1})</td>
<td>(d_{f1})</td>
<td>(\rho_{f1})</td>
</tr>
<tr>
<td>Layer B</td>
<td>(\mu_{g2})</td>
<td>(T_{g2})</td>
<td>(d_{f2})</td>
<td>(\rho_{f2})</td>
</tr>
</tbody>
</table>

\(d_{f1} > d_{f2}\)

Fig. 1 — Pore size cumulative distribution curve of Layer A and Layer B
A two-layered nonwoven geotextile may have two placing methods in practical uses. In one method (Method 1), the layer made up of coarse fibres is kept in contact with soil directly, as shown in Fig. 2(a) and in the other method (Method 2), the layer of fine fibres is kept in contact with soil directly, as shown in Fig. 2(b).

2.1.1 Filtration Performance of Geotextile for Method 1

A particle with radius $x$ may occur at three situations depending on the relationship between its size and the theoretical maximum radius $r_{M1}$ and $r_{M2}$ of Layer A and Layer B respectively, that is

(i) The particle is retained by the two-layered nonwoven geotextile if $x \geq r_{M1}$;

(ii) The particle may be clogged in the two-layered nonwoven geotextile if $r_{M2} \leq x < r_{M1}$; and

(iii) The particle may pass through the two-layered nonwoven geotextile if $x < r_{M2}$.

With Eq. (2), the probability of a particle with radius $x$ passing through the whole two-layered nonwoven geotextile can be expressed as

$$P_{gp(x)} = P_{gp(x)1} \cdot P_{gp(x)2}$$

Hence, the mass of soil passing through unit area of two-layered nonwoven geotextile can be obtained as follows:

$$U_P = \sum_{i=1}^{M} \left[ \int_{0}^{r_{M2}} (M - IP_{gp(x)1}P_{gp(x)2})^{-1} \cdot P_{gp(x)1}P_{gp(x)2} \cdot P_x \cdot d \right]$$

2.1.2 Filtration Performance of Geotextile for Method 2

A particle with radius $x$ may occur at two situations depending on the relationship between its size and the theoretical maximum radius $r_{M1}$ and $r_{M2}$ of Layer A and Layer B, that is

(i) the particle is retained by the two-layered nonwoven geotextile if $x \geq r_{M2}$; and

(ii) the particle may pass through the two-layered nonwoven geotextile if $x < r_{M2}$.

Hence, the mass of soil passing through unit area of two-layered nonwoven geotextile can also be expressed by Eq. (4).

The above analyses show that the theoretical masses of soil passing through two-layered nonwoven geotextile remain the same, no matter the coarse or fine fibres layer is contacting with soil directly. However, practically some soil particles can move with the fluid flow into the coarse layer whose pore sizes are relatively larger if it is contacting with soil directly, and encounter the fine layer whose pore sizes are relatively smaller. Hence, only a small part of soil particles can pass through this fine fibres layer and lose with the flow, but most of the particles are clogged in the two-layered nonwoven geotextile. When the fine fibres layer contacts with soil directly, the soil particles get to the smaller pores firstly so that only those with very smaller size can enter into or pass through it. These particles can now easily pass through coarse fibres layer that has larger pore size. For the former situation, there are more particles clogged in the two-layered nonwoven geotextile; but for the latter situation, there are more particles retained on the two-layered nonwoven geotextile. Although theoretical masses of soil passing through two-layered nonwoven geotextile are the same, the filtration behaviour would be different.

2.2 Characteristics of Two-layered Nonwoven Geotextile

Five kinds of two-layered needle-punched nonwoven geotextiles (Samples 1-5) were prepared from the polyester staple fibres (fibre density $p_f = 1380\text{g/m}^3$). The diameters of the fine and coarse fibres used in the above samples are 22.8 μm and 28.6 μm respectively. Firstly, the fine fibres and coarse fibres layers were needle punched at needle density of 160 punches/cm² respectively, and then the two layers were needle punched together at needle density of 140, 190, 260, 310, 360 punches/cm² for samples 1-5 respectively. The proportion of mass per unit area of fine fibres layer and coarse fibres layer approximates to 40/60. The characteristics of all the samples are shown in Table 2. The thickness values of
Table 2 — Nonwoven geotextile characteristics

<table>
<thead>
<tr>
<th>Sample</th>
<th>Needle density punches/cm²</th>
<th>$\mu_g$ g/m²</th>
<th>$T_g$ mm</th>
<th>$\varepsilon$ %</th>
<th>$N$</th>
<th>$\sigma$ mm⁻¹</th>
<th>$r_M$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer A</td>
<td>300</td>
<td>239</td>
<td>3.12</td>
<td>54</td>
<td>4.94</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>Layer B</td>
<td>160</td>
<td>2.08</td>
<td>94.4</td>
<td>46</td>
<td>6.19</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td>Whole fabric</td>
<td>399</td>
<td>5.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer A</td>
<td>350</td>
<td>235</td>
<td>2.91</td>
<td>50</td>
<td>5.19</td>
<td>0.49</td>
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<tr>
<td>Layer B</td>
<td>156</td>
<td>1.94</td>
<td>94.1</td>
<td>43</td>
<td>6.50</td>
<td>0.39</td>
<td></td>
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<tr>
<td>Whole fabric</td>
<td>391</td>
<td>4.85</td>
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<tr>
<td>3</td>
<td></td>
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<tr>
<td>Layer A</td>
<td>420</td>
<td>229</td>
<td>2.72</td>
<td>47</td>
<td>5.42</td>
<td>0.47</td>
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<tr>
<td>Layer B</td>
<td>153</td>
<td>1.82</td>
<td>93.9</td>
<td>40</td>
<td>6.78</td>
<td>0.38</td>
<td></td>
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<tr>
<td>Whole fabric</td>
<td>382</td>
<td>4.54</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Layer A</td>
<td>470</td>
<td>226</td>
<td>2.54</td>
<td>44</td>
<td>5.71</td>
<td>0.45</td>
<td></td>
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<tr>
<td>Layer B</td>
<td>150</td>
<td>1.70</td>
<td>93.6</td>
<td>37</td>
<td>7.15</td>
<td>0.36</td>
<td></td>
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<tr>
<td>Whole fabric</td>
<td>376</td>
<td>4.24</td>
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<td></td>
<td></td>
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<tr>
<td>5</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer A</td>
<td>520</td>
<td>223</td>
<td>2.62</td>
<td>45</td>
<td>5.51</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>Layer B</td>
<td>148</td>
<td>1.74</td>
<td>93.8</td>
<td>38</td>
<td>6.90</td>
<td>0.37</td>
<td></td>
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<tr>
<td>Whole fabric</td>
<td>371</td>
<td>4.36</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Layer A and Layer B are presumed to have the same proportion with their mass per unit area for simplification.

Layer A is constructed with coarse fibres; Layer B is constructed with fine fibres; Whole fabric is the two-layered nonwoven geotextiles. $\mu_g$ — the mass per unit area of geotextile; $T_g$, — thickness of geotextile; $\varepsilon$ — porosity of nonwoven geotextile; $N$ — number of all elementary plane in geotextile; $\sigma$ — specific length, the total fibre length per unit area of an elementary plane; and $r_M$ — theoretical maximal pore radius of nonwoven.

Layer A and Layer B are presumed to have the same proportion with their mass per unit area for simplification.

The theoretical maximum pore radius ($r_M$) can be obtained by the following equation:

$$r_M = \frac{7 + 4\sqrt{3}}{2\sigma(1 + \sqrt{3})} \quad \text{mm}$$

2.3 Soil characteristics

Soil (density 2.48g/cm³) used for the study was procured from the pumping station of Taipu River in Wuxi, China. The size cumulative distribution curve of the soil particles and its regression curve are shown in Fig. 3. A following regression equation can be obtained from Fig. 3:

$$Y = \left(1 + 1/P_1\right) / \left[1 + P_1 \cdot \exp(-P_2x)\right] \quad \text{...}(5)$$
where \( P_1 = 1306.191 \), \( P_2 = 104.956 \) and correlation coefficient \( R^2 = 0.988 \).

Based on the regression equation of the size distribution curve of soil particles [Eq. (5)], the proportion of particles with radius \( x \) in the entire soil can be obtained using the following relationship:

\[
\phi_x = \frac{P_1(1 + P_1) \exp(-P_2 x)}{[1 + P_1 \exp(-P_2 x)]^2}
\]

where \( P_1 = 1306.191 \) and \( P_2 = 104.956 \).

Hence, the average diameter of soil particles can be obtained as

\[
\bar{d} = 2 \int_0^\infty x \cdot \phi_x \cdot dx = 0.137 \text{ mm}
\]

The thickness of soil was selected as 5 mm in filtration test, then the number of unit soil layer will be

\[
M = \frac{5}{d} = 36
\]

### 2.4 Test Method

The experiment was carried on a filtering apparatus as shown in Fig.4. A circular piece of sample with diameter of 14cm was weighed and then put into water till fully saturation, so as to drive off air from it. The sample was then put on the center of ground platform, and a round rubber blanket with bore diameter of 14cm was kept on it, so as to avoid water seeping from the sample edge. A top cylinder was now put just on the rubber blanket in such a way that the sample, rubber blanket and top cylinder are homocentric. Tightened the six nuts and bolts that are evenly distributed along the edge of top cylinder and ground platform respectively, as shown in Fig. 4. According to the soil density and the inner diameter of the apparatus, weighed 191 g dry soil in order to have the soil on the sample with a thickness of 5 mm. The soil was kept into a beaker and then poured some water to dissolve it. The solution was poured onto the sample in the apparatus followed by the addition of air-free water till the water attains a height of 13cm; the same height was maintained during the whole experiment by adding water to ensure the height, as the water drops down through samples continually. The soil solution was stirred constantly. Stopped stirring and adding water after 3 h, and then allowed water in the apparatus drop freely. After the water on the sample fully dropped down, the sample was taken out along with soil remaining on it, dried and weighed. The mass of soil particles passing through the sample was calculated. The same procedure was repeated for another sample. Three specimens of each geotextile were tested and the average was taken.

### 3 Results and Discussion

The theoretical masses of soil passing through the five samples are calculated from Eq. (4). The
calculated results are found to be variable with different filtration time. According to Liu\textsuperscript{11}, filtration time is selected as 3000 times in theoretical calculation.

Table 3 shows the theoretical values of mass of soil particles passing through unit area of samples as well as the corresponding experimental results and their relative errors. Comparison of theoretical values with experimental ones shows that the average difference of five samples with coarse and fine fibres contacting with soil is about 6.8\%, i.e. the accuracy is 93.2\%, which indicates a good agreement between theoretical calculations and experimental results. Comparison of the five samples shows that with the increase in needle density, soil mass passing through unit area of fabric firstly decreases and then increases, reaching the smallest value at 470 punches/cm\(^2\). As the increase in needle density results in a more tangle structure, the pore size of fabric decreases, thereby decreasing the soil mass passing through it; when the needle density increases to 470 punches/cm\(^2\) and more, the barbs cause fibres to break and make the entanglement of fibres more difficult, ultimately causing a larger penetration hole and increasing the soil mass passing through fabric. Finally, the comparison of the same sample with different fibre layers contacting with soil shows that more soil particles can pass through fabric in which coarse fibres layer contacts soil. Since the finer fibres implies an increase in number of fibres in unit area, as well as a decrease in its stiffness, the fabric tangles more close and becomes more compact, thereby causing a smaller pore size in fabric. When two-layered nonwoven geotextile contacts soil with the fine fibres layer, some small soil particles can be retained on fabric surface. This blocks the movement of soil particles whose diameter is even smaller more than the pore size of fine fibres layer, causing the decrease in soil mass passing through the whole fabric. Moreover, it also decreases the passing of water flow. When the fabric contacts soil with coarse fibres layer, those small soil particles have the probability to enter into the coarse fibres layer. The soil particles whose diameter is smaller than pore size of fine fibres layer will also have the opportunity to enter into the fabric and then pass through the fine fibres layer as well as the whole fabric, and so more soil particles can pass through.

4 Conclusions
The theoretical mass of soil particles passing through unit area of two-layered nonwoven geotextile can be expressed as

\[ U_P = \sum_{i=1}^{n} \left[ \frac{U}{M_j} \int_{0}^{\phi x} (M - IP_{gr(x)}) P_{gr(x)} dx \right] \]

This remains the same no matter which single layer contacts the soil directly. The model has been validated using a slurry test. The comparison of experimental results with the theoretical results shows that the present model predicts to an accuracy of about 93.2\%.

The theoretical expression shows that the filtration performance of two-layered nonwoven geotextile is related to structures of the two layers (mass per unit area of geotextile, porosity, pore size distribution etc), soil characteristics (size distribution curve and maximal diameter of particles), flow conditions, and filtration time. In general, with the same soil characteristics and flow conditions, the thinner geotextile with higher porosity and greater pore sizes will permit more soil particles to pass through.

Although the theoretical soil mass passing through two-layered nonwoven geotextile is constant, the experimental results are different when different fibre layers are contacting with soil.

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References


