Grey trend relational model of production Cost for Torpedo

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For the same type of torpedo made in different batches, production cost will be different. In general, the cost is a downward trend with the increase of the batches. Production costs of different batches were divided into three data sequences with equal length, grey trend relational degree between the sequences is calculated. Equation based on trend consistent principle is established, and then the value of production cost can be predicted. A numerical example was provided to verify the properness of the model. Comparing with traditional methods, the result of the paper indicates that the grey trend relational model reflects the internal trend of production cost. It is suitable for predicting production cost of other productions that have a few batches.

[Keywords: Production cost, Grey trend relational degree, Learning curve, Torpedo]

Introduction

Production cost was studied in many aspects. The impact factors on production cost were studied in1. Some of the recent advances made in reducing the production cost were studied in2&3. Identification and collection of production cost data was studied in4. Estimation of production cost was studied in5,6,7&8, and estimation study was an important aspect of production cost.

Materials and Methods

There are many methods to estimate production cost of equipment. Engineering estimate method is estimation process based on engineering bills of materials and contract10. It is most accurate. But it needs exhaustive understanding of the product. A large amount of data, which spans a long time, should be collected. So it is fussy and time consuming, and can only be used in the last stage of production11&12. Expert estimate method13, which depends on the experience of the expert, is imprecise. Analogism estimate method14 can only be used when a similar product has been produced. Learning curve method15 is most commonly used. Learning curve was first bring forward by Wright in 1936 in the American aircraft industry16, and conformed by Crawford in 1940s17, which is still commonly used in estimate the production cost18,19,10&21.

The price of a torpedo depends heavily on the production cost, which is related to cumulative output and can be predicted by establishing a model according to the production cost of previous torpedo batches. Torpedo has only a few batches and less data that can be used to analyze. In the paper, a new method of grey trend relational model of production cost for torpedo is proposed.

Grey system theory is proposed by Deng in 198222, which is suitable for poor information systems. White system means the information of the system is well known. Black system means the information of the system is unknown. Grey system means the information of the system is somewhat known. Grey system theory is used in many aspects23, 24, 25&26. Grey system theory modeling needs less information (at least four series data can build a model27), and is fit for this problem.

The rest of the paper is organized as follows. Section 3 reviews the method of learning curve and presents grey trend relational model of production cost for torpedo. Then an example of how to use this method is given. The conclusions are given in Section 4.
Results and Discussion

Learning Curve of the Production Cost

The relationship between production cost and cumulative output is usually described by the learning curve, which indicates that production cost of the first product is the highest and will decline regularly at a certain rate with the increase of output under normal conditions. The decline is quick at first, then slow, eventually stabilizing approximately. When the output of a product doubles, the production cost per product will decline at a constant percentage.

The learning curve shows the decreasing law of production cost when production is carried out repeatedly and constantly. The curve can be built from collected historical data and can be used to predict subsequent production cost.

The learning curve currently has two forms: the Wright formula and the Crawford formula, both of which are empirical. The Wright formula is:

\[ C = C_1 N^b \]  

where \( C \) is the cumulative average cost, \( C_1 \) is the production cost of the first product, \( N \) is the cumulative output and \( b \) is the learning index.

The Crawford formula has the same form as Wright, with the difference being that \( C \) is the production cost of the \( N \)th product. With the previous data, specific relationship of the two formulas is shown in figure 1. In both instances, \( b = \log S / \log 2 \), represents the slope of the learning curve and \( S \) is the learning slope.

Fig. 1 — Relationship between the Wright and Crawford learning curves

Grey Trend Relational Model

Because torpedoes of the same type are made in few batches, it is not suitable to use traditional theory to build a production cost model because of the relatively low amount of data available for predicting subsequent production cost. Grey system theory is a large system theory. It reflects the nature rule of comprehensive influence of many comprehensive affecting factors. The relationship between production cost and cumulative output is influenced by many economical and societal factors. Those factors affect the relationship jointly. Using the Grey trend relational degree, which reflects the developing situation of a series, a new prediction model based on the Grey trend relational degree can be built through similarity system theory (i.e., the developing trend of the same system should be consistent).

Grey Trend Relational Degree

Given

\[ X_0 = (x_0(1), x_0(2), \ldots, x_0(n)) \]

as the system behavior characteristic reference series, the comparative series as

\[ X_1 = (x_1(1), x_1(2), \ldots, x_1(n)) \]
\[ X_2 = (x_2(1), x_2(2), \ldots, x_2(n)) \]
\[ \ldots \]
\[ X_m = (x_m(1), x_m(2), \ldots, x_m(n)) \]

and

\[ \sum_{k=1}^{n-1} |x_i(k+1) - x_i(k)| \neq 0, (i = 0, 1, 2, \ldots, m) \]

if

\[ (x_i(k+1) - x_j(k)) (x_j(k+1) - x_j(k)) \geq 0 \]

with

\[ k = 1, 2, \ldots, n-1; i, j = 0, 1, 2, \ldots, m \]

it can be regarded that \( x_i \) has the same developing trend as \( x_j \).

Given

\[ D_i = \frac{1}{n-1} \sum_{k=1}^{n-1} |x_i(k+1) - x_i(k)|, i = 0, 1, 2, \ldots, m. \]
\[ y_i(k) = \frac{1}{D_j} x_i(k), k = 1, 2, \ldots, n; i = 0, 1, 2, \ldots, m. \]

\[ \Delta y_i(k + 1) = y_i(k + 1) - y_i(k), \]

\[ k = 1, 2, \ldots, n - 1; i = 0, 1, 2, \ldots, m. \]

if

\[ \Delta y_i(k + 1) \Delta y_j(k + 1) = 0, \ z_{ij}(k + 1) = 0 \]

\[ \Delta y_i(k + 1) \Delta y_j(k + 1) \neq 0 \]

\[ z_{ij}(k + 1) = \text{sgn} \left( \frac{\Delta y_i(k + 1) \Delta y_j(k + 1)}{2 \max (|\Delta y_i(k + 1)|, |\Delta y_j(k + 1)|)} \right) \]

then

\[ \gamma(X_i, X_j) = \frac{1}{n - 1} \sum_{k=1}^{n-1} z_{ij}(k + 1) \]

is the Grey trend relational degree of \( X_i \) and \( X_j \), and \( z_{ij}(k + 1) \) is the relational coefficient of \( X_j \) relative to \( X_i \) on point \( k + 1 \). Thus, the Grey trend relational degree reflects the developing situation of the series and has no relationship with its curve shape.

Torpedo Production Cost Model

Because torpedoes are produced in batches, only the average production cost per batch can be obtained. In order to eliminate the influence of production cost per batch caused by the difference of output, the cumulative average cost of each batch can be gotten by the average production cost and output of each batch:

\[ \hat{y}_k = \frac{\sum_{l=1}^{k} (x_l \gamma_l(k))}{\sum_{l=1}^{k} x_l} \]

where \( x_l \) is the output of the \( l \) batch, \( y_l \) is the average production cost of the \( l \) batch and \( \hat{y}_k \) is the cumulative average production cost of \( k \) batches.

The cumulative average production cost of the subsequent batch can be predicted by building a model using the previous cumulative average production cost sequences.

The cumulative average cost data series can be rearranged into three series: \( X_0, X_1, X_2 \), which includes the data being forecasted. \( X_0, X_1, X_2 \) can be inputted respectively into formula (3) to calculate \( r(X_0, X_1) \) and \( r(X_1, X_2) \), which are the Grey trend relational degree of the two adjacent series. Because \( X_0, X_1, X_2 \) are from the same data series, their Grey trend relational degree should be approximately equal according to similarity system theory:

\[ r(X_0, X_1) = r(X_1, X_2) \]

This can be used to establish equations. In order to simplify the model, some definite intervals are analyzed, given that the production cost of each batch does not change widely. Using MATLAB\textsuperscript{35}, equations can be solved. The cumulative average production cost of the next batch can then be predicted.

The constraint of the Grey trend relational model is that the trend of the two series must stay uniform. Therefore, the series should be monotonous or change only periodically. In this paper, we use only a monotonous series for demonstration.

Example

Using historical data for a torpedo manufactured in six batches, the output, cumulative output and average production cost of batches one through six is as table 1:

<table>
<thead>
<tr>
<th>Batch</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>35</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>Cumulative output</td>
<td>10</td>
<td>25</td>
<td>45</td>
<td>80</td>
<td>120</td>
<td>180</td>
</tr>
<tr>
<td>Production Cost ($)</td>
<td>4400</td>
<td>4240</td>
<td>4100</td>
<td>4000</td>
<td>3900</td>
<td>3820</td>
</tr>
</tbody>
</table>
The average production cost of the sixth batch can be predicted using the Grey trend relational model with data from the previous five batches.

The learning curve model can also be used to predict the cost. The effectiveness of the methods can be compared with the historical value of the sixth batch to validate the models.

Using equation (4), the cumulative average production cost of batches one through six is as follows:

<table>
<thead>
<tr>
<th>Batch</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>4400</td>
<td>4304</td>
<td>4213.3</td>
<td>4120</td>
<td>4046.7</td>
<td>3971.1</td>
</tr>
</tbody>
</table>

(1) Grey Trend Relational Method

According to the Grey trend relational degree, three series can be structured:

\[ X_0 = (4400, 4304, 4213.3, 4120) \]
\[ X_1 = (4304, 4213.3, 4120, 4046.7) \]
\[ X_2 = (4213.3, 4120, 4046.7, s) \]

where \( s \) is the predicted cumulative average production cost of the sixth batch.

According to equation (3), with \( X_0 \) and \( X_1 \), the relational coefficient and the Grey trend relational degree of the cumulative average production cost can be obtained:

\[ \zeta_{01}(2) = 0.9863, \zeta_{01}(3) = 0.9467, \zeta_{01}(4) = 0.9275 \]

\[ r(X_0, X_1) = \frac{1}{3} \sum_{k=1}^{3} \zeta_{01}(k + 1) = 0.9535. \]

According to equation (3), with \( X_1 \) and \( X_2 \), the expression of the Grey trend relational degree of the cumulative average production cost can also be obtained by the following analyses:

\[ r(X_1, X_2) = \frac{1}{3} \sum_{k=1}^{3} \zeta_{12}(k + 1) \]

\[ |\Delta y_1(k + 1)| = (1.0575, 1.0878, 0.85464) \]

\[ |\Delta y_2(k + 1)| = \begin{pmatrix} 279.9 \\ 4213.3 - s \\ 219.9 \\ 4213.3 - s \\ 3(4046.7 - s) \\ 4213.3 - s \end{pmatrix} \]

Because \( X_0, X_1, X_2 \) are all obtained from the same data series, then the equation (5) must be met.

According to the learning curve, the average production cost of the sixth batch is less than that of the fifth batch.

\[ \Delta y_1(k + 1) \Delta y_2(k + 1) (k = 1, 2, 3) \]

is larger than zero, so

\[ \text{sgn}\left[ \Delta y_1(k + 1) \Delta y_2(k + 1) \right] = 1. \]

Because \( |\Delta y_2(k + 1)| \) contains the unknown \( s \), it is difficult to determine which is larger between \( |\Delta y_1(k + 1)| \) and \( |\Delta y_2(k + 1)| \),

so, the search range \( s \in [0, +\infty] \) is separated into four sections.

(a) Given \( 0 < s \leq 3948.6 \) or \( s \geq 4213.3 \)

\[ \max\left( |\Delta y_1(k + 1)|, |\Delta y_2(k + 1)| \right) = \begin{pmatrix} 1.0575 \\ 1.0878 \\ 3(4046.7 - s) \\ 4213.3 - s \end{pmatrix} \]

According to equation (5), an equation can be obtained, and the equation can be solved with MATLAB. The solutions are:

\[ s_1 = 3963.3 \notin \{[0,3948.6] \ or \ [4213.3, +\infty]\} \]

\[ s_2 = 4085.6 \notin \{[0,3948.6] \ or \ [4213.3, +\infty]\} \]
These results are not reasonable.

(b) Given $3948.6 < s \leq 3980.3$
\[ \max \left( \left| \Delta y_1 (k + 1) \right|, \left| \Delta y_2 (k + 1) \right| \right) = \left( \frac{279.9}{4213.3 - s} \right) \times 1.0878 \times \frac{3(4046.7 - s)}{4213.3 - s} \]

According to equation (5), an equation can be obtained, and the solutions are:
\[ s_1 = 4137.6 \notin [3948.6, 3980.3] \]
\[ s_2 = 3977.7 \in [3948.6, 3980.3] \]
\[ s_3 = 3713.1 \notin [3948.6, 3980.3] \]

$s_2$ is reasonable, Other results are not reasonable.

(c) Given $3980.3 < s \leq 4011.1$
\[ \max \left( \left| \Delta y_1 (k + 1) \right|, \left| \Delta y_2 (k + 1) \right| \right) = \left( \frac{279.9}{4213.3 - s} \right) \times 1.0878 \times 0.85464 \]

According to equation (5), an equation can be obtained, and the solutions are:
\[ s_1 = 3982.8 \in [3980.3, 4011.1] \]
\[ s_2 = 4652.7 \notin [3980.3, 4011.1] \]

$s_1$ is reasonable. $s_2$ is not reasonable.

(d) Given $4011.1 < s < 4213.3$
\[ \max \left( \left| \Delta y_1 (k + 1) \right|, \left| \Delta y_2 (k + 1) \right| \right) = \left( \frac{279.9}{4213.3 - s} \right) \times 219.9 \times \frac{4213.3 - s}{4213.3 - s} \times 0.85464 \]

According to equation (5), an equation can be obtained, and the solutions are:
\[ s_1 = 4521.3 \notin [4011.1, 4213.3] \]
\[ s_2 = 3995.7 \notin [4011.1, 4213.3] \]

These results are not reasonable.

According to the analysis of these four situations, it can be concluded that two results are reasonable, $s = 3977.7$ and $s = 3982.8$. To integrate the two forecast results, the average of 3980.3 is adopted. i.e., the forecast of cumulative average production cost of the sixth batch is 3980.3. Therefore, the forecast of average production cost of the sixth batch is 3847.6.

(2) Learning Curve Method

The average cost of the first batch is regarded as the production cost of the first product, $C_1$. The output and cost of the second through fifth batches are inputted respectively into
\[ C = C_1 N^b \]
\[ b = \log S / \log 2 \]

The formula of the learning curve using the Wright formula is:
\[ C = 4400 \times N^{-0.0138} \]

The formula of the learning curve using the Crawford formula is:
\[ C = 4400 \times N^{-0.0207} \]

(3) Inspection, Contrast of Prediction Results

The results of the models in prediction values, actual values and relative error are as table 3:

<table>
<thead>
<tr>
<th>Prediction Method</th>
<th>Predicted Value ($)</th>
<th>Actual Value ($)</th>
<th>relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crawford Method</td>
<td>3,952.50</td>
<td>3,820.00</td>
<td>3.47</td>
</tr>
<tr>
<td>Wright Method</td>
<td>4,095.50</td>
<td>3,971.10</td>
<td>3.13</td>
</tr>
<tr>
<td>Grey Trend Relational Method</td>
<td>3,847.6</td>
<td>3,820</td>
<td>0.723</td>
</tr>
</tbody>
</table>

**Conclusion**

Given that the norm is for torpedo types to be produced in small, few batches, the resulting data for a production cost model is also low. To circumvent the low-data issue, we used the Grey trend relational theory to analyze the production cost trend of five continuous batches and predict
the production cost of the sixth batch, then validated the predicted cost against the actual value. We have contrasted this with the learning curve model, which results in greater relative error.

Compared with traditional methods, the result of the study indicates that the Grey trend relational model can predict cumulative average cost more accurately.

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