A systematic performance evaluation of indigenously developed
Shack-Hartmann wavefront sensor

Vikash Porwal\textsuperscript{a}, Awakash Dixit\textsuperscript{a,b}, Aditya Kumar Mangain\textsuperscript{a}, Sanjay Kumar Mishra\textsuperscript{a,*} & Arun Kumar Gupta\textsuperscript{a}

\textsuperscript{a}Adaptive Optics Division, Instruments Research & Development Establishment, Defence Research & Development Organisation, Dehradun 248 008, India
\textsuperscript{b}Uttarakhand Technical University, Dehradun 248 007, India

Received 5 May 2015; revised 11 May 2016; accepted 2 June 2016

Shack-Hartmann wavefront sensor (SH-WS) is the key subsystem of an adaptive optical system (AOS). The systematic performance evaluation is the prerequisite for the optimal utilization of SH-WS in AOS. This requires generation and sensing of pure (Zernike modes) and random aberrations in laboratory. The Zernike modes, tilt and defocus are generated in the laboratory through a simple experimental arrangement. Other Zernike modes astigmatism, coma, spherical aberration, and trefoil are generated through a phase-only spatial light modulator. The random wavefront errors have been generated through indoor convective turbulence. Further, the aberrated wavefronts have been sensed by SH-WS and calibrated standard methods, simultaneously. The recorded data is analyzed to estimate the performance parameters, which are discussed in terms of calibration, accuracy, precision, response of various Zernike modes, and sensing of indoor convective turbulence.

Keywords: Adaptive optics, Shack-Hartmann wavefront sensor, Phase-only Spatial light modulator, Zernike modes

1 Introduction

A Shack-Hartmann wavefront sensor (SH-WS) is the key subsystem of an adaptive optical system (AOS). The AOS compensates the atmospheric turbulence, with the help of SH-WS, multichannel deformable mirror (DM), relay optics and a wavefront controller. In AOS fast changing wavefront errors (of the order of milliseconds), caused by atmospheric turbulence, need to be sensed precisely in real time. This requirement can be accomplished by a fast and accurate SH-WS. The SH-WS is a simple, compact, robust and relatively vibration insensitive sensor, which makes wavelength-independent passive measurements, thus giving an alternative to interferometry\textsuperscript{1-4}. For moderate atmospheric turbulence the dynamic range is not an issue. In some applications, dynamic range becomes more important (up to several lambdas) over accuracy, but there is always a tradeoff between accuracy and dynamic range. The SH-WS, indigenously developed at Instruments Research and Development Establishment (IRDE), Dehradun, India\textsuperscript{5}, is shown in Fig. 1.

The accuracy of the SH-WS for measuring a wavefront error is mainly dependent upon the measuring accuracy of the centroid estimation of each focal spot\textsuperscript{6}. The main sources of errors are the photon noise, the readout noise of the detector, the background noise and the sampling. These errors were widely investigated theoretically and experimentally\textsuperscript{6-8}. The SH-WS precision and accuracy analysis\textsuperscript{4}, calibration\textsuperscript{9} for absolute wavefront measurements, and measurement of aberrations in micro-lenses by SH-WS\textsuperscript{10} were previously carried out by employing single mode fiber as point source.

The systematic performance evaluation, though a prerequisite for the optimal utilization of SH-WS, is not reported yet to the best of author’s knowledge. The current communication deals with the systematic performance evaluation by realizing different experiments and analysis of recorded data. The study also attempts to consolidate the earlier studies and techniques used elsewhere. The calibration of SH-WS in terms of linearity is the very first step before using it in AOS. Accuracy reflects the ability of the sensor to measure a given known wavefront. Precision signifies the repeatability of the sensor, which are the variations in the centroid data of unchanged incident wavefront. The efficacy of sensor can be thought of in terms of the response of various Zernike modes (modal reconstruction), and the sensing of random wavefront errors. Thus,
it is very important to determine the accuracy, precision and efficacy of the sensor so as to know the performance parameters.

Zernike polynomials are a complete set of orthogonal functions that represent balanced aberrations over the circular pupil. An advantage of using orthogonal set of functions is that the aberration coefficients are uniquely specified regardless of truncation, although some errors occur due to series truncation. Thus, the random wavefront errors induced by atmospheric turbulence can be expressed as the linear combinations of various Zernike modes.

The estimation of performance parameters necessitates the generation and sensing of quantified Zernike modes (pure aberrations) in the laboratory. The requirement is met by producing the axial aberrations, tilt and defocus in the laboratory with known resolutions by a simple experimental arrangement. Further, they are sensed by SH-WS and a calibrated mechanism simultaneously. Zernike modes especially after defocus are very difficult to generate in predetermined and repeatable manner by simple experimental arrangement. However, it is reported that phase only Liquid-crystal (LC) spatial light modulators (SLMs) can be used for generating the Zernike modes in quantified manner. So the phase-only SLM is used to generate Zernike modes (astigmatism, coma, spherical aberration, and trefoil) with a large dynamic range, in steps of known resolution. The Zernike modes are sensed by SH-WS and the far-field diffraction patterns are recorded, simultaneously. The random wavefront errors are also generated in the laboratory through indoor convective turbulence by placing multiple heaters in the beam propagation path; further these errors are sensed by the SH-WS. Finally, the recorded data is analyzed to estimate the performance parameters, which are discussed in terms of calibration, accuracy, precision, response of various Zernike modes, and analysis of random wavefront errors.

### 2 Shack-Hartmann Wavefront Sensor and Spatial Light Modulator

A SH-WS optically samples the beam by means of a lenslet array to produce a spot pattern at detector plane. The displaced spots (aberrated wavefront) with respect to reference spots (plane wavefront), provide the values of local tilts by calculating the slopes. A numerical integration of the slope information allows the wavefront reconstruction, which generally involves modal/or zonal approaches. In modal approach slope errors are fitted into a complete set of Zernike modes; this approach has a distinct advantage in providing phase quantization of wavefront error in terms of Zernike aberrations. The zonal method estimates a phase value in a local zone (different geometries such as Fried, Hudgin, Southwell), while the modal method is based on a coefficient of aberrations of the aperture function. The specifications of SH-WS are presented in Table 1. The components used to develop the indigenous SH-WS are lenslet arrays (144 μm pitch, 8.2 mm focal length, 16x16 square lenses, λ/8 surface flatness, fused silica), CMOS sensor (Monochrome, 10.6 μm pixel size), CamLink output with windowed resolution of

<table>
<thead>
<tr>
<th>SH-WS specifications</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectral range</td>
<td>VIS</td>
</tr>
<tr>
<td>Sampling</td>
<td>16 x 16</td>
</tr>
<tr>
<td>Spatial resolution</td>
<td>144 μm</td>
</tr>
<tr>
<td>Tilt accuracy</td>
<td>1.78 µrad</td>
</tr>
<tr>
<td>Defocus accuracy</td>
<td>0.138 µm</td>
</tr>
<tr>
<td>Tilt range</td>
<td>± 4 mrad</td>
</tr>
<tr>
<td>Defocus range</td>
<td>1.78 m – ∞</td>
</tr>
<tr>
<td>Fastness</td>
<td>&gt; 1000 fps</td>
</tr>
<tr>
<td>Zernike terms</td>
<td>10 terms</td>
</tr>
</tbody>
</table>

![Sensor optical head](image-url)
256 × 256 @ 2200 frame/second and fast FPGA based framegrabber (compatible with camera frequency 85MHz clock rate and continuous data transfer over 200 MB/s).

Spatial light modulator (SLM) is the transducer that modulates incident light by optical and electronic addressing. The light can be modulated in terms of phase, intensity or the polarization. SLMs are being used for various applications such as astronomical instrumentation, ophthalmology, apodization, focus tracking, optical image processing etc. The SLM under use is a phase only reflective micro-display (1920×1080 resolution, pixel size 8 µm, and 8-bit addressable phase levels). The device is optimized for phase shift 2π in visible spectrum with 87% fill factor. Initial aberrations, larger than 2λ are common for a given SLM and this would degrade the performance of optical setup for which it is being employed. So a good and complete calibration of SLM is an essential step, particularly if SLM is to be used in applications that require precise wavefront control. For this two important calibration steps must be performed; first, the mapping of optical phase to the command values chosen, and second, the measurement and compensation of the initial aberrations of SLM.

3 Methodology

The source (He-Ne laser 633nm) is arranged at the point of best collimation and no tilt in the wavefront, for recording of centroid data by SH-WS. The size of the focal spot is kept moderate, as accuracy will be reduced both in the case of too small and too large a focal spot. At this position, the data of 1000 frames are averaged to construct the reference centroid data, so as to minimize the effects of lenslet fabrication errors and to reduce the effects of air turbulence. Sensing of aberrations by SH-WS include; firstly, the recording of test beam wavefront in the optical setup without any introduced aberration, secondly, different strengths of aberrations are introduced in the test beam to record the wavefronts, finally these wavefronts are subtracted with that of the zero aberration test beam in order get the introduced aberrations. Subsequently, the data is processed to estimate and analyze the performance parameters, all analysis being performed at 20% intensity threshold. Methodology for the generation and sensing of pure aberrations (Zernike modes) and random aberrations is discussed in subsequent subsections.

3.1 Tilt and defocus (Z1, Z2 and Z3)

A simple experimental arrangement (Fig. 2) is realized to generate known axial aberrations tilt and defocus and their sensing. The scheme employs a pair of lenses L1 (f = 380 mm) and L2 (f = 75 mm) mounted on translational stages in collimated laser beam. The transverse motion of lens L1 generates the tilt in the wavefront, while the longitudinal motion of either lens L1 or L2 imparts the defocus in wavefront. Generated tilts (in steps of 20µm translation of L1) are simultaneously measured by SH-WS and a standard centroid measuring system to calibrate SH-WS and know its accuracy and precision. For defocus, spot diameters (in steps of 1mm) are measured in the plane of SH-WS on a screen, subsequently sagor stroke (S) and corresponding wavefront curvature (R) are calculated from the spot size at screen and the geometry of experiment (Fig. 3) by using the given equations:

\[
S = \frac{D^2}{8 \left( f^2 - (x - f) \right)} \quad \text{and} \quad R = \frac{S}{2} + \frac{D^2}{8S}
\]

\[
\cdot (1)
\]
where, \( D \) is the diameter of the beam spot, \( f \) is the focal length of the lens, \( d \) is the applied shift, \( x \) is the distance between lens (\( L2 \)) and the screen.

3.2 Zernike modes (\( Z4 \) to \( Z10 \))

An experimental setup (Fig. 4) is realized, that mainly consists of a phase-only SLM for the generation of Zernike modes, SH-WS for their sensing, and CCD for the recording of PSFs. The SLM is illuminated by a plane wavefront (4 mm beam diameter through aperture A1) polarized along the long axis of SLM display. SLM is kept at a small tilt (less that 6°) with the incident wavefront, so as to ensure effective phase modulation and diffraction efficiency21. Additionally, small tilt also avoids the use of a beam splitter and the associated energy loss. A blazed grating is encoded with the aberration functions, through a MATLAB routine, to see the effects of aberrations in 1st order. Aberration phases are displayed on the phase-only SLM in a quantified manner (\( -0.25\lambda \): 0.05\( \lambda \): +0.25\( \lambda \)), through a cloned computer monitor interface. The range of aberration strength and the resolution (step size) is adequate for AOS designed for moderate atmospheric turbulence, although the SLM can produce the higher aberration strengths too. Zernike modes are generated, when wavefronts get reflected from the SLM. Zernike modes include; astigmatism sine (\( Z4 \)), astigmatism cosine (\( Z5 \)), coma sine (\( Z6 \)), coma cosine (\( Z7 \)), trefoil sine (\( Z8 \)), trefoil cosine (\( Z9 \)), and spherical (\( Z10 \)) terms. The Zernike modes, through an aperture A2 in the 1st order, are sensed by SH-WS and the far-field diffraction pattern of aberrated wavefronts are recorded by the CCD with a Fourier transform lens \( L2 \) (\( f=380 \) mm). Finally, the responses of various Zernike modes are estimated through sensed Zernike coefficients.

The SLM used in the experiment is characterized for its inherent aberration by a phase-shifted Fizeau interferometer. The corresponding interferogram is shown in Fig. 5. The initial aberrations are measured on 512x512 pixels (4 mm diameter, 8 \( \mu \)m pixel size) as 194 nm (RMS), and 800 nm (PV). Corresponding Zernike coefficients for the dominating defocus is found to be 335 nm, while other aberrations are less than 63 nm22.

Fig. 3 – Defocus formulation

Fig. 4 – Experimental schematic for lower order Zernike mode generation and detection

Fig. 5 – SLM initial surface
3.3 Random wavefront errors

An experimental setup is realized for generating the indoor convective turbulence (random wavefront errors) in laboratory (Fig. 6). A collimated laser beam (633 nm) of 50 mm diameter, after being passed over a set of heaters, is reflected from the two mirrors (M1, M2). The beam later passes through a 20X beam reducer, which resizes the beam to 2.5 mm, so as to match the SH-WS aperture. Further, the beam is split in two parts; one for SH-WS, and another for the CCD through a focusing lens. Centroid shift data of sampled wavefronts are recorded with SH-WS for the cases of ambient condition, heaters on (random wavefront errors), and heaters off. Finally, the data is analyzed to estimate the mean and the RMS pixel movements.

4 Results and Discussion

The results and analysis performed over the recorded data to estimate various performance parameters are discussed in the subsequent subsections for tilt and defocus, Zernike modes (Z4 to Z10) and random wavefront errors.

4.1 Tilt and defocus (Z1, Z2 and Z3)

A one-to-one correspondence has shown that the SH-WS possesses a high degree of linearity as shown in Fig. 7 (a). The SH-WS is calibrated in the linear range, with a tilt measuring error of 1.1%. The tilt measuring accuracy is found to be 1.78 µrad (~λ/43). The variations in the measured tilts can be attributed to finite pixelization, cross talk, background light etc, thresholding also affects.
the accuracy. Sensor precision is found to be 0.78 µrad (≈λ/99) as shown in Fig. 7(b). Finite precision comes about as a result of detector SNR, CMOS readout noise, synch jitter etc. Dynamic range for measuring the defocus is from ∞ to 1.7 m (Radius of curvature) as shown in Fig. 7(c), and the sag estimation error is found to be 0.1376 µm (≈λ/4.6), within the dynamic range as shown in Fig. 7(d). In order to distinguish the sag measurements between λ/4.6 and λ/4.7, it is necessary to calculate the errors up to the third decimal place, at least.

4.2 Zernike modes (Z4 to Z10)

Pure Zernike modes (Z4 to Z10) are generated by SLM for modal strengths (−0.25λ: 0.05λ: +0.25λ). Modal strengths are the magnitude of Zernike modes; they are the root mean square (RMS) values. Efficacy of Zernike modes is established by comparing the PSFs of SLM generated Zernike modes (Z4 to Z10) and the simulated PSFs on MATLAB platform, for various strengths. A clear qualitative agreement is achieved in comparison. A typical result for modal strength of +0.25λ is given in Table 2. The generated Zernike modes are sensed by the SH-WS, the data fit shows that the sensor behaves almost linearly in the given range; the results are shown in Figs 8-11. The modal strengths are sensed with fairly good accuracy, the mean accuracy of SH-WS (for Z4 to Z10) is found to be 0.0245 (≈λ/41). The variations can be attributed to the fact that possibly the SLM aberrations (excluding defocus) interfere with the step size of generated modal strength. The results of the sine terms seem to be better sensed than cosine terms. Possibly this may be due to the inherent aberrations of experimental setup and the SLM.

4.3 Random wavefront errors

The indoor convective turbulence (random wavefront errors) generated in the laboratory is sensed by SH-WS. The shift data for each frame is used to calculate the shift rms error for that frame, such errors for 250 frames are calculated and plotted in Fig. 12. The plot shows that the mean (amplitude) and the variance (randomness) of shift rms error are increased from 0.1398 to 0.3096 and 0.045 to 0.112, respectively. The increases in the values are because of the change in the turbulence conditions from ambient to random wavefront errors (stimulated by switching on heaters). Further, the values decrease to 0.1332 and 0.036 (all values in pixels) when heaters are switched off and sufficient time is given to settle the convective turbulence; the values are very close to that of ambient conditions. The results are presented in Table 3.

Table 2 – The simulation and experimental PSFs for generated Zernike modes Z4 to Z10

<table>
<thead>
<tr>
<th>Zernike mode</th>
<th>Phase on SLM</th>
<th>PSF (Sim)</th>
<th>PSF (Exp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z4</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>Z5</td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td>Z6</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
<tr>
<td>Z7</td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
<tr>
<td>Z8</td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td><img src="image15.png" alt="Image" /></td>
</tr>
<tr>
<td>Z9</td>
<td><img src="image16.png" alt="Image" /></td>
<td><img src="image17.png" alt="Image" /></td>
<td><img src="image18.png" alt="Image" /></td>
</tr>
<tr>
<td>Z10</td>
<td><img src="image19.png" alt="Image" /></td>
<td><img src="image20.png" alt="Image" /></td>
<td><img src="image21.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Fig. 8 – Astigmatism aberration detected: (a) Z4 and (b) Z5

Fig. 9 – Coma aberration detected (a) Z6 and (b) Z7

Fig. 10 – Trefoil aberration detected (a) Z8 and (b) Z9
5 Conclusions

The Zernike modes tilt and defocus are generated in the laboratory through a simple experimental arrangement. The SH-WS possesses a high degree of linearity for tilt measurements and is calibrated with a slope error of 1.1%. The accuracy and precision of SH-WS are found to be $\lambda/43$ and $\lambda/99$ µrad, respectively. Dynamic range for measuring the defocus is found to be from $\infty$ to 1.7 m (radius of curvature). The sag estimation error is found to be 0.1376 µm within the dynamic range. The Zernike modes astigmatism, coma, spherical aberration, and trefoil are generated through a phase-only SLM. Further, the modes are sensed, and their phase maps and point spread functions (PSFs) are recorded by the SH-WS and a CCD camera, respectively. The Zernike modes (Z4 to Z10) are sensed with fairly good accuracy, of the order of $\lambda/41$ and the SH-WS behaves almost linearly in the given range ($-0.25\lambda; 0.05\lambda; +0.25\lambda$). The random wavefront errors are generated through indoor convective turbulence and subsequently sensed by the SH-WS. A qualitative sensing for different strengths of indoor convective turbulence (random wavefront errors) is also achieved. The results presented in the communication correspond to the indigenously developed SH-WS for AOS, however, the procedures discussed can be applied to any SH-WS.

Acknowledgement

The authors wish to acknowledge, Dr. S S Negi, Director, IRDE, for permitting to publish the research work, Mr Durga Singh and Dr Ajay Kumar, IRDE, for encouragement, and members of Adaptive Optics Division for cooperation. In addition, authors are also thankful to DRDO–India for awarding the research fellowship to Mr Awakash Dixit.

References