Position and Velocity control of Remotely Operated Underwater Vehicle using Model Predictive Control

M.P.R. Prasad & Akhilesh Swarup,
Department of Electrical Engineering, National Institute of Technology Kurukshetra, Haryana, India
[ Email Id: mprprasad@nitkkr.ac.in : aswarup@nitkkr.ac.in ]

Received 30 October 2015; revised 05 December 2015

Present paper consists of the attempt to apply of model predictive control on the position and velocity control of Remotely Operated Underwater Vehicle (ROUV). Linear Quadratic Regulator (LQR) method has been applied to stabilize the ROV model. Further Model Predictive Control has been applied to improve the position and velocity trajectories of the ROV. Simulation results carried out on ROUV shows that good performance and stability are achieved by the MPC algorithm, whereas sliding mode control lose its stability when ocean currents are high.

[Keywords: Model Predictive Control, LQR, ROUV, control]

Introduction

Ecosystem is by definition, the relationship between living organisms, particularly human beings and their environment—that is external surroundings such as oceans, seas, rivers, the hills, mountains and the fields which give us food and also help us in transporting merchandise of all these, let us look at the oceans in the context of underwater robots. The underwater vehicles play important roles in a number of commercial, scientific and military tasks. ROVs are useful in many ways. They help us in avoiding human labour because they are highly automated and reliable [5]. The offshore oil and gas industry depends mostly on ROV’s for inspection, installation and servicing of platform, pipelines and facilities of subsea production [9].

Underwater Vehicles are broadly classified into three types: a) Human operated b) Operated by remote control c) Automatically-operating of these ROV’s is a type of underwater unmanned vehicle and is less expensive and also capable of doing operations that are too dangerous or even impossible for human beings [7].

ROVs are technically advanced underwater vehicles. The vehicle operator remains in comfortable zone while the ROV works in critical and dangerous environments. ROVs are available in various shapes and sizes. The ideal applications of ROVs are within the onshore, offshore and inshore environments [4].

The use of ROVs in ports and defence are increasing rapidly so the dynamic position and velocity control becomes an important topic in research. In this paper Model Predictive Control has been applied to ROV for control of position and velocity trajectories of ROV [19-20]. Underwater Vehicle Dynamics are multivariable, non-linear and strongly coupled due to many factors such as hydrostatic and hydrodynamic forces acting on the vehicle. An advanced control techniques must be used in ROV for stable cruising, maintaining altitude or depth, maintaining direction and so on. [3,16].

The mathematical model of a vehicle dynamics is very important for design of the good performance controller. The dynamic model of a vehicle in six
degrees of freedom is represented by two coordinate frames (body and earth fixed frame) as indicated in fig.1. The position and velocity coordinates are given by equations 1 & 2 respectively

\[
\eta = [x, y, z, \phi, \theta, \psi]^T \tag{1}
\]

\[
\nu = [u, v, w, p, q, r]^T \tag{2}
\]

The linear, angular positions and velocity vectors are given by equation 3. These are sub-vectors of the generalized coordinates.

\[
p = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad v = \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \theta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, \quad w = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{3}
\]

where \( p \in \mathbb{R}^3 \times 1 \) is the linear position, \( v \in \mathbb{R}^3 \times 1 \) is the linear velocity, \( \theta \in \mathbb{R}^3 \times 1 \) is the angular position and \( w \in \mathbb{R}^3 \times 1 \) is the angular velocity.

The force and moment vector with elements corresponding to the degrees of freedoms are given by equation 4.

\[
\tau = [Fx, Fy, Fz, Mx, My, Mz]^T \tag{4}
\]

Here ‘\( \tau \)’ is the vector of forces and moments acting on the vehicle and ‘\( \nu \)’ is the vector of linear and angular velocities with respect to body fixed frame and ‘\( \eta \)’ is the vector of position and orientation of vehicle with respect to the earth fixed frame [10].

**Notations Used for Underwater Vehicles**

The notations used in this paper are adopted from the SNAME convention and Fossen’s Robotic-like vectorial model listed in table-I. Motion, position and orientation of a vehicle are given by vectors and generalized coordinates [17].

**Table-I: Notations used in Underwater Vehicles [17]**

<table>
<thead>
<tr>
<th>Degrees of Freedom</th>
<th>Surge</th>
<th>Sway</th>
<th>Heave</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forces and Moments</td>
<td>X</td>
<td>Y</td>
<td>Z</td>
<td>K</td>
<td>M</td>
<td>N</td>
</tr>
<tr>
<td>Linear and Angular Velocity</td>
<td>u</td>
<td>v</td>
<td>w</td>
<td>p</td>
<td>q</td>
<td>r</td>
</tr>
<tr>
<td>Position and Attitude</td>
<td>x</td>
<td>y</td>
<td>z</td>
<td>( \phi )</td>
<td>( \theta )</td>
<td>( \psi )</td>
</tr>
</tbody>
</table>

In literature MPC has applied to either dynamic or kinematic control of Underwater Vehicles whereas here both the dynamic and kinematic equations are considered for simulating the model. This paper is organised as follows Section-II presents a state space model of the ROUV and Linear Quadratic Regulator (LQR) method has been developed for stabilization of states for ROUV in section-III. Section-IV describes about Model Predictive Control design. The simulation details and results have been developed in Section V. The section VI highlights the conclusion of the paper.

**Materials and Methods**

### II Modelling of a ROV

The equations of the motion developed for the ROV are written in a vectorial form [5]. The generalized coordinates and matrices to describe the six degrees of freedom differential equations of motion are seen in [6].The control modeling of a ROV system is generally classified into two groups

1) Kinematic Model
2) Dynamic Model

The kinematic model provides the mathematics of motion of a ROV without considering the forces responsible for the motion. On the other hand, the dynamic model takes into account the forces/torques causing the ROV to move [4, 6].

**Kinematic Model:**

In a kinematic model, the control inputs to the system are generally linear and angular velocity variables. The kinematic model of a ROV system is given by the following equation

\[
\dot{\eta} = J(\eta) \cdot \nu \Leftrightarrow \begin{bmatrix} \dot{\eta}^1 \\ \dot{\eta}^2 \end{bmatrix} = \begin{bmatrix} J_1(\eta_2) & 0 \\ 0 & J_2(\eta_2) \end{bmatrix} \begin{bmatrix} \eta^1 \\ \eta^2 \end{bmatrix} \tag{5}
\]

Where, \( J(\eta) \) is the kinematic transformation matrix.

\[
J_1(\eta_2) = 
\begin{bmatrix}
S(\phi) \cdot C(\theta) & C(\phi) \cdot S(\theta) & C(\phi) \cdot C(\theta) & S(\phi) \\
S(\phi) \cdot S(\theta) & C(\phi) \cdot C(\theta) & -S(\phi) \\
C(\phi) \cdot S(\theta) & C(\phi) \cdot C(\theta) & S(\phi) \cdot C(\theta) & S(\phi) \\
S(\phi) & C(\phi) & 0 & 0
\end{bmatrix}
\]

\[
J_2(\eta_2) = 
\begin{bmatrix}
1 & S(\phi) \cdot t(\theta) & C(\phi) \cdot t(\theta) & 0 \\
0 & C(\phi) & -S(\phi) & 0 \\
0 & S(\phi) / C(\phi) & C(\phi) / C(\theta) & 0
\end{bmatrix}
\]

Where, \( s(\cdot) = \sin(\cdot) \), \( c(\cdot) = \cos(\cdot) \) and \( t(\cdot) = \tan(\cdot) \)

**Dynamic Model:**

In a dynamic model, the control inputs to the system are generally force and torque
variables. The dynamics of a nonholonomic system is given by the following equation

\[ M\ddot{\mathbf{u}} + C(\mathbf{u})\dot{\mathbf{u}} + D(\dot{\mathbf{u}}) + g(\mathbf{q}) = \tau \]  

(6)

where \( J \in \mathbb{R}^{6 \times 6} \) is the rotation matrix from the vehicle frame to reference frame. \( M \in \mathbb{R}^{6 \times 6} \) is the mass matrix, \( C \in \mathbb{R}^{6 \times 6} \) is the Coriolis and centripetal force matrix and \( D \in \mathbb{R}^{6 \times 6} \) is the damping matrix, \( g \in \mathbb{R}^{6 \times 1} \) is a vector with restoring forces. Equation 5 is Newton’s second law expressed in a moving coordinate frame, hence the need to compensate for Coriolis and centripetal forces.

Defining \( x_1 = \Delta \mathbf{u} \) and \( x_2 = \Delta \mathbf{q} \), yields the following linear time varying model [7]

\[ \dot{x}_1 + C(t)x_1 + D(t)x_1 + Gx_2 = \tau \]  

(7)

\[ \dot{x}_2 = Jx_1 \]  

(8)

From the above equation we can write the state model as

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
-M^{-1}[C + D] & -M^{-1}G \\
J & 0_{6 \times 6}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
-M^{-1}\tau \\
0_{6 \times 6}
\end{bmatrix} u
\]

(9)

which can be written in state equation form as

\[ \dot{x} = Ax + Bu \]  

(10)

III Stabilization of vehicle states using Linear Quadratic Regulator (LQR)

LQR is an optimal control strategy which has been widely developed and used in various applications. LQR design is based on the selection of feedback gain \( K \) such that the cost function \( J \) is minimized. This ensures that the gain selection is optimal for the cost function specified [18].

The state model of the system is considered as

\[ \dot{x} = Ax + Bu \]

\[ y = Cx + Du \]

where \( A \) is system matrix, \( B \) is input matrix, \( C \) is output matrix and \( D \) is direct transmission matrix between input and output.

The performance index is defined [8] as

\[ J = \frac{1}{2} \int_0^\infty (x^TQx + u^TRu)dt \]  

(11)

where \( Q \) is positive semi definite Hermitian matrix, and \( R \) is the input weight matrix and \( I \) is the identity matrix.

The feedback control is a linear function of state as

\[ u = -Kx \]

\( K \) is given by

\[ K = R^{-1}B^TP \]

where, \( P \) can be found by solving the continuous time algebraic Riccati equation [21]

\[ PA + A^TP - PB^{-1}B^TP + Q = 0 \]  

(12)

An advantage of using the quadratic optimal control scheme is that the system designed will be stable and robust.

Simulations have been carried out in MATLAB for stabilization of all the states of ROUV where model is taken from [1]. The LQR has been applied for this system. Fig.2 presents the position and velocity states with respect to body fixed frame and vehicle fixed frame. It is evident from the responses that all the responses of the states are stabilized and reach steady state quickly.

IV Model Predictive Controller

Model predictive control (MPC) is an important model based control strategy devised for large multi input and multi output control problems with inequality constraints on the inputs and or outputs. The models generally used in MPC are intended to represent the behaviour of complex dynamical systems. This control has already been in the industry for more than 15 years serving as an effective means to deal with multivariable constrained control problem [2, 19].

MPC offers several advantages compare to other control systems. One such advantage is that the ROUV captures the dynamic and static interactions between input, output and disturbance variables. Another advantage is constraints on inputs and outputs are considered in a systematic manner. Control calculation can also be coordinated with the calculation of optimum set point. An
accurate model prediction by this control system can provide early warning of potential problems [12].

MPC refers to a class of algorithms that compute a sequence of manipulated variable adjustments in order to optimise the future behaviour of a ROUV system [11].

The use of MPC in this application is due to its ability to handle multiple degrees of freedom using a model of the system with constraints, where the control algorithm calculates the optimal control inputs over a finite number of time steps in order to reduce the error. The constraints are specified within the controller, thus enabling the optimal control to be calculated within the stated constraints.

Following steps are required to design and implement the model predictive controller will be described. First the continuous-time state space model is sampled to a result in a discrete state space model with state, input and output matrices denoted by $A_d$, $B_d$ and $C_d$ respectively. The MPC algorithm embeds an integrator to deal with model inaccuracies and ensures zero steady state error for set point following. This is done by replacing the system input vectors $u$ and $\Delta u$, constructed by taking the difference between the current and previous state and input vectors and likewise for the state vector, that is, in terms of $\Delta u$ and $\Delta x$.

$$\Delta x(k) = x(k) - x(k-1)$$
$$\Delta u(k) = u(k) - u(k-1)$$

where $k$ denotes the sampling instant. The state space model used for MPC design is

$$\begin{bmatrix} \Delta x(k+1) \\ y(k+1) \end{bmatrix} = A \begin{bmatrix} \Delta x(k) \\ y(k) \end{bmatrix} + B \Delta u(k)$$

$$y(k) = C \begin{bmatrix} \Delta x(k) \\ y(k) \end{bmatrix}$$

where

$$A = \begin{bmatrix} A_d & 0 \\ G_d & 1 \end{bmatrix}; B = \begin{bmatrix} B_d \\ C_d B_d \end{bmatrix}; C = [0 \\ I]$$

and for the remainder of this paper the null and identity matrices with compatible dimensions are denoted by 0 and I respectively.

In this model it is $\Delta u(k)$ that is optimized by the predictive control algorithm. In the steady state, all entries in $\Delta x_n(k)$ are zero and steady state values
of the output vector \( y(k) \) will be taken as the set-point signals. Therefore, with the inclusion of an integrator in the predictive controller algorithm, the steady state values are not required, leading to simplification at the implementation stage [19].

The Concept of moving horizon is shown in fig.4. The predictive controller is designed using the receding horizon control principle, where future state vector is calculated for a prediction horizon of \( N_p \) samples for a future control trajectory of \( N_c \) samples where \( N_c \leq N_p \). The prediction will be denoted as starting from sample number \( K_i > 0 \).

Let \( N_c \) and \( N_p \) denote the control and prediction horizons, respectively, where \( N_c \leq N_p \). Also introducing

\[
U = [\Delta u^T(k) \Delta u^T(k+1) \ldots \Delta u^T(k+Nc-1)]^T
\]

\[
x(k) = [x^T(k+1)k] \ x^T(k+2)k \ldots x^T(k+Npk)]^T
\]

Then the state space model can be used to recursively compute the future state vectors and output vectors, or in a more compact form,

\[
Y = FX(k) + \Phi U
\]  \hspace{1cm} (17)

where

\[
F = \begin{bmatrix} CA & 0 & 0 & \ldots & 0 \\
CA & CB & CB & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
CA^{Nc-1} & CA^{Nc-2} & \ldots & \ldots & \ldots \end{bmatrix}
\]

\[
\Phi = \begin{bmatrix} I & I & \ldots & I \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots \\
\vdots & \vdots & \ddots & \ddots \end{bmatrix}
\]

Introducing the set-point or reference vector of length \( N_p \) as

\[
R_k^T = [I \ I \ \ldots \ I]r(k)
\]

where \( r(k) \) is the reference vector at sample instant \( k \) and \( I \) is the identity matrix. The cost function for MPC design is

\[
J = (R_s - Y^T) (R_s - Y) + U^T R U
\]  \hspace{1cm} (18)

where \( R \) is a symmetric positive definite matrix to be selected.

Routine analysis now gives the minimizing control in the absence of constraints as [14]

\[
U = (\phi^T \phi + R)^{-1} (\phi^T R_s - \phi^T F x(k))
\]  \hspace{1cm} (19)

where the required matrix inverse is assumed to exist.

Using receding horizon control, only the entries in \( \Delta U \) corresponding to \( \Delta u(k) \) are used and the actual control signal applied to the plant is computed using

\[
u(k) = u(k-1) + \Delta u(k)
\]

where, both the current optimal control \( \Delta u(k) \) and the past value \( u(k-1) \) are used. Since the current and past control signals have the same steady state value, if the first sample of the control signal is taken as the actual plant input signal before the closed loop controller is in operation, the computation of the control signal using above equation leads the actual control signal for direct implementation. Hence the control signal has included its steady state value, which is also part of the simplification in the implementation of the predictive controller [14, 15].

Magnitude and rate constraints on the control signals can be achieved by minimizing the cost function \( J \) of eq.(18) in real-time with constraints imposed.

Selection of Design and Tuning parameters

A number of design parameter must be specified in order to design the MPC system. The following are some of the key design issues and recommended values for the parameters.

Sampling Period \( \Delta t \) and model horizon \( N \):

Sampling period and model horizon should be chosen so that \( N \Delta t = t_s \) where \( t_s \) is the settling time of the response. This choice ensures that the model reflects the full effect of a change in an input variable over the time required to reach steady state. If the output variable responds on different time scales, different values of \( N \) can be used for each output. Also, different model horizons can be used for the inputs and disturbances [11].

Control M and Prediction P horizons:

As control horizon \( M \) increases, the MPC tends to become more aggressive and the required computational effect increases. However, computational effort can be reduced by input blocking. The prediction horizon \( P \) is often selected to be \( P = N + M \) so that the full effect of last input move is taken into account. Decreasing value of \( P \) tends to make controller more aggressive. A different value \( P \) can be selected for each output if their settling times are quite different. An infinite prediction horizon can also be used and has significant advantages [15].

Weighting Matrices \( Q \) and \( R \)
The output weighting matrix $Q$ allows the output variables to be weighted according to their real values. Thus $mP \times mP$ diagonal $Q$ matrix allows the output variables to be weighted individually, with the most important variables having the largest weights. For example, if the Yaw angle is more important than pitch, then the Yaw angle will be assigned a larger weighting factor [2].

Similarly, $R$ in equation (19) allows input variables to be weighted according to their relative importance. This $rM \times rM$ matrix is referred to as the input weighting matrix or the move suppression matrix [11].

**Reference trajectory $\alpha_t$**

In MPC applications, the desired future output behaviour can be specified in several different ways as a set point, high or low limits, a reference trajectory or a funnel. Both the reference trajectory and funnel approaches have a tuning factor that can be used to adjust the desired speed of response for each output. As $\alpha_t$ increases from zero to one, the desired reference trajectory becomes slower. Alternately, the performance ratio concept can be used to specify reference trajectories. The performance ratio is defined to be the ratio of desired closed loop settling time to the open loop settling time [19].

**V Results & Discussion**

The model of ROUV and system parameters are taken from [1] for implementing the proposed MPC. The simulation results are obtained and presented here in fig.5. Fig.5 shows the trajectories of position and velocity of ROUV.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction Horizon P</td>
<td>10</td>
</tr>
<tr>
<td>Control Horizon M</td>
<td>1</td>
</tr>
<tr>
<td>Q</td>
<td>1</td>
</tr>
<tr>
<td>R</td>
<td>0.1</td>
</tr>
<tr>
<td>Sampling Period $\Delta t$</td>
<td>0.2</td>
</tr>
<tr>
<td>Model Horizon</td>
<td>1</td>
</tr>
</tbody>
</table>

![Fig.5 Position and Velocity trajectories of Kaxan Underwater Robot](image-url)
From the results it is obvious that the position and velocity trajectories are following their set points (step input) tracking in the presence of ocean currents (constant disturbances). All the trajectories are having good transient and steady state response. The MATLAB Simulink diagram of Remotely Operated Underwater Vehicle is used in the simulation shown in fig.6. It represents the dynamics and kinematics of the ROUV. Controller Parameters used in simulation are listed in Table-II.

VI Conclusions
In this paper, the motion control problem for remotely operated underwater vehicle “Kaxan” developed at CIDESI, Mexico [1] has been considered. Remotely Operated Underwater Vehicle model is unstable and therefore LQR method is applied to stabilize the vehicle states. Further MPC method is applied to follow the reference trajectories of position and velocity control of ROUV in the presence of constant disturbances (ocean currents). The presented MPC algorithm is based on a state space model of the plant and is therefore flexible to be used for MIMO systems. Finally, a simulation result demonstrates the effectiveness of the proposed control method. Further the performance can be improved by considering the higher level of MPC.

Appendix
The dynamic model of ROV is described by the following state equation
\[ x = A x + B u \]

Where \( A = 12 \times 12 \) matrix

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2.366 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.823 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.192 & -0.176 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.1 & -0.085 & -0.336 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.06 & 0.055 & -0.039 & -0.28 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.02 & 0.016 & -0.018 & -0.16 & -0.034 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0.01 & 0 & 0 & 0.085 & -0.085 & -0.176 & -0.012 & 0 & 1 & 0 & 0 & 0 \\
0.01 & 0 & 0 & 0 & 0 & 0 & 0.01 & -0.085 & -0.176 & -0.012 & 0 & 0 \\
0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & -0.085 & -0.176 \\
0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & -0.085 \\
0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01
\end{bmatrix}
\]

\[ B = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^{T} \]

References


