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Free vibration analysis of elastically supported Timoshenko columns with attached masses using fuzzy neural network

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This study presents elastically supported Timoshenko column with attached masses for free vibration analysis using fuzzy neural network. Neuro fuzzy frequency estimation (NFEE) models were developed to compute vibration frequencies using fuzzy logic toolbox of software Matlab 7.0. Gaussian membership functions and adaptive neuro fuzzy inference system (ANFIS) were used in NFFE model. Hybrid learning rule was applied for quantifying output variables in NFFE model. Frequency values of column with 1, 5 and 10 attached masses were computed. Training sets for NFFE models used transfer matrix method (TMM). During testing of NFFE model, good agreement was observed with results obtained using TMM as reduction in computation effort.

Keywords: Elastic support, Free vibration, Neuro-Fuzzy, Timoshenko model, TMM

Introduction

Timoshenko columns are columns of multistory frames with attached masses of floors at different heights from base support. Support is modeled by elastic rotational spring with end fixity factor in order to reflect relative stiffness of column. Governing equation of free vibration includes effects of bending and shear deformation with rotary inertia of columns. Eigen frequencies are determined by transfer matrix method (TMM) using separation of variable method[1]. Studies[2-11] are available on vibration of beam-columns with attached masses using conventional methods; most of these studies require much computing effort and time.

In fuzzy modeling, determination of best fitting boundaries of membership functions and number of rules, which are generally determined by trial-and-error approaches, are very difficult. Therefore, employing neural networks[12-17], a new fuzzy neural, neuro-fuzzy or adaptive network based system has been developed. Key properties of neuro-fuzzy systems are accurate learning and adaptive capabilities of neural networks, together with generalization and fast-learning capabilities of fuzzy logic systems. Adaptive network based fuzzy inference system[12] (ANFIS) has been used in fuzzy logic toolbox of Matlab software.

In present study, neuro-fuzzy algorithm is used to obtain frequencies of model by establishing neuro fuzzy frequency estimation (NFFE) models. Nondimensionalized attached mass and its rotary inertia values, and fixity factors are inputs and natural frequencies are outputs for neuro-fuzzy algorithm.

Neuro-Fuzzy Modeling

Mathematical model[10] of n uniform Timoshenko columns with n attached masses (Fig. 1) has been used in this study for multistory frames. First order Sugeno model with following rules is taken into account:

Rule 1: If (x is A_1) and (y is B_1) then ($f_1 = p_1 x + q_1 y + r_1$)
Rule 2: If (x is A_2) and (y is B_2) then ($f_2 = p_2 x + q_2 y + r_2$)

where x and y are inputs; A_i and B_i are fuzzy sets; f_i are outputs within fuzzy region specified by fuzzy rule, p_i; q_i and r_i are design variables that are ascertained during training process. ANFIS architecture (Fig. 2) that implements these two rules shows a circle indicating a fixed node, whereas a square indicates an adaptive node.

In first layer, all nodes are adaptive. Outputs of layer 1 are fuzzy membership grade of inputs and given as

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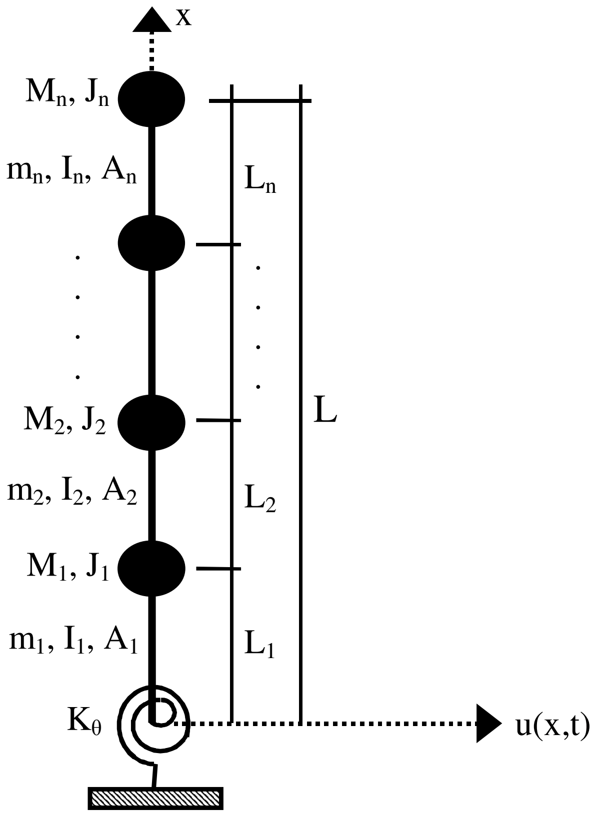

Fig. 1—Mathematical model of n-storey frame

$$O_i^1 = \mu_{A_i}(x) \quad I=1,2 \qquad \ldots(1)$$

$$O_i^1 = \mu_{B_{i-2}}(y) \quad I=3,4 \qquad \ldots(2)$$

where $\mu_{A_i}(x)$, $\mu_{B_{i-2}}(y)$ can adopt any fuzzy membership function. For instance, if Gaussian function is employed, $\mu_{A_i}(x)$ is given by

$$\mu_{A_i}(x) = \exp\left[-\left(\frac{x - c_i}{a_i}\right)^2\right] \qquad \ldots(3)$$

where ai and ci are variables of membership function. As values of these variables change, Gaussian function varies accordingly, thus exhibiting various forms of membership functions on linguistic label A_i. Variables in this layer are referred to as premise variables.

In second layer, nodes are fixed and labeled with Z, which multiplies incoming signals and sends product out. Outputs of this layer can be represented as

$$O_i^2 = w_i = \mu_{A_i}(x)\mu_{B_i}(y), \quad I=1,2 \qquad \ldots(4)$$

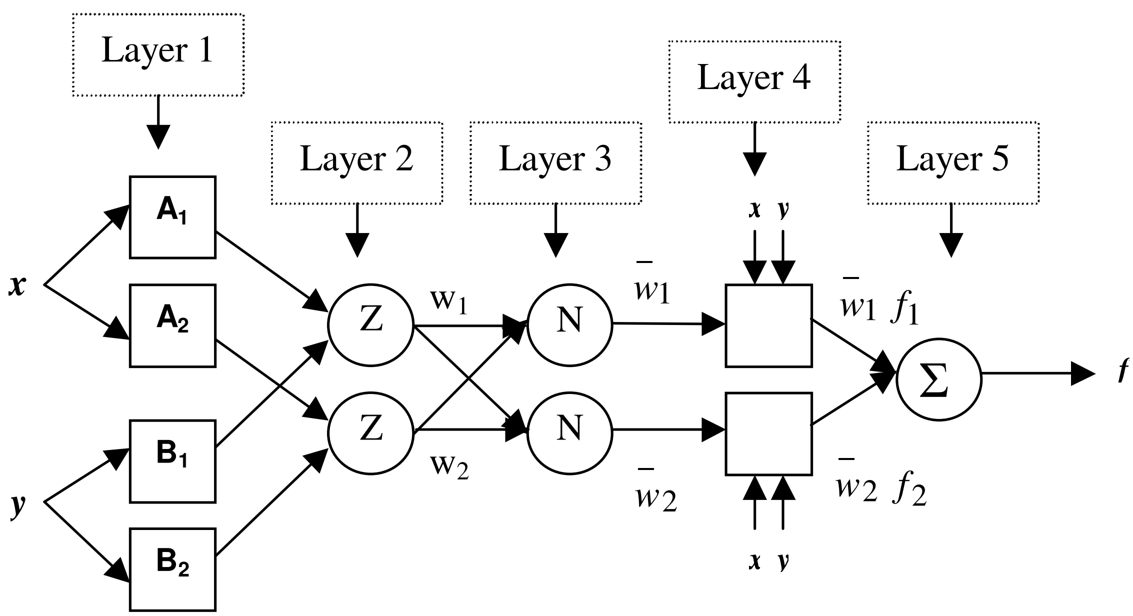

Fig. 2—ANFIS architecture

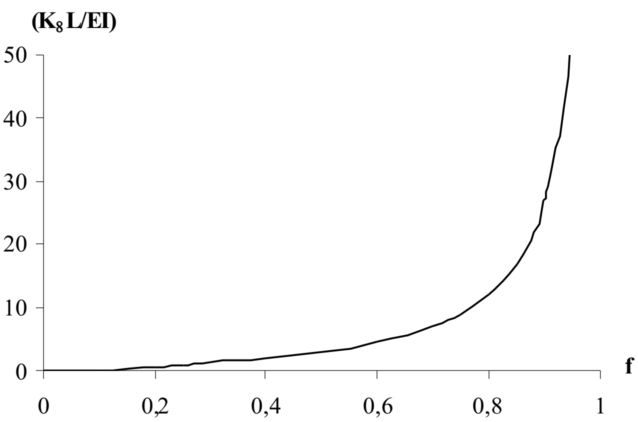

Fig. 3—Relationship between connection stiffness ($K_\theta L/EI$) and fixity factor (f)

which are firing strengths of a rule.

In third layer, nodes are also fixed and labeled with N, which play a normalization role to firing strengths from previous layer. Outputs of this layer can be represented as

$$O_i^3 = \bar{w}_i = \frac{w_i}{w_1 + w_2}, \quad I=1,2 \qquad \ldots(5)$$

which are so-called normalized firing strengths.

In fourth layer, nodes are adaptive. Output of each node in this layer is simply product of normalized firing strength and a first order polynomial (for a first order Sugeno model). Thus, outputs of this layer are given as

$$O_i^4 = \bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i) \qquad \ldots(6)$$

In fifth layer, there is only one single fixed node labeled with £, which performs summation of all incoming signals. Hence, overall output of the model is given as

$$O_i^5 = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \qquad \ldots(7)$$

In order to tune premise (a_i, c_i) and design variables (p_i, q_i, r_i), hybrid-learning algorithm[18-20], which combines gradient descent and least square methods and is faster than a back propagation algorithm, was proposed. Least squares method (forward pass) is used to optimize consequent variables with premise variables fixed. Once optimal consequent variables are found, backward pass starts immediately. Gradient descent method (backward pass) is used to adjust optimally premise variables corresponding to fuzzy sets in input domain. By this passing process, optimum variables are determined.

Numerical Analysis

Natural frequencies for first three modes of an elastically supported Timoshenko column with 1, 5 and 10 attached masses are computed by both TMM and Neuro-Fuzzy approaches for parameters f=0.1-0.25-0.5-0.75-0.99-0.999, $\overline{M}_i$=0.1-0.5-1-2.5-5-7.5-10, $\overline{J}_i$=0.1-0.5-1-5-10. Values mi=0.32 kNs2/m2, Li=1 m, EI=1353870 kNm2, AG=3240000 kN, k=2.426, Sx=0.00743 m3, A=0.04 m2 and I=0.006447 m4 are characteristics of IPB profile column used for numerical analysis. Relationship between connection stiffness (KθL/EI) and fixity factor (f) is approximately linear when f values are between 0.0 and 0.5 and nonlinear from 0.5 to 1 (Fig. 3). As f approaches unity, curve increases asymptotically to infinity since f as 1 is used for theoretically ideal fixed support.

NFFE Models

NFFE models were developed for estimating vibration frequencies of an elastically supported Timoshenko column with attached masses for different conditions. Models (NFFE1, NFFE2, NFFE3), (NFFE4, NFFE5, NFEE6) and (NFFE7, NFFE8, NFEE9) are first, second and third modes of 1, 5 and 10 attached masses system, respectively. Effective variables of vibration phenomenon are determined considering previous studies and models. Attached Mass (AM), Rotary Inertia (RI) and Fixity Factor (FF) are fuzzy logic vibration estimation model variables.Increases in spring coefficient values cause increase in frequencies. FF concept is used to formulate elastic support behavior. Theoretically, zero for FF value denotes a pinned support whereas infinity denotes a fixed support. Therefore, variation of FF is considered one of the effective variables on frequency values. Number and value of AMs are directly related to frequency values of the column. Increasing number and value of AMs decreases frequency values. An increase in RI value of AM causes, also, a decrease in frequency values. Nondimensional parameters for AM and its RI are selected as effective variables on vibration frequency. Membership functions of variable are determined using data obtained by ANFIS, which has three input variables and one output variable.

NFFE model (Fig. 4) is developed using fuzzy logic toolbox of software Matlab 7.0. As a process used by ANFIS systems, initial values of antecedents' variables

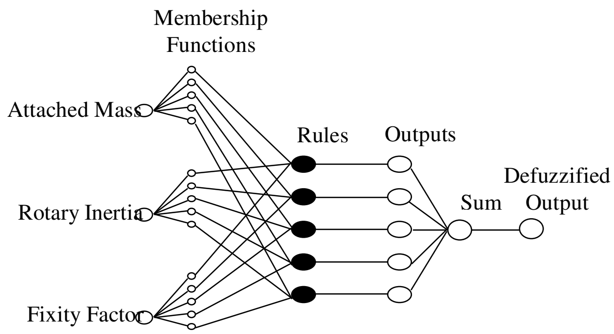

Fig. 4—NFFE model structure

can be defined in a way that centers of membership functions are equally spaced along the range of each input variable. Then, variables of fuzzy rules are optimized to get final membership range. Gaussian membership functions are used in definition of NFFE model variables. Tuned membership functions of input variables are shown in Fig. 5. Sugeno fuzzy inference system is used in NFFE model. In Sugeno fuzzy inference models, crisp output function (or value) is described using input fuzzy variables. The Coefficients and intercepts (design variables for Layer 4) of output membership functions are determined after training

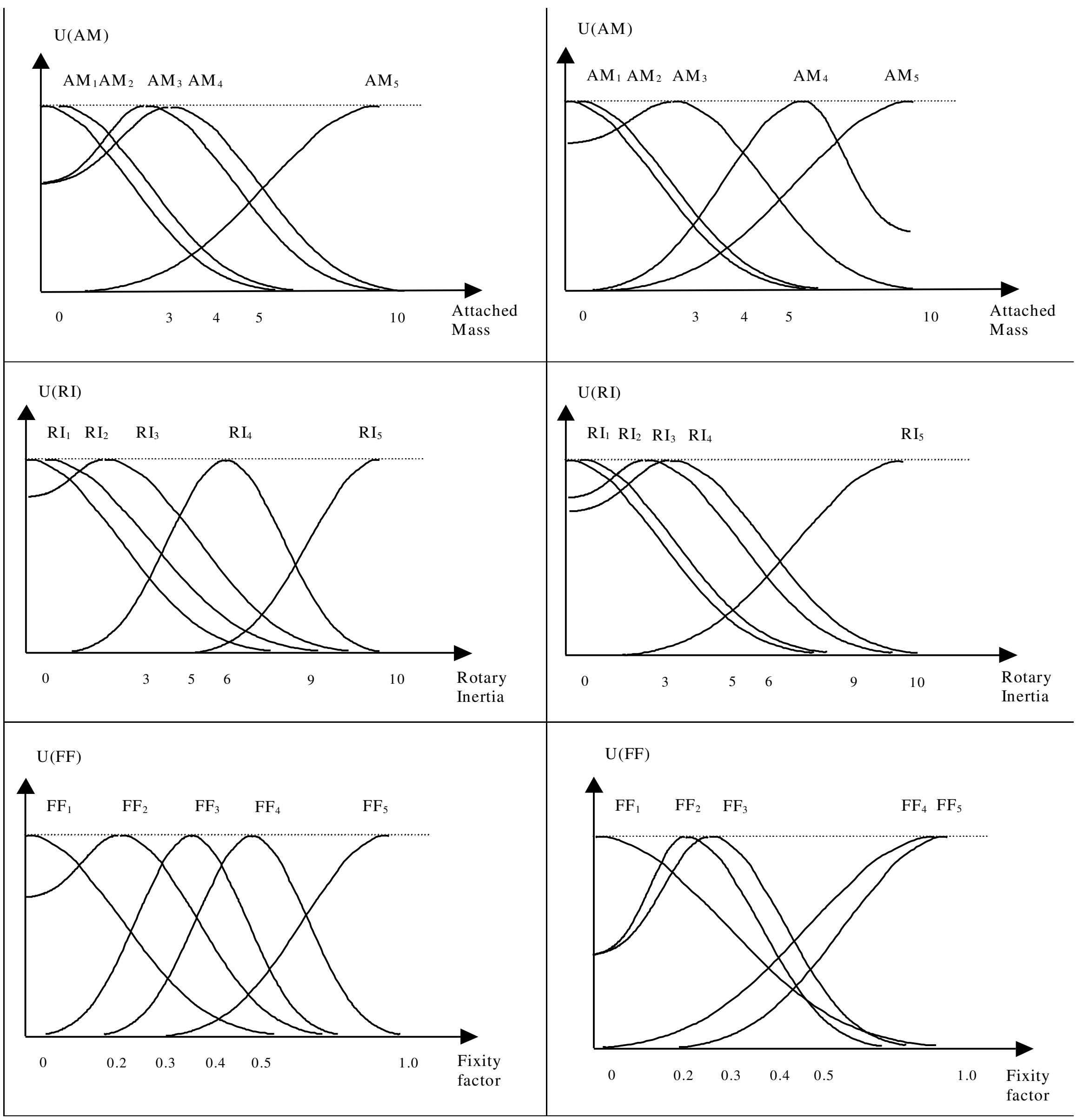

Fig. 5—Membership functions of NFFE1 and NFFE4 models

process in NFFE model (Table 1). General form of output function (linear) used in NFFE model is given as

$$fi = pi\ AM + qi\ RI + si\ FF + ri \quad \ldots(8)$$

Hybrid learning rule, which was applied for identifying output variables in neuro-fuzzy optimization process, combines steepest descent and least squares estimator for identifying variables of consequent part of inferential rules15,18. Variables of antecedent part of fuzzy inference rules are set up based on evaluation of characteristics of input data set. Rule base of the model is formed considering membership functions of variables. In NFFE models, rule base is ascertained by ANFIS (Table 2).

Table 1—Output membership functions coefficients and intercepts of NFFE4 model

Output Membership Function	p_i	q_i	s_i	r_i
f_1	-39.83	-5.969	114	113.3
f_2	-9.27	0.0688	191.1	45.27
f_3	-6.073	-0.750	46.87	81.04
f_4	-6.427	4.615	98.64	2.599
f_5	-2.276	0.149	67.62	37.14

Table 2—NFFE4 model rule bases

1 IF AM is AM_1 and RI is RI_1 and FF is FF_1 THEN O is f_1
2 IF AM is AM_2 and RI is RI_2 and FF is FF_2 THEN O is f_2
3 IF AM is AM_3 and RI is RI_3 and FF is FF_3 THEN O is f_3
4 IF AM is AM_4 and RI is RI_4 and FF is FF_4 THEN O is f_4
5 IF AM is AM_5 and RI is RI_5 and FF is FF_5 THEN O is f_5

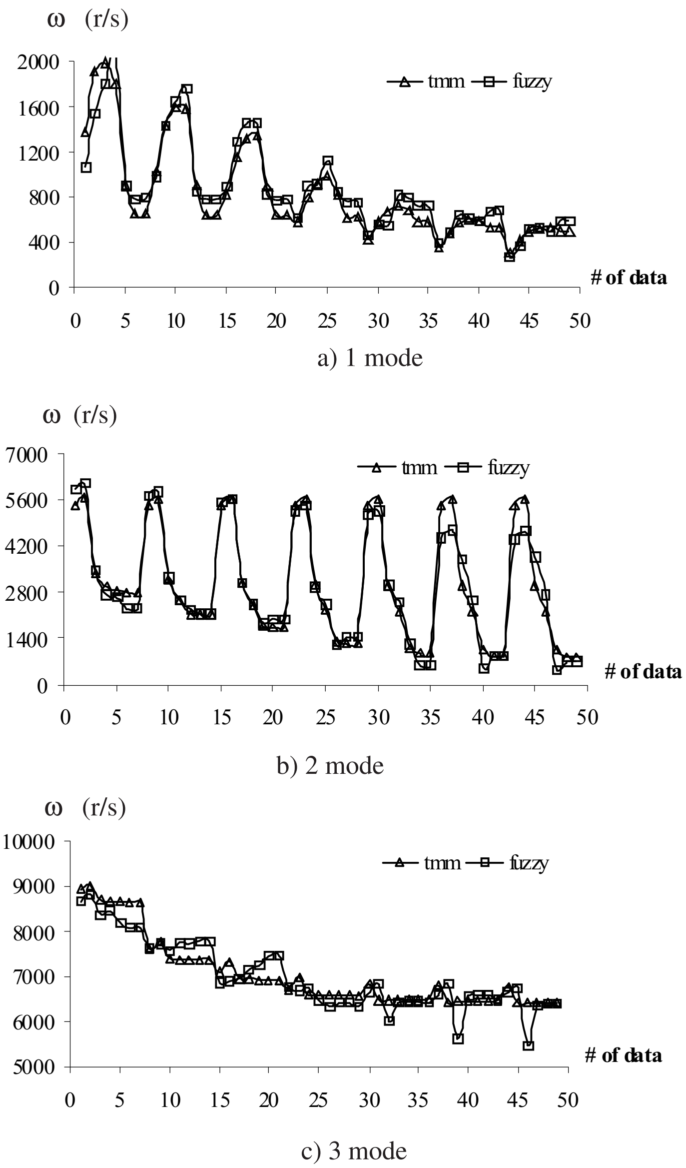

Fig. 6—Comparing frequency values of model with 1 attached mass obtained from TMM and fuzzy

Table 3—Error rates

Models	Train errors			Test errors		
	MAE	MSE	ARE, %	MAE	MSE	ARE, %
NFFE1	35.73	3610.57	4.22	99.64	16335.4	12.55
NFFE2	238.57	115495.92	11.07	300.85	171364.31	12.39
NFFE3	256.26	101422.78	3.57	351.45	180805.52	5.0
NFFE4	5.61	62.34	6.54	6.18	79.66	6.65
NFFE5	9.08	181.48	1.7	18.5	1032.96	3.54
NFFE6	34.71	3283.39	2.95	43.68	5891.18	3.58
NFFE7	0.73	0.996	3.44	1.18	3.59	4.29
NFFE8	3.53	24.49	2.09	6.003	92.24	3.58
NFFE9	7.82	128.37	1.81	17.27	539.95	4.25

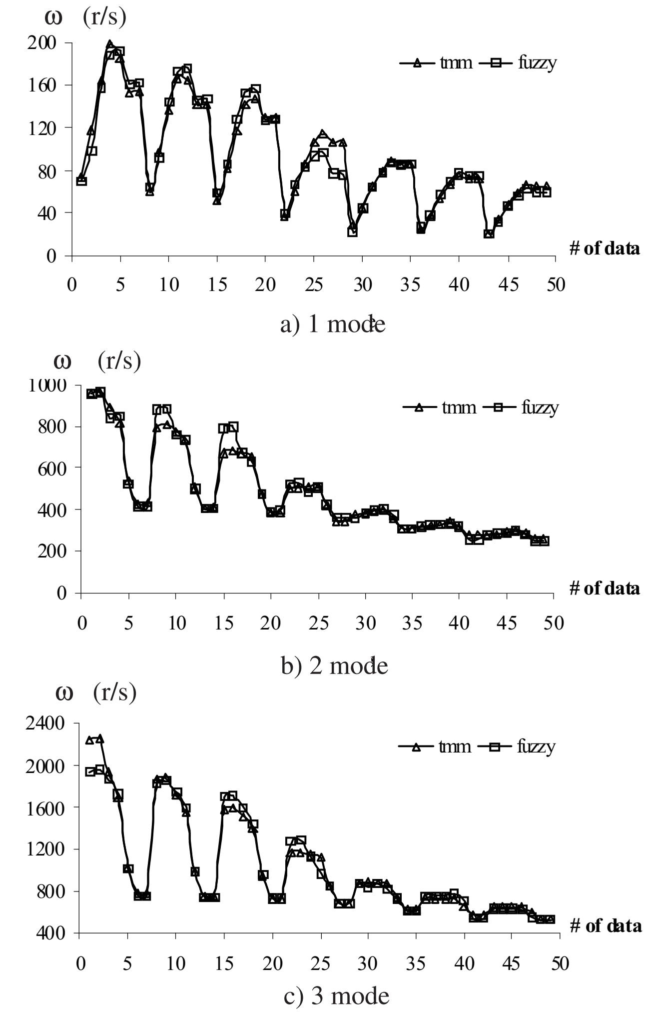

Fig. 7—Comparing frequency values of model with 5 attached masses obtained from TMM and fuzzy

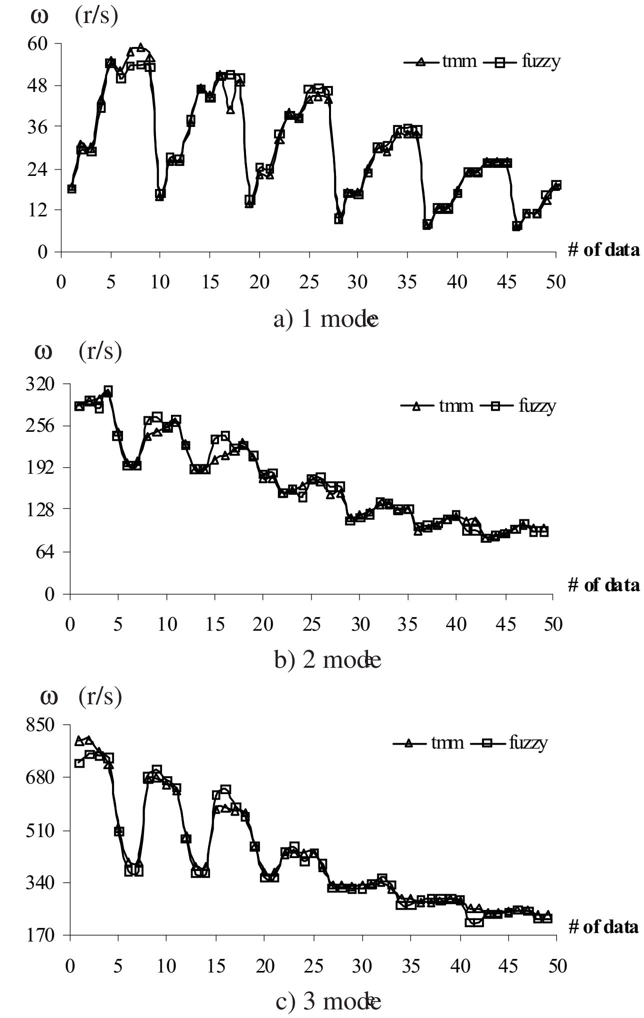

Fig. 8—Comparing frequency values of model with 10 attached masses obtained from TMM and fuzzy

Weighted average method is used for defuzzification in NFFE models. Mean absolute error (MAE), mean squared error (MSE) and average relative error (ARE) rates of nine neuro-fuzzy models are presented in Table 3. Frequency values of elastically supported Timoshenko column with 1, 5 and 10 attached masses are computed by TMM and estimated by NFFE model. Comparison graphs of frequency values obtained for models with 1, 5 and 10 attached masses are presented for first (Fig. 6), second (Fig. 7) and third (Fig. 8) modes. The data in X-axis of graphs is the number of natural frequency values used for testing phase of NFFE models.

Conclusions

Elastically supported Timoshenko column with attached masses was analyzed to obtain free vibration natural frequencies using transfer matrix method and fuzzy neural approach. NFFE model reduces computational effort and time to obtain free vibration frequencies. NFFE model computed values are generally close to the values obtained from TMM. Thus, ANFIS can be applied for vibration frequency estimation. MAE value is decreasing as number of attached mass is increasing. It means that NFFE model gives better results for the model with 5 attached masses than with 1 and for the model with 10 attached masses than with 5.

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