Depth estimation of an underwater target by the method of time reversal mirror

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Time reversal mirror (TRM) refocuses the received signal back to the original source location regardless of the complexity of the medium of propagation. When the medium contains several reflectors, the time reversal process can be used to focus on the desired target. Two arbitrary scatterers are positioned in the oceanic waveguide at different ranges and depths. Assuming that range of the focused scatterer is known approximately, the converged transmission vector through TRM may be matched against the computed eigenvector corresponding to the largest eigenvalue of the modeled channel transfer matrix between the transmitter and a test scatterer (at test depth) in order to determine the depth of the focused scatterer. Note that this approach of matching with the focused transmission vector leads to an improved depth estimation performance due to better SNR gain over a conventional matched field processor without TRM.

Keywords: Time Reversal Mirror (TRM), Matched Field Processing (MFP)

Introduction

Depth of a target in a shallow water waveguide is an important location parameter that aids in classifying it as an underwater vessel or a surface vessel. In this work, it is proposed to determine the depth of the target using iterative TRM based target focusing. The method proposed here is similar to the Matched Field Processing (MFP). However, unlike the MFP where the matching is done between the received pressure field vector and the modeled channel transfer vector, the present method does a matching between the focused transmission vector obtained from iterative TRM method and the dominant singular vector of the modeled channel transfer matrix.

When the medium contains several reflectors, the time reversal process can be iterated in order to focus on the most reflective one as demonstrated in ultrasonic laboratory acoustic experiments. The theory of iterative time reversal mirror has been presented by Prada et al. They further extended the iterative time-reversal process to focus on a specific target by a decomposition of time-reversal operator called DORT method. Iterative time reversal in the ocean has been demonstrated in an experiment conducted in the Mediterranean Sea. The focusing quality improves in a situation where the wave traverses a random medium or in cases where the wave propagates via multiple paths in a medium with reflecting boundaries such as in a waveguide. Multiple scattering or multiple reflections in the medium are exploited in TRM by redirecting/refocusing a part of the transmitted wave towards the receiver array which would have normally missed the transducer array in other approaches. TRM appears to have an aperture which is much larger than its actual physical size. Based on the time invariance and the reciprocity principle, focusing of a pulsed source in an acoustic waveguide is achieved. The focusing capability suggests potential applications of TRM to sonar system concepts such as reverberation mitigation and target detection without a priori environmental knowledge. Here we propose a method for estimating the depth of an underwater acoustic target through selective focusing using TRM in an acoustic waveguide.

Section 2 describes the construction of the field matrix for an oceanic waveguide based on normal mode theory. Section 3 describes the concept of iterative TRM technique used to focus on the desired target. The last section outlines numerical simulations carried out to establish this concept and introduces the method of depth estimation based on TRM.
Acoustic waveguide field transfer matrix

We consider a range-independent, horizontally stratified water layer of constant depth $h$ meter overlying a horizontally stratified bottom. There are $M$ narrowband sources of center frequency $\Omega$ located at cylindrical range $r_{sm} = (x_{sm}^2 + y_{sm}^2)^{\frac{1}{2}}$, depth $z_{sm}$ and azimuth $\phi_{sm}$; $m=1, \ldots, M$. There are $L$ hydrophones located at range $r_{sl}$, depth $z_{sl}$ and azimuth $\phi_{sl}$; $l=1, \ldots, L$. The sound speed variation along depth in the water layer $(z<h)$ is $c(z)$ and $c_s(z)$ is the sound speed variation in the bottom layer $(z>h)$. Fig. 1 shows the source-receiver geometry in the oceanic waveguide considered here. The pressure field $p_{lm}$ is computed using the normal mode theory of wave propagation in an oceanic waveguide.

$$ B_s = (\sqrt{2\pi}) \exp(\frac{i\pi}{4}) / \rho(z_{sl}) $$

and the number of discrete propagation modes is finite.

The mode functions and the eigenvalues are computed using the “KRAKEN” program. Given the sound speed profile (SSP), KRAKEN computes the mode-functions and the eigenvalues associated with the range-invariant ocean. Then, according to the normal mode solution, the resulting pressure signal amplitude can be expressed as a weighted sum of several mode functions. Thus once the mode functions and the mode values are computed for a particular medium, it is straightforward to compute the field at any point due to any point source in that medium.

Iterative Time Reversal Mirror

An array of $M$ elements (array-1) insonifying a scattering medium and an array of $L$ receivers (array-2) are considered. The system is assumed to be a linear and time invariant multiple-input-multiple-output (MIMO) system consisting of $M$ inputs and $L$ outputs. Thus it is characterized by an $M\times L$ matrix of inter-element impulse response functions. Let $k_{lm}(t)$ be the signal output of the receiver $l$ when a unit impulse $\delta(t)$ is transmitted by source $m$. These $M\times L$ functions provide a complete description of the transmit–receiver process. Indeed, if $e_m(t), 1 < m < M$ are the transmitted signals, then the received signals are given by the equation.

$$ r_l(t) = \sum_{m=1}^{N} k_{lm}(t) \otimes e_m(t) $$

In the frequency domain it is given by $\gamma$,$$
R(\omega) = K(\omega)E(\omega)
$$

where, $E(\omega) \in \mathbb{C}^{M\times L}$ is the Fourier Transform of $e(t) = [e_1(t), \ldots, e_M(t)]^T \in \mathbb{R}^{M\times L}$, $R(\omega) \in \mathbb{C}^{L\times L}$ is the Fourier Transform of $r(t)=[r_1(t), \ldots, r_L(t)]^T \in \mathbb{R}^{L\times L}$, and $K(\omega) \in \mathbb{C}^{L \times M}$ is the field transfer matrix from array-1 ($M$ elements) to array-2 ($L$ receivers) whose $(lm)^{th}$ element $k_{lm}(\omega)$ is the Fourier Transform of $k_{lm}(t)$. The element $K_{lm}(\omega)$ of the field transfer matrix is the Green’s function of the channel at frequency $\omega$ for the $l^{th}$ receiver (of array-2) and the $m^{th}$ source element (of array-1). By virtue of spatial reciprocity, we have, $K_{lm}(\omega) = K_{ml}(\omega)^H$. A time reversal operation is equivalent to phase conjugation in the frequency domain.

![Fig. 1–Source-receiver geometry in oceanic waveguide: $m^{th}$ source and $l^{th}$ receiver are shown.](image)
Thus in an iterated time reversal process, if $E_1(\omega)$ is the first transmitted signal (applied to array-1) of the time reversal process and $R_{12}(\omega)\equiv K(\omega)$ $E_1(\omega)$ is the received signal at array-2, then the second transmitted signal (applied to array-2) $E_2(\omega)$ is the phase conjugate of the received signal $R_{12}(\omega)$, i.e., $E_2(\omega) = R_{12}(\omega)^* K(\omega) E_1(\omega)$.

After the transmission of $E_2(\omega)$ from array-2, the signal received by array-1 is

$$R_{21}(\omega) = \mathbf{K}(\omega) K^*(\omega) E_1(\omega)$$

where, $\mathbf{K}(\omega) \in \mathbb{C}^{M \times M}$ represents the matrix transpose of $K(\omega) \in \mathbb{C}^{M \times M}$, which due to reciprocity is the field transfer matrix from array-2 to array-1. Note that array-2 is the one acting as a TRO. The received signal $R_{21}(\omega)$ is linked to the first transmitted signal $E_1(\omega)$ through a phase conjugation and a product with the matrix $\mathbf{K}(\omega) K^*(\omega) \in \mathbb{C}^{M \times M}$. The matrix $\mathbf{K}(\omega) K^*(\omega)$ allows any time reversal process to be described and is therefore called the time reversal operator (TRO). An important property of the time reversal operator is that it is Hermitian, i.e., $[\mathbf{K}(\omega) K^*(\omega)]^H = \mathbf{K}(\omega) K^*(\omega)$. Therefore all the eigenvalues $\lambda(\omega)$ of the TRO $\mathbf{K}(\omega) K^*(\omega)$ are real and non-negative, i.e., $\lambda(\omega) = \bar{\lambda}^*(\omega)$. Let the first transmitted signal be $V_1(\omega) \in \mathbb{C}^{M \times 1}$, an eigenvector of $\mathbf{K}(\omega) K^*(\omega)$ associated with an eigenvalue, say, $\lambda_1(\omega)$. Then after a time reversal process the received signal is $\lambda_1(\omega) V_1^*(\omega)$, which is proportional to the conjugate of $V_1(\omega)$. Consequently one can say that the eigenvectors of $\mathbf{K}(\omega) K^*(\omega)$ corresponds to waveforms that are invariants of the time reversal process. The Singular Value Decomposition (SVD) of the complex matrix $K(\omega) \in \mathbb{C}^{M \times M}$ is given by

$$K(\omega) = U(\omega) \Sigma(\omega) V^H(\omega)$$

where $\Sigma(\omega) \in \mathbb{C}^{M \times M}$ is a real diagonal matrix of non-negative numbers, $U(\omega) \in \mathbb{C}^{M \times M}$ and $V(\omega) \in \mathbb{C}^{M \times M}$ are unitary matrices whose columns correspond to the left and the right singular vectors of $K(\omega)$ respectively. The SVD of $K(\omega)$ can now be interpreted. It follows from (6) that

$$K^H(\omega) K(\omega) = V(\omega) \Sigma^H(\omega) \Sigma(\omega) V^H(\omega)$$

where $\Lambda(\omega) = \Sigma^H(\omega) \Sigma(\omega)$ is a diagonal matrix with real-valued entries, and hence on post-multiplying both sides with $V(\omega)$ we have

$$[K^H(\omega) K(\omega)] V(\omega) = V(\omega) \Lambda(\omega)$$

(7).

Therefore the eigenvalues of $K^H(\omega) K(\omega)$ are the square of the singular values of $K(\omega)$ and its eigenvectors are the columns of $V(\omega)$. We may conclude that the right singular vectors of $K(\omega)$ (found through SVD) corresponds to waveforms that are the invariants of a time reversal process. Moreover, had we transmitted the unit norm singular vector $V_1(\omega) \in \mathbb{C}^{M \times 1}$ corresponding to the largest singular value $\lambda_1(\omega)$ of $K(\omega)$ we would get back $\lambda_1(\omega) V_1^*(\omega)$ whose norm is given by $\lambda_1(\omega)$. Therefore the right singular vector of $K(\omega)$ corresponding to its largest singular value is the forward transmission that will result in maximum return response. Now suppose we started with an arbitrary transmission vector $E_1(\omega)$ (of unit energy) at array-1 so that after its reception, and time reversed transmission at array-2 (TRM), we get a return response vector $R_1(\omega) = \mathbf{K}(\omega) K^*(\omega) E_1^*(\omega)$ at array-1. We then transmit $E_2(\omega) = R_1^*(\omega) = [K^H(\omega) K(\omega)] E_1^*(\omega)$, the conjugate of return response, from array-1 and continue the process to result in $R_2(\omega) = \mathbf{K}(\omega) K^*(\omega) E_2^*(\omega) = \mathbf{K}(\omega) K^*(\omega) R_1(\omega)$. After $N$-iterations the return response is given by

$$R_N(\omega) = [K(\omega) K^*(\omega)]^N E_1^*(\omega)$$

and its conjugate

$$R_N^*(\omega) = [K^H(\omega) K^*(\omega)] R_{N-1}^*(\omega)$$

For $N$ sufficiently large, we have

$$[V(\omega) \Lambda^N(\omega) V^H(\omega)] E_1^*(\omega) \cong \lambda_1^N(\omega) V_1^*(\omega) V_1^H(\omega) E_1^*(\omega)$$

(8)

where $\lambda_1(\omega)$ is the square of the largest singular value of $K(\omega)$ and $\cong$ means the two vectors on its either side are close-by in the sense of $L_2$ norm. Therefore we find that

$$R_N^*(\omega) = [K^H(\omega) K(\omega)] R_{N-1}^*(\omega)$$
\[ \lambda_1(\omega) \lambda_1^{N-1}(\omega) V_1(\omega) V_1^H(\omega) E_1(\omega) \]
\[ \cong \lambda_1(\omega) R_N^{*}(\omega) \]  
\hfill (9).

From the last equation we may interpret that for sufficiently large \( N \) (iterations), the conjugate of the return response vector \( R_N(\omega) \) converges to the eigenvector of \( K_H(\omega) K(\omega) \) corresponding to largest singular value. In other words through an iterative TR process described above we can find the optimum transmission vector between two transmit-receive arrays that results in maximum power transfer (i.e. minimum transmission loss) as well as least waveform distortion due to channel without the explicit knowledge of the field transfer matrix \( K(\omega) \). The maximum power transfer is due to spatial focusing while the least waveform distortion is due to temporal focusing of the TRM.

**Depth estimation of target in a waveguide by time reversal mirror**

The objective of this simulation is to selectively focus on the desired target and estimate its depth upon focusing. The simulation environment is a Pekeris waveguide channel shown in Fig. 2. The water column is iso-velocity with a bottom sound speed of 1600 m/sec, density of 1500 kg/m\(^3\) and an attenuation of 0.2 dB/\( \lambda \).

Two targets, target A with a range 5800 m at a depth of 80 m and target B with a range of 6200 m at a depth of 20 m, are placed in the water column. An array with 17 elements at 5 m interval along the water column acts as the trans-receiver. A pulse with center frequency of 300 Hz and a pulse width of 23.33 ms is transmitted from the array Fig. 3(a). The incident signal on the targets after traveling through the waveguide is shown in Fig. 3(b). The signal arrives later at target B due to an additional spatial separation of 400 m with respect to the transmitter array compared to target A. The incident signal on the target gets scattered and is received back at the sensor as shown in Fig. 4(a). The received signal at the trans-receiver is used to construct the next signal to be transmitted so as to focus on the desired target as described in section 3. Continuing this process, the transmission vector and the received vector at the transceiver array converges eventually to the eigenvector corresponding to the largest eigenvalue of the transmitter-scatterer-receiver transfer matrix. Fig 4(b) shows the return signal upon focusing on the weaker scatterer.

Assuming that the range of the focused scatterer is known approximately (through, say, range-tracking of the target designated on the range-bearing display of an active sonar), the converged transmission vector may be correlated with the computed right-singular vector corresponding to the largest singular value of the modeled transfer matrix between the transmitter and a scatterer (for every value of the depth parameter), in order to determine the depth of the focused scatterer. Note that this approach of correlating with the focused transmission vector gives a significant SNR gain over conventional matched field processing without TRM. In our simulations, the pressure field is computed for a known range based on Eq. 2. The pressure field is computed for all depths varying from \( 0 \leq z \leq h \), in steps of \( \Delta z \), where \( \Delta z \) is the incremental depth and \( h \) is the channel depth. The transfer function between the source and the target for all depth
points in \([0, h]\) is constructed based on the Eq. 5, i.e.,
\[
K(\omega z) = H_2(\omega z) C(\omega) H_1(\omega z), \quad 0 \leq z \leq h,
\]
(10)
\(C(\omega) \in \mathbb{C}^{D \times D}\), where \(D\) is the number of point like scatterers in the medium, is a complex scalar scattering function (unknown) for a point target. The propagation matrix from the transmit array toward scatterers is denoted by \(H_1(\omega) \in \mathbb{C}^{D \times M}\). The propagation matrix from scatterers toward the receive array is represented by \(H_2(\omega) \in \mathbb{C}^{M \times D}\).

From Eq. 10, one can see that the rank of the transfer matrix must be less than or equal to the number of scatterers. If multiple scattering is negligible, then \(C(\omega)\) is a diagonal matrix. Furthermore if the scatterers have distinct reflectivities, and are ideally resolved by both transmit and receive arrays, then there is a one to one correspondence between the scatterers and the singular vectors\(^1\). In other words, each vector corresponds to the response of one scatterer to the array. The eigenvalue associated with a scatterer is proportional to the reflectivity of the scatterer and its position relative to the array. The eigenvector \(V_i(\omega)\) corresponding to the eigenvalue \(\lambda_i(\omega)\) gives the phase information necessary to focus on the associated target.

Now, SVD of the \(K(\omega z)\) matrix yield singular vectors with their corresponding singular values (Eq. 6). This process is repeated for all along the depth in steps of \(\Delta z\). The modeled right singular vector \((V_M)\) corresponding to the largest singular value for each depth \((z \text{ in steps of } \Delta z)\) is correlated with the converged transmission vector \((V_E)\) obtained using the experimental iterated TRM technique. The inner product of the two vectors given by,
\[
D(z) = \left| \langle V_M^*(\omega), V_E(\omega) \rangle \right|,
\]
(11)
is maximum near the location of the target depth. The best matching depth of the focused scatterer is thus:
\[
z_0 = \arg \max_z D(z)
\]
(12).

Fig. 5 shows the correlation processor output against different values of depth. It is seen that the peak of the processor output occurs at the depth value corresponding to that of the target of interest. During the process of iterated TRM experiment, the transmission vector \(V_E\) progressively focuses higher acoustic power at the target of interest. The converged transmission vector, in the case of a plane wave field, would have focused as much as \(M^2\) times the power transmitted by the single projector of an \(M\)-element array. Since more energy is focused on the selected target after each iterative process, better estimates of target parameters can be obtained. In the case of matched field processing such focusing does not happen, leading to a comparatively lower signal to noise ratio at the receiver.

**Conclusion**

Depth estimation of an underwater target based on selective focusing technique (TRM) has been demonstrated. The method proposed here is similar to the Matched Field processing (MFP). However, unlike the MFP where the matching is
done between the received pressure field vector and the modeled channel transfer vector, the present method does a matching between the focused transmission vector obtained from iterative TRM method and the dominant singular vector of the modeled channel transfer matrix.

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