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Supersymmetric structure of fractional quantum Hall effect in graphene

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Graphene shows unusual low-energy electronic excitations described in terms of Dirac fermions. The fractional quantum Hall effect in a bilayer graphene in different Landau levels within the supersymmetric formalism has been studied. The quantized Hall conductivity is found to be

$\sigma_{xy} = \pm (n+1)\nu \frac{e^2}{h}$, where $n = 0, 1, 2, 3 \ldots$

Keywords: Supersymmetry, Fractional quantum Hall effect, Graphene, Landau levels

1 Introduction

Graphene is the recently[1,2] discovered two-dimensional allotropic from of carbon. Before graphene three-dimensional (diamond, graphite), one-dimensional (nanotubes) and zero-dimensional[3] (fullerenes) allotropes of carbon were known. It is a monolayer of carbon atoms packed into a dense honeycomb crystal structure. In graphene, carbon atoms are arranged in planar and hexagonal form. One of the most striking features of graphene is that electrons in graphene are obeying a linear dispersion relation (i.e. the electron energy is linearly proportional to the momentum, $E=\hbar k v_F$) and behave as massless relativistic particles, called Dirac fermions[3], where $\hbar=h/2\pi$, h is the Planck's constant and v_F is the Fermi velocity of electron in the graphene. In ordinary metals and semiconductors the electronic energy and momentum are related quadratically via effective mass m^*, $\left(E = \hbar^2 k^2 / 2m^*\right)$. Due to the linear dispersion relation, the effective mass in graphene is zero, which gives an unusual electrodynamics. In fact, graphene can be described mathematically by the two-dimensional Dirac equation, whose elementary excitations are electrons and holes (or particles and antiparticles). Recently, it is found that electrons in graphene move with the velocity of the order of $10^6 m s^{-1}$ (i.e. $v_F \sim 10^6 m s^{-1}$). Although this is a factor of 300 slower than the velocity of light in vacuum, it is still much faster than the velocity of electrons in an ordinary conductor ($10^{-3} m s^{-1}$). The electrons in most of the conductors can be described by non-relativistic quantum mechanics whereas the electrons in graphene are treated as relativistic particles.

For Dirac particles with mass m, there is a gap between the minimal electron energy, $E_0 = mc^2$, and the maximal positron energy, $-E_0$ (c is the velocity of light in vacuum). When the electron energy $E >> E_0$, the energy is linearly dependent on the wave vector. For massless Diarc fermions, the gap is zero and this linear dispersion law holds at any energy. In this case, there is an intimate relation between the spin and motion of the particles: spin can only be directed along the propagation (for particles) or only opposite to it (for antiparticles). It is shown that graphene has a minimum electrical conductivity of the order of the quantum unit e^2/h, even when the concentration of charge carriers is zero[4,5] . This is a peculiar property of graphene because in all other systems the conductivity is zero if no charge carriers are present.

Besides high mobility and minimum conductivity, graphene shows very interesting behaviour in the presence of a magnetic field. Graphene shows an anomalous quantum Hall effect with the sequence shifted by 1/2 with respect to the standard sequence. The basic fact about the quantum Hall effect (QHE) is

that the diagonal electric conductivity of a two-dimensional electron system in a strong magnetic field is vanishingly small $\sigma_{xx} \to 0$, while the non-diagonal conductivity is quantized in multiples[6] of e^2/h: $\sigma_{xy} = n\,e^2/h$, when n is an integer (the integral quantum Hall effect, IQHE) and when n is a fractional number (the fractional quantum Hall effect[7,8], FQHE) . We have discussed both the IQHE and FQHE briefly in (Ref. 9). In recent experiments[4,5], the integral quantum Hall effect is observed in graphene. But in this case, it is found[6,10] that $\sigma_{xy} = \pm 4\left(n+\frac{1}{2}\right)e^2/h$. That is why; it is characterized as half-integer quantum Hall effect. This anomalous QHE is the direct evidence for Dirac fermions in graphene. The quantum Hall effect in graphene is also discussed in (Refs 11 and 12). Although the FQHE has not yet been observed in graphene, it has been explored theoretically in a number of papers [13-19]. Possible fractional quantum Hall (FQH) states in graphene have been discussed there. Further, it is found that room temperature[10,20] QHE can be observed in graphene. It is also shown that superconductivity[21,22] can be induced in graphene.

The IQHE can be understood solely in terms of individual electrons in a magnetic field whereas the FQHE reflects new physics arising from the collective behaviour of all the electrons. The IQHE do not depend upon interactions between electrons whereas the FQHE depends upon the combined effects of the magnetic field and Coulomb interaction between electrons. The relativistic nature of the energy dispersion relation of electrons in graphene modifies the interelectron interactions. The problem of electron-electron interactions in the presence of a large magnetic field in a honeycomb lattice is a complex problem that deserves a rigorous study. In this paper, we have studied the fractional quantum Hall effect in a bilayer graphene in different Landau levels using the properties of supersymmetry (SUSY). We found the quantized Hall conductivity to be:

$$\sigma_{xy} = \pm\,(n+1)\,\nu\,\frac{e^2}{h}, \qquad \text{where } n = 0, 1, 2, 3 \ldots$$

2 Fractional Quantum Hall Effect

In the Hall Effect, one takes a system of electrons in the xy plane, subject to a magnetic field in the z-direction. A current density j_x is driven in the x-direction and the voltage V_y is measured in the y-direction. If E_y is the Hall field in the y-direction, the force due to this field on a carrier of charge e is eE_y. The average Lorentz force on a carrier is ev_xB, where v_x is the drift velocity in x-direction. At equilibrium, the force due to Hall field balances the Lorentz force and the Hall conductance[8] is:

$$\sigma_{xy} = \frac{j_x}{E_y} = \frac{n_e\,e\,v_x}{v_xB} = \frac{n_e\,e}{B}, \qquad \ldots (1)$$

where n_e is the electron density. From Eq. (1), it is clear that the Hall conductance varies linearly with $n_e e/B$. This is the classical Hall effect. But it is found that at low temperatures of only a few kelvin and high magnetic field (up to 30 T), the Hall conductance did not vary linearly with $n_e e/B$. Instead, it varied in a stepwise fashion:

$$\sigma_{xy} = \frac{p}{2ps+1}\left(\frac{e^2}{h}\right) \equiv \nu\left(\frac{e^2}{h}\right) \qquad \ldots (2)$$

where

$$p = 1, 2, ... \qquad s = 0, 1, 2..... \qquad \ldots (3)$$

where ν is the filling factor. When s = 0, $\nu = p$ corresponds to the integral quantum Hall effect discovered by von Klitzing *et al*[23]. When $s > 0$ corresponds to the FQHE discovered by Tsui *et al*[24] . The case $p=1$, $\nu = 1/(2s+1)$ was explained by Laughlin[25]. The general case $s \geq 0$, $p \geq 1$, $\nu = \frac{p}{2ps+1}$ was explained by Jain using the idea of composite fermions[26] (CF). $\nu = \frac{p}{2ps-1}$ is a trivial case, where the effective field felt by composite fermions is antiparallel to the real external field *B*. Thus, in general, we can write the expression for the filling factor as:

$$\nu = \frac{p}{2ps \pm 1} \qquad \ldots (4)$$

From Eqs (1) and (2), we can define:

$$\nu = \frac{n_e h}{eB} = \frac{n_e}{B/(h/e)} = \frac{n_e}{n_B} \qquad \ldots (5)$$

Thus, the filling factor is defined as the ratio between the electron density (n_e) to the flux[18] density $\left(n_B \right)$. It shows how many energy levels are filled up.

3 Energy Spectrum of Graphene in Magnetic Field

Graphene has two atoms per unit cell, which results in two conical points K and K' $(\tau = \pm)$ per brillouin zone. The low-energy band structure of graphene is described by cones located at these two points. These points[27,28] can be defined:

$$\pm K = (\pm 4\pi/\sqrt{3a}\,, 0) \qquad \ldots (6)$$

where a is the lattice constant, + sign is for the point K and – sign is for the point K'. Near the points K and K', the electrons have a linear Dirac-Weyl (relativistic) type dispersion relation[29,30]. The point K can be transformed into the point K' under the mirror reflection.

Let us apply the magnetic field to a graphene sheet taken in the xy plane, we assume that the magnetic field is homogeneous and given by:

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \ldots (7)$$

where $\vec{A}$ is the magnetic vector potential. In the presence of magnetic field, the Hamiltonian matrix[13] can be written as:

$$H = v_F \begin{pmatrix} 0 & P_x - iP_y \\ P_x + iP_y & 0 \end{pmatrix} \qquad \ldots (8)$$

where the covariant momentum is

$$P_i \equiv -i\hbar\partial i + eA_i \qquad \ldots (9)$$

In the zero field, the Hamiltonian[8] has a linear dispersion for both K and K' points.

Let z = x – iy and symmetric gauge $\vec{A} = \left(-B_y/2\,, B_x/2 \right)$. In magnetic field, electrons undergo cyclotron motion and fill Landau levels, successively. The Landau level (LL) raising and lowering operators can be written as:

$$a^\dagger = \frac{l_B (P_x - iP_y)}{\sqrt{2}\hbar} \qquad \ldots (10)$$

and

$$a = \frac{l_B (P_x + iP_y)}{\sqrt{2}\hbar}. \qquad \ldots (11)$$

where l_B is the magnetic length and the commutation relation $\left[a\,, a^\dagger \right] = 1$ gives $\left[P_x\,, P_y \right] = i\hbar^2/l_B^2$.

Now, the Hamiltonion becomes:

$$\mathrm{H} = \frac{\sqrt{2}}{il_B} \hbar v_F \begin{pmatrix} 0 & a \\ -a^\dagger & 0 \end{pmatrix}, \qquad \ldots (12)$$

and

$$H^2 = \frac{2\hbar^2 v_F^2}{l_B^2} \begin{pmatrix} a^\dagger a + 1 & 0 \\ 0 & a^\dagger a \end{pmatrix}. \qquad \ldots (13)$$

Eigen vector of H^2 can be represented by $\psi = \begin{pmatrix} \alpha\eta_{n-1,m_1} \\ \beta\eta_{n,m_2} \end{pmatrix}$, where $\eta_{n,m}$ are the standard LL eigen functions in graphene, n = 0, 1, 2 …… is the LL index and m is the angular momentum index. Each Landau level is fourfold degenerate. The corresponding wave functions for an electron can be written as:

$$\psi(n \neq 0, m) = \frac{1}{\sqrt{2}} \begin{pmatrix} -\mathrm{sgn}(n) i\eta_{|n|-1.m} \\ \eta_{|n|,m} \end{pmatrix} \qquad \ldots (14)$$

and

$$\psi(0, m) = \begin{pmatrix} 0 \\ \eta_{0,m} \end{pmatrix}. \qquad \ldots (15)$$

The corresponding Landau level energies:

$$E_n = \pm \sqrt{\frac{2\hbar v_F^2 eB|n|}{c}}\,, \qquad \ldots (16)$$

where the negative energy states corresponds to the Dirac hole states or the valence band states in the graphene[19].

4 Supersymmetry and FQHE in Graphene

Supersymmetry (SUSY) is a symmetry between fermions and bosons. It introduces a fermionic (bosonic) partner for every boson (fermion) differing by half a unit of spin quantum number. This partner is called superpartner (sparticle) of the original particle. A supersymmetric quantum mechanical system[31,32] is one in which there are operators that commute with the Hamiltonian:

$$[H^{\tau},\ Q_{\tau}] = 0 \qquad \ldots (17)$$

and satify the algebra

$$\frac{1}{2}\{Q_{\tau},Q_{\tau}\} = H^{\tau} \qquad \ldots (18)$$

where Q_{τ} $(\tau = \pm)$ is the supercharge[27,28] :

$$Q_{+}=\begin{pmatrix} 0 & A^{\dagger} \\ A & 0 \end{pmatrix},\ Q_{-}=\begin{pmatrix} 0 & A \\ A^{\dagger} & 0 \end{pmatrix} \qquad \ldots (19)$$

where A is an arbitrary operator where $A \neq A^{\dagger}$ and is given by $A = \hbar\omega_c a^j$ for j –layer graphene. Using Eq. (19), the Hamiltonian can be written as :

$$H^{+} = \begin{pmatrix} A^{\dagger}A & 0 \\ 0 & AA^{\dagger} \end{pmatrix} \equiv \begin{pmatrix} H^{+\uparrow} & 0 \\ 0 & H^{+\downarrow} \end{pmatrix} \qquad \ldots (20)$$

and

$$H^{-} = \begin{pmatrix} AA^{\dagger} & 0 \\ 0 & A^{\dagger}A \end{pmatrix} \equiv \begin{pmatrix} H^{-\uparrow} & 0 \\ 0 & H^{-\downarrow} \end{pmatrix}. \qquad \ldots (21)$$

These two Hamiltonians $H^{\tau\uparrow}$ and $H^{\tau\downarrow}$ are superpartners at the K point and the K' point separately.

In order to calculate the accurate value of the Hamiltonian let us consider the K point (where τ =+). For eigenvalue problems, we have to study the spin components separately (i.e. up-spin and down-spin):

$$H^{+\uparrow}\left|\psi_n^{+\uparrow}\right\rangle = E_n^{+\uparrow}\left|\psi_n^{+\uparrow}\right\rangle , \qquad \ldots (22)$$

$$H^{+\downarrow}\left|\psi_n^{+\downarrow}\right\rangle = E_n^{+\downarrow}\left|\psi_n^{+\downarrow}\right\rangle , \qquad \ldots (23)$$

where the energy spectrum $E_0^{+\uparrow} \le E_n^{+\uparrow} < E_{n+1}^{+\uparrow}$ and $E_0^{+\downarrow} \le E_n^{+\downarrow} < E_{n+1}^{+\downarrow}$. We assume $E_0^{+\uparrow}$=0. The existence of the zero energy state is an intriguing property of the SUSY theory, where the bosonic and fermionic zero point energies are canceled out[33] . This shows the supersymmetry is a good symmetry in graphene. It is shown that for nonzero-energy states at the point K, the up-spin behaves as the bosonic sector and the down-spin behaves as the fermionic sector, and they constitute supermultiplets. Similarly, at the K' point the down-spin behaves as the bosonic sector and the up-spin behaves as the fermionic sector.

Now, we consider the bilayer graphene. A bilayer system is made up of two coupled hexagonal lattices with the Bernel stacking[34,35] . In the absence of magnetic field, the low-energy spectrum of the bilayer graphene is known to be parabolic[34-36]:

$$E(k) \propto |k|^2 \qquad \ldots (24)$$

near the K and K' points. Using the theory of quantum physics, we can write the generalized Dirac Hamiltanion in the presence of magnetic field as:

$$H_D^{\pm} = diag\left(\sqrt{Q_{\pm}Q_{\pm}}, -\sqrt{Q_{\pm}Q_{\pm}}\right). \qquad \ldots (25)$$

The above Hamiltonian reproduces the energy spectrum without magnetic field when A=$\hbar\omega_c a^2$. For generalization of the model, we can write:

$$A = \hbar\omega_c a^{\dagger j\downarrow} a^{j\uparrow} , \qquad \ldots (26)$$

and

$$A^{\dagger} = \hbar\omega_c a^{\dagger j\uparrow} a^{j\downarrow} . \qquad \ldots (27)$$

Here $j\uparrow$ and $j\downarrow$ are the integers and $j\uparrow > j\downarrow \ge 0$. For bilayer graphene $j\uparrow$ = 2 and $j\downarrow$ = 0. For the K point , we can write the zero energy up-spin states as $A|n\rangle = 0$, where $|n\rangle$ is $|0\rangle$, $|1\rangle$ both are degenerate states. Similarly, zero-energy down-spin states can be determined by using the condition $A^{\dagger}|n\rangle = 0$. Since $j\downarrow$ = 0, there are no zero-energy states. Hence, we can write the degenerate states as 2-fold including the both up-spin and down-spin. Similar analysis can be done for the K' points.

The eigenvalue of the Hamiltanion ($H_D^{\pm}$) can be written as:

$$H_D^{\pm}|n\rangle = (\varepsilon_n^{\pm\uparrow}, \varepsilon_n^{\pm\downarrow}, -\varepsilon_n^{\pm\uparrow}, -\varepsilon_n^{\pm\downarrow})|n\rangle , \qquad \ldots (28)$$

where n is the Landau level index. For energy spectrum including spin operators, we can write:

$\varepsilon_n^{+\uparrow} = \varepsilon_n^{-\downarrow} = \hbar\omega_c\sqrt{n(n-1)}$ for bosonic sector

and

$\varepsilon_n^{+\downarrow} = \varepsilon_n^{-\uparrow} = \hbar\omega_c\sqrt{(n+2)(n+1)}$ for fermionic sector $\qquad \ldots (29)$

where n = 0, 1, 2…….. .

Now consider the Coulomb interactions. The zero energy state contains both electrons and holes. Due to the Coulomb attraction, electron-hole pairs make bound states and condense into the excitonic states which produce an excitonic gap. According to the SUSY spectrum, a single energy level contains up-spin and down-spin electrons belonging to different Landau levels. The form factor mixes these Landau levels[37,38] . Coulomb interactions are different for electrons in different Landau levels. The effective Coulomb potential depends on spins in each energy level. Due to the Coulomb Hamiltonian, the degeneracy is modified.

We can define $\nu-$FQHE state at *n*th Landau level as the state corresponding to the $\nu-$filling factor of *n*th Landau level, while all the lower energy Landau levels are completely occupied. Considering the contribution from both electrons and holes including the interactions among themselves at the K and K' point, we can write the quantized Hall conductivity as:

$$\sigma_{xy} = \pm\,(n+1)\,\nu\,\frac{e^2}{h},$$

where $n = 0, 1, 2 \ldots\ldots$ … (30)

Eq. 30 represents the quantized Hall conductivity of fractional quantum Hall effect for bilayer graphene in the presence of supersymmetric theory. We have used this formula in the next section for obtaining our results.

5 Results and Discussion

The quantized Hall conductivity of fractional quantum Hall effect in a bilayer grapheme has been calculated. We have used the standard value of $\frac{h}{e^2} = 25813\ \Omega$. Hence, $\frac{e^2}{h} = 3.874\times10^{-5}\ \Omega^{-1}$. The variation of σ_{xy} with respect to different values of filling factors for $n = 0$ and $n = 1$ Landau level is shown in Fig. 1. The unusual electronic dispersion of graphene is reflected in the variation of Hall conductance staircase. So long as electrons are confined to the $n = 0$ Landau level, they have no memory of the Dirac nature of the zero field dispersion, with some surprising consequences for the FQHE. Each electron captures an even number ($2p$) of quantized vertices to become a composite fermion. The dynamics of composite fermions are described by the reduced magnetic field $B^* = B - 2\,p\,\rho\,\varphi_0$, where ρ is the particle density in the $n = 0$ Landau level and $\varphi_0 = h/e$ is the flux quantum. The composite particles do not feel the external field B but instead an effective field B^*. Therefore, the FQHE of electrons can be considered as an IQHE of these composite particles.

From our result, it is shown that the fractional quantum Hall effect in the $n = 1$ graphene Landau level resembles with that of $n = 0$ graphene Landau level. But in this case, the value of Hall conductivity is higher (twice) than $n = 0$ graphene Landau level. Thus, the lowest Landau level is significantly narrower than other Landau levels.

Recently, it is discovered that graphene[4,5] has a minimum conductivity of the order of $\frac{e^2}{h}$ even when the concentration of charge carriers tends to zero. But our calculated values of conductivity for $n = 0$ and $\nu = \pm\frac{1}{3}., \pm\frac{2}{5}$, $\pm\frac{3}{5}$ and $\pm\frac{2}{3}$ is less than this minimum value. Similarly, for $n = 1$ and $\nu = \pm\frac{1}{3}$ and $\pm\frac{2}{5}$ conductivity is less than the minimum value. But for $n = 1$, and $\nu = \pm\frac{2}{3}$ and $\pm\frac{3}{5}$ our calculated value of conductivity is greater than the minimum value.

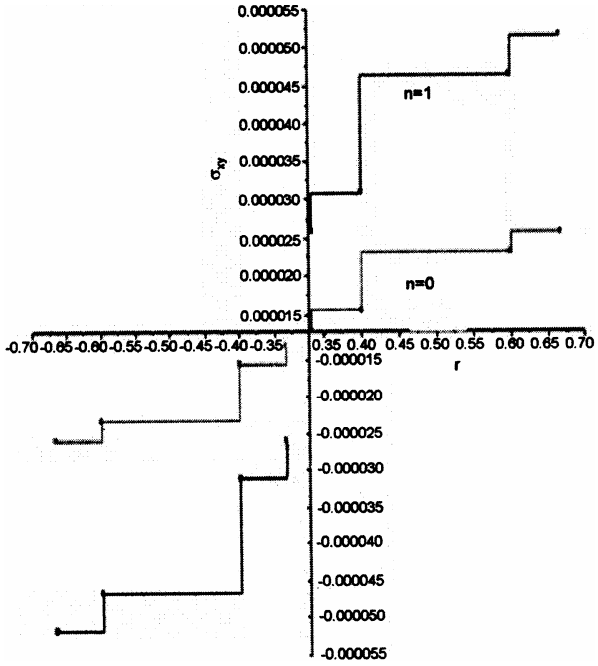

Fig. 1–Quantized Hall conductivity in graphene for different filling factor (ν) in $n = 0$ & $n = 1$ Landau levels

Our theoretical study makes new predictions that may be checked experimentally. This fascinating result is either due to the highly unusual nature of charge carriers in graphene or due to the presence of supermultiplets. These facts lead to enrichment in the phenomenology of FQHE in graphene. The study of FQHE in graphene is a very challenging field both theoretically and experimentally in condensed matter physics as well as quantum field theory.

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