Analytical simulation of mixed convection between two parallel plates in presence of time dependent magnetic field

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Heat transfer enhancement in various energy systems is vital because of the increase in energy prices. In recent years, nanofluids technology is proposed and studied by some researchers experimentally or numerically to control heat transfer in a process. In this paper, an unsteady incompressible three-dimensional mixed convection rotating flow of viscous fluid between two infinite vertical plane walls is investigated analytically using Galerkin optimal homotopy asymptotic method (GOHAM). We also consider the viscous dissipation effects. Such a consideration is significant, because the viscous dissipation effects (the generation of heat due to friction caused by shear in the flow) are important when the fluid is largely viscous or flowing at a high speed. The effects of the emerging parameters on the flow and heat transfer characteristics are studied and examined.

Keywords: Heat transfer enhancement, viscous dissipation effects, Mixed Convection, GOHAM

1 Introduction

As the need for modern technologies and multi-tasking systems rise, the production of high heat dissipating devices increases. In order to better remove this huge amount of heat in such devices, using effective cooling techniques seems inevitable. Some of these techniques include heat sinks, jet impingements, microdroplets, heat pipes and spray cooling. One of the most effective ways for achieving this amount of heat dissipation is by adding nanofluids to the base fluid.

Most industrial fluid processing includes non-Newtonian liquids like multi-grade oils, liquid detergents, paints, polymer solutions and polymer melts. In recent years the analysis of the effect of rotating concentric cylinders using non-Newtonian liquids is a popular area of research, not only due to its geophysical and technological importance but also in view of the interesting mathematical features presented by the equations governing the flow.1-3

Azimi and Riazi4 investigated the magnetohydrodynamic squeezing flow of graphene oxide water nanofluid between parallel disks using Galerkin optimal homotopy asymptotic method (GOHAM).

Three-dimensional heat and mass transfer in a rotating system using a nanofluid was investigated by Sheikholeslami and Ganji5. They concluded that the Nusselt number has a direct relationship with the Reynolds number while it has a reverse relationship with rotation parameter, magnetic parameter, Schmidt number, thermophoretic parameter and Brownian parameter.

Heat transfer enhancement in various energy systems is vital because of the increase in energy prices. In recent years, nanofluids technology is proposed and studied by some researchers experimentally or numerically to control heat transfer in a process. The nanofluid can be applied to engineering problems, such as heat exchangers, cooling of electronic equipment and chemical processes. Almost all of the researchers assumed that nanofluids treated as the common pure fluid and conventional equations of mass, momentum and energy are used and the only effect of nanofluid is its thermal conductivity and viscosity which are obtained from the theoretical models or experimental data.6

In the heart of all the different engineering sciences, everything shows itself in the mathematical relation that most of these problems and phenomena are modeled by ordinary or partial differential equations. In most cases, scientific problems are inherently of nonlinearity that does not admit analytical solution, so these equations should be solved using special techniques. Some of these methods are reconstruction of variational iteration
method (VIM), differential transformation method (DTM), homotopy perturbation method (HPM) and optimal homotopy asymptotic method (OHAM) and others. The aim of this paper is to derive simple accurate polynomial expressions of heat transfer of 3D mixed convection rotating flow of MHD fluid between two infinite vertical plane walls using Galerkin optimal homotopy asymptotic method (GOHAM).

2 Problem Formulation

Here, we consider an unsteady incompressible 3D mixed convection rotating flow of MHD fluid between two infinite vertical plane walls. The plane positioned at $y=0$ is stretched with a time-dependent velocity $u_y(t)$ in the $x$-direction. The plane is located at a variable distance $h = \frac{v(0)}{a}$, and it squeezes the fluid with a time dependent velocity $V_h = \frac{dh}{dt}$ in the negative $y$-direction. The fluid and the channel are rotating about the $y$-axis with an angular velocity $\Omega = \frac{\omega}{1-\eta}$, and the plate $y=0$ sucks the flow with a velocity $-V_0$. A magnetic field $B_0$ is applied along the $y$-axis. The viscous dissipation effects are also considered. The flow configuration and the coordinate system are shown in Fig. 1. The governing equations for the velocity and temperature fields are as following form:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + 2\omega \frac{\partial u}{\partial \eta} &= - \frac{1}{\rho_{nf}} \frac{\partial P}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y} &= - \frac{1}{\rho_{nf}} \frac{\partial P}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\
\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} &= - \frac{2\omega}{1-\eta} \frac{\partial \omega}{\partial \eta} + \frac{\sigma B_0^2}{\rho_{nf}(1-\eta)} \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \\
(\rho C_p) \frac{\partial T}{\partial t} + \nu \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) &= \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu_c \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial \omega}{\partial x} \right)^2 + \left( \frac{\partial \omega}{\partial y} \right)^2 \right)
\end{align*}
\]

where $u$, $v$ and $\omega$ are, respectively, the $x$-, $y$- and $z$-components of the velocity field. $\rho_{nf}$, $\nu_{nf}$, $p$, $T$, $g$, $\mu_{nf}$, $(c_p)_{nf}$, $k$, $B_0$ and $\gamma$ are, respectively, the nanofluid density, the kinematic viscosity, the pressure, the fluid temperature, the magnitude of acceleration due to gravity, the dynamic viscosity, the specific heat at constant pressure, the coefficient of thermal conductivity, the magnetic field and the characteristic parameter.

The boundary conditions are:

\[
\begin{align*}
u(x, y, t) &= u_0 = \frac{ax}{1-\eta}, \quad \nu(x, y, t) = -V_0 = \frac{V_0}{1-\eta}, \quad \omega(x, y, t) = 0, \quad T(x, 0, t) = T_0, \\
u(x, y, t) &= 0, \quad \nu(x, y, t) = V_h = \frac{V_0}{2} \sqrt{\frac{v(0)}{a(1-\eta)}}, \quad \omega(x, y, t) = 0 \\
u(x, h(t), t) &= T_0 + \frac{T_0}{1-\eta}
\end{align*}
\]

Here, $a$ is the stretching rate of the wall $y = 0$, and $V_0$ is the suction/injection velocity. Introducing following non-dimensional parameters:

Fig. 1—Geometry of problem

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\[
\eta = \frac{y}{h(t)}, \quad u = U_0 F(\eta), \quad v = -\frac{1}{\sqrt{1-\eta^2}} F(\eta), \quad a = U_0 G(\eta), \quad T = \frac{T_0}{1-\eta^2} \theta(\eta)
\]...

(7)

Substituting Eq. (7) into Eqs (1-5), we have:

\[
F^\prime + \left(\left(1-\phi^2\right) + \frac{P_r \phi}{\rho_f} \theta^2\right) - \frac{S_{o f}}{2} (\eta^2 + 3\eta^4) - 2\Omega G = 0
\]

(8)

\[
G^* + FG - FG = \frac{k_f}{k_f} \left(\frac{1-\phi^2}{\rho_f} \phi \right) \left( F\theta - S_{o f} \left( \theta + \frac{\eta}{2} \theta' \right) \right)
\]

(9)

\[
\frac{\theta^*}{\theta_f} + \frac{Pr \left(1-\phi^2\right)}{\rho_f} \phi \left( \frac{k_f}{k_f} \right) \left( G^2 + \frac{F^*}{2} \right) = \frac{Pr Ec}{1-\phi^2} \left( \frac{k_f}{k_f} \right) \left( G^2 + 4F^* \right) = 0
\]

(10)

With the boundary conditions:

\[
F(0) = S, \quad F(0) = 1, \quad F(1) = \frac{S_0}{2}, \quad F(1) = 0, \quad G(0) = 0, \quad G(1) = 0,
\]

(11)

Here,

\[
S = \frac{V_s}{2akh}, \quad Pr = \frac{\mu_f}{\rho_f}, \quad M = \sqrt{\frac{\mu_f}{\rho_f}}, \quad \Omega = \frac{\omega}{\Omega}, \quad Re = \frac{Gr}{Re}, \quad Gr = \frac{g H o^2}{\nu^2 (1-\eta^2)}, \quad \text{and} \ Re = \frac{U_0^2}{v}
\]

are, respectively, squeezing parameter, the suction/injection parameter, the Prandtl number, the magnetic parameter, the rotation parameter, the Richardson number, the modified Grashof number and the Reynolds number. It is worth mentioning that the plane at \( y = h(t) \) moves with the velocity \( V_s < 0 \) for \( S_q > 0 \) towards the plane at \( y = 0 \). For \( S_q < 0 \) the plane at \( y = h(t) \) moves apart with respect to the plane \( y = 0 \), and \( S_q = 0 \) corresponds to the steady case or the stationary plane. The skin friction coefficient and the Nusselt number are defined as:

\[
\tau_{y y} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad C_f = \frac{(\tau_{y y})_{y=0}}{\rho u^2}, \quad Nu = -\frac{1}{T_0} \sqrt{\frac{v}{\partial T}} \frac{\partial T}{\partial y}_{y=0}
\]

(12)

Inserting Eq. (7) into Eq. (12), we obtain:

\[
2Re^{0.5} C_f = F^*(1), \quad (1-\eta^2)^2 Nu = -\theta'(1)
\]

(13)

By introducing \( \phi \) as a solid volume fraction, fluid density, dynamic viscosity, the kinematic viscosity, thermal diffusivity and thermal conductivity of nanofluid can be written as follows:

\[
\rho_{nf} = \rho_f (1-\phi) + \rho_s \phi,
\]

(14)

\[
\mu_{nf} = \frac{\mu_f}{(1-\phi)^{1.5}},
\]

(15)

\[
\nu_{nf} = \frac{\nu_f}{\rho_f (1-\phi)^{1.5}},
\]

(16)

\[
\alpha_{nf} = \frac{k_f}{\rho_f C_p_f} (1-\phi)^{1.5},
\]

(17)

\[
\frac{k_{nf}}{k_f} \left[ (k_s + 2k_f) - 2\phi(k_s - k_f) \right] + \phi(k_s - k_f)
\]

(18)

3 GOHAM Solution

Following differential equation is considered:

\[
L(u(t)) + N(u(t)) + g(t) = 0, \quad B(u) = 0
\]

where \( L \) is a linear operator, \( \tau \) is an independent variable, \( u(t) \) is an unknown function, \( g(t) \) is a known function, \( N(u(t)) \) is a nonlinear operator and \( B \) is a boundary operator. By means of OHAM one first constructs a set of equations:

\[
(1 - p) [L(\phi(\tau, p) + g(\tau))] = H(p)
\]

(16)

where \( p \in [0, 1] \) is an embedding parameter, \( H(p) \) denotes a nonzero auxiliary function for \( p \neq 0 \) and \( H(0) = 0 \), \( \phi(\tau, p) \) is an unknown function. Obviously, when \( p = 0 \) and \( p = 1 \), it holds that:

\[
\phi(\tau, 0) = u_0(\tau), \quad \phi(\tau, 1) = u(\tau)
\]

(17)
Thus, as \( p \) increases from 0 to 1, the solution \( \phi(\tau, p) \) varies from \( u_0(\tau) \) to the solution \( u(\tau) \), where \( u_0(\tau) \) is obtained from Eq. (16) for \( p = 0 \):

\[
L(u_0(\tau)) + g(\tau) = 0, \quad B(u_0) = 0 \quad \text{ ... (18)}
\]

We choose the auxiliary function \( H(p) \) in the form:

\[
H(p) = p_1 C_1 + p_2 C_2 + \cdots \quad \text{ ... (19)}
\]

where \( C_1, C_2, \ldots \) are constants which can be determined later. Expanding \( \phi(\tau, p) \) in a series with respect to \( p \), one has:

\[
\sum_{i=0}^k u_k(\tau, C_i) p_1, \quad i = 1, 2, \ldots \quad \text{ ... (20)}
\]

Substituting Eq. (20) into Eq. (16), collecting the same powers of \( p \), and equating each coefficient of \( p \) to zero, we obtain set of differential equation with boundary conditions. Solving differential equations by boundary conditions \( u_0(\tau), u_1(\tau, C_1), u_2(\tau, C_2), \ldots \) are obtained. Generally speaking, the solution of Eq. (15) can be determined approximately in the form:

\[
\sum_{i=0}^k u_k(\tau, C_i) p_1, \quad i = 1, 2, \ldots \quad \text{ ... (21)}
\]

\[
\sum_{i=1}^m u_k(\tau, C_i) \quad \text{ ... (22)}
\]

Note that the last coefficient \( C_m \) can be function of \( \tau \). Substituting Eq. (21) into Eq. (15), there results the following residual:

\[
R(\tau, C_i) = L(\tilde{u}(\tau, C_i)) + g(\tau) + N(\tilde{u}(\tau, C_i)) \quad \text{ ... (23)}
\]

If \( R(\tau, C_i) = 0 \) then \( \tilde{u}(\tau, C_i) \) happens to be the exact solution. Generally such a case will not arise for nonlinear problems, but we can minimize the functional by Galerkin method:

\[
w_i = \frac{\partial R(\tau, C_1, C_2, \ldots, C_m)}{\partial C_i}, \quad i = 1, 2, \ldots, m \quad \text{ ... (24)}
\]

The unknown constants \( C_i(i = 1, 2, \ldots, m) \) can be identified from the conditions:

\[
J(C_1, C_2) = \int_a^b w_i R(\tau, C_1, C_2, \ldots, C_m) d\tau = 0 \quad \text{ ... (25)}
\]

where \( a \) and \( b \) are two values, depending on the given problem. With these constants, the approximate solution (of order \( m \)) (Eq. (25)) is well determined. It can be observed that the method proposed in this work generalizes these two methods using the special (more general) auxiliary function \( H(p) \).

### 4 Results and Discussion

The results for \( f' \) and \( f \) represent the radial and axial velocities, respectively. The physical properties of GO- water nanofluid are given in Table 1.

Figure 2 shows the effect of solid volume fraction on temperature profile. As it can be seen more solid volume fraction cause more heat transfer rate. For energy applications of nanofluids, two remarkable properties of nanofluids are utilized, one is the higher thermal conductivities of nanofluids, enhancing the heat transfer, and another is the absorption properties of nanofluids.

Figure 3 shows the effect of Eckert number on temperature profile. This figure demonstrates that changing the \( Ec \) number will have a moderate effect on temperature profile. Increasing the \( Ec \) number actually makes the thermal boundary layer to grow faster and so that it will be thicker by increasing the \( Ec \) number.

<table>
<thead>
<tr>
<th>Table 1—Thermo physical properties of water and GO nanoparticle</th>
<th>( \rho ) (kg/m³)</th>
<th>( C_p ) (j/kgk)</th>
<th>( k ) (W/m.k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure water</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
</tr>
<tr>
<td>Graphene Oxide</td>
<td>1800</td>
<td>717</td>
<td>5000</td>
</tr>
</tbody>
</table>

![Fig. 2—Effect of graphene oxide solid volume fraction on temperature profile when \( \Omega = 10, \ Re = Ec = M = 1, \ Pr = 7, \ S_g = 0.8, \ s = 1, g_f = 1\)]
Figure 4 shows the variation of dimensionless velocity profile denoted by \( f' \) with similarity parameter \( \eta \) for GO-water as a nanofluid and considering that the volume fraction, Reynolds number and Prandtl number equal to \( \phi = 0.15 \), \( Re = 0.4 \) and \( Pr = 7 \), respectively. The magnetic number is varied from 0 to 3 to see how the power of magnetic field will affect the velocity distribution and flow profile. As it can be seen the increase in \( M \) will decrease the thickness of the boundary layer near the stationary plate while the reverse effect has been illustrated near the rotating one. This is true because the applied magnetic field behaves like a suction boundary condition so that it will absorb the fluid toward the surface. It is also important to note that this decrease reaches its minimum when there is no magnetic field (i.e. \( M = 0 \)). In other words this shows that by increasing the magnetic the momentum of the flow will be more damped so we recognize a smaller value of dimensionless velocity magnitude in high \( M \) numbers at the same similarity parameter.

In Fig. 5, the effect of rotation parameter on velocity component \( F \) has been plotted against the dimensionless parameter. According to Fig. 5, with the increase in rotation parameter, the magnitude of velocity profile decreases for \( 0 \leq \eta < 0.6 \) and this parameter will not significantly change the velocity profile for \( 0.6 < \eta \leq 1 \). This is because the fact that, the permeable nature of the right disk allows the fluid particles to move closer to the boundary, which makes the boundary layer thinner, so the effect of rotation parameter is higher near right plate.

The effect of mixed convection parameter on temperature profile has been depicted in Fig. 6. The temperature decreases by increasing the mixed convection parameter. This is because of the reality
that the increase in the buoyancy force will cause an increment in the mixed convection parameter. It means that increase in the buoyancy force will accelerate the fluid through plates by which the temperature decreases.

The comparison of approximate results achieved from Galerkin Homotopy asymptotic method with numerical solutions obtained by fourth order Runge-Kutta have been presented in Fig. 7. The high agreement assures about the accuracy and validity of our results.

5 Conclusions

The unsteady mixed convection squeezing flow of an incompressible graphene oxide water nanofluid between two vertical parallel planes is discussed. The buoyancy force due to thermal and molecular diffusion is taken as the source of the convective flow. The non-similar nature of the transformed equations in terms of time helps to study the unsteady nature of the problem. Based upon the results presented here, the following conclusions can be drawn:

(i) When graphene oxide solid volume fraction increases, the rate of heat transfer increases.

(ii) Eckert number has significant effect on temperature profile and it can increase the rate of heat transfer by increasing.

(iii) The temperature field $T$ decreases by increasing the mixed convection parameter.

(iv) Runge Kutta method, revealed that GOHAM can be simple, powerful and efficient techniques for finding analytical solutions in science and engineering non-linear differential equations.

References